

TEACHER AND STUDENT RESOURCE

Completing the Square to Reveal Extreme Values



NC Math 2 • NC.M2.A-SSE.3

Seeing Structure in Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-SSE.3 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Write an equivalent form of a quadratic expression by completing the square, where a is an integer of a quadratic expression, $ax^2 + bx + c$, to reveal the maximum or minimum value of the function the expression defines.

T1 Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

T2 Use a signed square and constant to justify a maximum or minimum and its location.

T3 Interpret contextual or restricted-domain extrema and repair equivalence errors.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M02, O02, M03, O03, M04, O04, M05, O05, M06, O06, M11, O11, M12, O12, M16, O16, M19, O19, O22, M24, O24, O27, M29, O29

T2: M07, O07, M08, O08, M13, O13, M14, O14, M17, O17, M20, O20, M22, M23, O23, M26, O26, M27, M30, O30

T3: M09, O09, M10, O10, M15, O15, M18, O18, M21, O21, M25, O25, M28, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Learning target: Use a signed square and constant to justify a maximum or minimum and its location.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Use a signed square and constant to justify a maximum or minimum and its location.

Learning target: Interpret contextual or restricted-domain extrema and repair equivalence errors.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Learning target: Use a signed square and constant to justify a maximum or minimum and its location.

Learning target: Interpret contextual or restricted-domain extrema and repair equivalence errors.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Expand squared binomials, factor a common coefficient, calculate fractions, and understand that a real square is nonnegative.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Quadratic leading coefficients are nonzero integers in numerical tasks. A completed-square form is equivalent for every input in the original domain. The vertex supplies an extremum only when it lies in the allowed domain; integer and bounded domains may require additional checks. No calculus is used.

TEACHER MODEL

Keep the expression equivalent while making a square

Rewrite $2x^2+8x+1$ by completing the square, then justify the smallest value over all real x .

Factor 2 from the variable terms: $2(x^2+4x)+1$. Inside, $x^2+4x=(x+2)^2-4$. Substitute and distribute: $2[(x+2)^2-4]+1=2(x+2)^2-7$. The square is nonnegative, so the expression is at least -7; equality occurs at $x=-2$. Expand to check: $2x^2+8x+8-7=2x^2+8x+1$.

VISUAL ANCHOR

Build a square and compensate

The expression x^2+6x contains a square and two strips. Adding 9 closes the corner, but equivalence requires removing the same 9.

Step	Equivalent expression	Reason
Start	x^2+6x+5	Original expression
Make the square	$(x^2+6x+9)-9+5$	Add and subtract 9
Group	$(x+3)^2-4$	Recognize a square
Interpret	Minimum -4 at $x=-3$	Square is nonnegative

With a leading factor, the added inside constant is scaled. For $2(x^2+6x)$, adding 9 inside adds 18 to the whole expression, so subtract 18 to preserve equivalence.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Expand a squared binomial to expose its two cross terms. Ask learners to identify the half-coefficient and then square it; distinguish those two steps.

Core task

Find c so $x^2+10x+c$ is a perfect square. Give the factored form and explain why the cross term is $10x$, not $5x$.

Teacher solution and discussion

Half of 10 is 5, so $c=25$ and $x^2+10x+25=(x+5)^2$. Expansion gives $x^2+5x+5x+25$; the two cross terms total $10x$. The half-coefficient must be squared to supply the corner term.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model an equality chain. Factor the leading coefficient first, add and subtract the needed inside square, then distribute the compensation. Name the input where the square vanishes.

Core task

Complete the square in $2x^2+12x+11$. Explain why adding 9 inside the factor 2 changes the entire expression by 18 and must be compensated outside. State the minimum and where it occurs.

Teacher solution and discussion

$2x^2+12x+11=2(x^2+6x)+11=2[(x+3)^2-9]+11=2(x+3)^2-7$. The internal 9 is multiplied by 2, so the full compensation is -18. The minimum is -7 at $x=-3$.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Interpret contextual or restricted-domain extrema and repair equivalence errors.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Pause before every compensation and ask how much was added to the whole expression. Verify the completed form by expansion before interpreting its constant as an extremum.

Core task

A student completes $x^2 - 6x + 2$ as $(x - 3)^2 + 2$. Locate the exact equivalence error, repair it, and use one input to show why the two original expressions were unequal.

Teacher solution and discussion

The square $(x - 3)^2$ already contains $+9$, which was added without compensation. The correct form is $(x - 3)^2 - 7$. At $x = 0$, the original is 2 while the incorrect form is 11, so they are unequal. The corrected form gives $9 - 7 = 2$.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Use a signed square and constant to justify a maximum or minimum and its location.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign one partner to complete the square, one to expand-check, and one to test the extremum against the domain. Compare different constructions or bounds and justify disagreements from equivalence.

Core task

Compare the minimum values and the locations of $f(x)=x^2+2x+5$ and $g(x)=2x^2-8x+10$. Explain why comparing only the original constant terms 5 and 10 would give the wrong basis for the decision.

Teacher solution and discussion

$f=(x+1)^2+4$ has minimum 4 at $x=-1$. $g=2(x-2)^2+2$ has minimum 2 at $x=2$. The original constants are values at $x=0$, not generally extrema. Thus f has the larger minimum despite its smaller original constant.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Preserve the first response without directing the half-coefficient or compensation step. Require exact values and a nonnegativity or nonpositivity argument, not only a memorized vertex label.

Core task

Complete the square in $4x^2-4x+3$, keeping exact fractions. Identify the minimum and verify the constant term by expansion.

Teacher solution and discussion

$4x^2-4x+3=4(x^2-x)+3=4[(x-1/2)^2-1/4]+3=4(x-1/2)^2+2$. The minimum is 2 at $x=1/2$. Expansion gives $4x^2-4x+1+2=4x^2-4x+3$.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Diagnose a sign error, unscaled compensation, incorrect half-coefficient, or confusