

TEACHER AND STUDENT RESOURCE

Linear and Quadratic Models Meeting in Context



NC Math 2 • NC.M2.A-REI.7

Reasoning with Equations and Inequalities

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-REI.7 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.

T1 Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

T2 Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

T3 Use tables and successive approximations to locate intersections, distinguish exact from rounded agreement and report contextual precision.

T4 Use graphs to interpret intersection counts, tangency, relative values and window limitations, connecting graphical and algebraic evidence.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M04, O04, M10, O10, M12, O12, M15, O15, M18, O18, M20, O20, M21, O21, M24, O24, M27, O27, M30, O30

T2: M02, O02, M06, O06, M09, O09, M14, O14, M17, O17, M29, O29

T3: M03, O03, M07, O07, M11, O11, M16, M23, O23, M28, O28

T4: M05, O05, M08, O08, M13, O13, O16, M19, O19, M22, O22, M25, O25, M26, O26

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Learning target: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Learning target: Use tables and successive approximations to locate intersections, distinguish exact from rounded agreement and report contextual precision.

Learning target: Use graphs to interpret intersection counts, tangency, relative values and window limitations, connecting graphical and algebraic evidence.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Learning target: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Learning target: Use tables and successive approximations to locate intersections, distinguish exact from rounded agreement and report contextual precision.

Learning target: Use graphs to interpret intersection counts, tangency, relative values and window limitations, connecting graphical and algebraic evidence.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Quadratic and linear functions, factoring and the quadratic formula, evaluating ordered pairs, tables, graph scales, nonnegative and integer domains, and unit conversion.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Contexts are invented instructional models. A system solution is an ordered pair satisfying both original equations at the same input; equal outputs at different inputs do not count. Apply the stated physical or discrete domain. A finite table or limited graph window may miss intersections. Mark approximations explicitly and keep exact forms until rounding is requested. A repeated algebraic root yields one distinct graph point.

TEACHER MODEL

Agreement is a shared ordered pair

A project's cost is $C=q^2+8$ dollars and its funding is $F=6q$ dollars for $q \geq 0$ kg. Use algebra and a short table to find all agreement points and interpret them.

Set $q^2+8=6q$, so $q^2-6q+8=(q-2)(q-4)=0$. At $q=2$ both amounts are 12 dollars; at $q=4$ both are 24 dollars. Thus the intersections are (2, 12) and (4, 24), meaning 2 kg at \$12 or 4 kg at \$24. A table at $q=1, 2, 3, 4, 5$ gives costs 9, 12, 17, 24, 33 and funding 6, 12, 18, 24, 30. The two matching columns confirm the pairs; factoring proves there are no additional real inputs. Between the crossings, funding exceeds cost.

VISUAL ANCHOR

Three representations of the same agreement

For models $y=f(x)$ and $y=g(x)$, a meeting is a shared pair (x, y) , not merely two equal numbers found somewhere in a table.

Representation	What to inspect	What it does not automatically prove
Table	Compare both outputs in one input column	A coarse or rounded table can miss a crossing
Graph	Locate where the original curves meet	One viewing window may hide other intersections
Algebra	Solve $f(x)-g(x)=0$, then recover y	An algebraic input may violate the context

Two distinct intersections, one tangent intersection and no real intersection are all possible for a genuine quadratic and a line. Check original equations and units, then interpret which pairs are allowed.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask what each coordinate measures. Require the same input in both models and check a proposed ordered pair before launching into a procedure.

Core task

A workshop has cost $C=q^2+3$ dollars and revenue $R=4q$ dollars for whole-number production counts $0 \leq q \leq 4$. Find every break-even ordered pair, check both models and interpret both coordinates.

Teacher solution and discussion

Set $q^2+3=4q$, giving $q^2-4q+3=(q-1)(q-3)=0$. Thus $q=1$ or 3 , both allowed. At 1 , $C=4=R$; at 3 , $C=12=R$. The break-even points are $(1, 4)$ and $(3, 12)$: one item with \$4 of cost and revenue, or three items with \$12 of each.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T4: Use graphs to interpret intersection counts, tangency, relative values and window limitations, connecting graphical and algebraic evidence.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Equate the original outputs, solve the resulting quadratic, recover a common output for every root and apply the domain. Compare the two original graphs rather than only the difference graph.

Core task

For a small production run, cost is $C=q^2$ dollars and funding is $F=q+2$ dollars, with continuous production amount $0 \leq q \leq 3$ kilograms. Graph both models, solve for their intersections algebraically and interpret all physically allowed agreement points.

Teacher solution and discussion

Set $q^2=q+2$, so $(q-2)(q+1)=0$ and $q=2$ or -1 . The context retains $q=2$ only, with $C=F=4$ dollars. The plotted curves on $0 \leq q \leq 3$ meet at $(2, 4)$. The negative root -1 gives the abstract pair $(-1, 1)$, but it is outside the allowed mass interval. Label horizontal mass in kg and vertical dollars.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Use a table that misses a noninteger crossing and a graph that touches at one repeated root. Separate evidence for approximate location from evidence that a solution list is complete.

Core task

A project's cost is $C=q^2+6$ dollars and its revenue is $R=4q$ dollars for $q \geq 0$. Decide whether any break-even point exists. Give an algebraic bound on the cost-revenue difference and interpret the smallest gap.

Teacher solution and discussion

$C-R=q^2-4q+6=(q-2)^2+2 \geq 2$. Equality in this bound occurs at $q=2$, which is allowed. There, $C=10$ and $R=8$, a \$2 gap. Since the difference is never zero, no real or whole-number nonnegative production amount breaks even. The cost graph stays strictly above the revenue line.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign model interpreter, algebra solver, table checker and graph-window reviewer roles. Construct a line from prescribed agreements and challenge assumptions about negative coordinates or rounded displays.

Core task

Cost is $C=q^2+2$ dollars. Construct a linear net-receipts model $R=mq+b$ that breaks even exactly at $q=1$ and $q=4$. Find m and b , report both dollar values, and prove there are no additional break-even inputs.

Teacher solution and discussion

The required points are $(1, 3)$ and $(4, 18)$. Their slope is $(18-3)/(4-1)=5$, so $R=5q-2$. Hence $m=5$ and $b=-2$. Equality $q^2+2=5q-2$ becomes $q^2-5q+4=(q-1)(q-4)=0$, proving the only inputs are 1 and 4. Both common dollar values check. The negative intercept can represent a fixed deduction from net receipts.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Use graphs to interpret intersection counts, tangency, relative values and window limitations, connecting graphical and algebraic evidence.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require new systems, explicit units and domain decisions. Include parameter cases, reverse construction and context creation so success cannot come from repeatedly substituting into one memorized template.

Core task

Cost is $C=q^2+5$ and revenue is $R=6q$ dollars for $q \geq 0$. Find all break-even points, determine the real interval where revenue is greater, and list the profitable whole-number counts. Connect these conclusions to the relative positions of the graphs.

Teacher solution and discussion

Equality gives $q^2-6q+5=(q-1)(q-5)=0$, so points are (1, 6) and (5, 30). Revenue exceeds cost when $(q-1)(q-5)<0$, or $1<q<5$. The profitable whole-number counts are 2, 3, 4. Between the intersections the line is above the parabola; outside them it is below. At the endpoints profit is zero, so they are not strictly profitable.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Address the observed issue: output recovery, moved-term sign, different-input table matches, a lost zero branch or solving for zero instead of equality. Check the corrected idea in a fresh original model.

Core task

Cost is $C=q^2+2$ and funding is $F=3q+6$ dollars for $q \geq 0$ kg. A learner moves terms to get $q^2+3q-4=0$. Correct the rearrangement, solve the original system and check which pair is physically meaningful.

Teacher solution and discussion

Equality is $q^2+2=3q+6$. Subtract $3q$ and 6 to get $q^2-3q-4=(q-4)(q+1)=0$, not the learner's sign pattern. Candidate $q=4$ gives $C=F=18$, while $q=-1$ gives $C=F=3$ but violates $q \geq 0$. The feasible point is $(4, 18)$, meaning 4 kg and \$18. The moved linear term must be negative.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Use tables and successive approximations to locate intersections, distinguish exact from rounded agreement and report contextual precision.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Collect table, graph and algebra evidence with interpretation. Require exact forms where possible, labeled approximation where needed, and an explanation that agreement does not require either original output to be zero.

Core task

Two height readings are $A = -t^2 + 4t + 2$ and $B = t + 2$ meters for $0 \leq t \leq 4$ seconds. Make a table at $t = 0, 1, 2, 3, 4$, graph both readings and solve the system algebraically. Interpret every agreement and explain how the three representations support one another.

Teacher solution and discussion

The A values are 2, 5, 6, 5, 2; B values are 2, 3, 4, 5, 6. Equal columns occur at (0, 2) and (3, 5). Algebra gives $-t^2 + 4t + 2 = t + 2$, so $t(3 - t) = 0$ and exactly those two inputs. The graph shows intersections at the initial 2 m reading and the later 5 m reading after 3 s. The table finds both sampled agreements, algebra proves completeness, and the graph displays their locations and the intervening height comparison.

Evidence and response

Assess equation setup, completeness of input roots, recovered common outputs, original-model verification, representation quality and contextual meaning separately. Tables support numerical location but do not automatically prove completeness; graphs show relative position but depend on scale and window. Record unsupported rounding or answer-supplying graph cues separately from independent reasoning.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Production cost is $C=q^2+2$ dollars and revenue is $R=3q$ dollars for q items.

Which ordered pair (q , common dollars) satisfies both models?

- A. (2, 4)
- B. (3, 9)
- C. (0, 2)
- D. (2, 6)

Reasoning: _____

M02 | Warm-up | T2

A classroom ball-height model is $H=-t^2+4t$ meters for $0 \leq t \leq 4$ seconds. A horizontal marker is $H=3$ m. At what times do the models meet?

- A. $t=0$ and $t=4$ seconds
- B. $t=1$ and $t=3$ seconds
- C. $t=2$ seconds only
- D. $t=1$ second only

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

A workshop has cost $C=q^2+3$ dollars and revenue $R=4q$ dollars for whole-number production counts $0 \leq q \leq 4$. Find every break-even ordered pair, check both models and interpret both coordinates.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

The modeled height of a moving object is $H = -t^2 + 6t + 1$ meters for $0 \leq t \leq 6$ seconds. Find all meetings with a horizontal 6 m level using algebra. Report ordered pairs and explain the two times in context.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Answer: D. (2, 6)

At $q=2$, cost is $4+2=6$ and revenue is $3 \times 2=6$. A system solution uses one input and the same output in both equations.

Check each alternative

A. At $q=2$ both models give 6, not 4.

B. Revenue is 9 at $q=3$, but cost is 11.

C. Cost is 2 at $q=0$, but revenue is 0.

D. At $q=2$, cost is $4+2=6$ and revenue is $3 \times 2=6$. A system solution uses one input and the same output in both equations.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02

M02 • T2: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Answer: B. $t=1$ and $t=3$ seconds

Set $-t^2+4t=3$, so $t^2-4t+3=(t-1)(t-3)=0$. Both 1 and 3 lie in the time interval and give height 3 m.

Check each alternative

A. These are the zero-height times, not meetings with the 3 m marker.

B. Set $-t^2+4t=3$, so $t^2-4t+3=(t-1)(t-3)=0$. Both 1 and 3 lie in the time interval and give height 3 m.

C. At $t=2$ the height is 4 m, not 3 m.

D. This omits the later return to the same height.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T1: Represent and interpret a contextual system solution as one allowed input with the same output in both original models.

Set $q^2+3=4q$, giving $q^2-4q+3=(q-1)(q-3)=0$. Thus $q=1$ or 3 , both allowed. At 1 , $C=4=R$; at 3 , $C=12=R$. The break-even points are $(1, 4)$ and $(3, 12)$: one item with \$4 of cost and revenue, or three items with \$12 of each.

Evidence to assess

Require equating the two models, both allowed roots, both common outputs, checks and quantity/unit interpretation of each coordinate.

WORKED TEACHER KEYS

002

002 • T2: Find exact linear-quadratic system solutions algebraically, recover every ordered pair and check the original models and domains.

Solve $-t^2+6t+1=6$, or $t^2-6t+5=(t-1)(t-5)=0$. Both $t=1$ and 5 are allowed, and each gives $H=6$. The points are $(1, 6)$ and $(5, 6)$. The object first reaches the level while rising and returns to it while falling; the vertex occurs between them at $t=3$.

Evidence to assess

Require the correct equality, both checked times, height coordinates with units and a valid rising/falling interpretation.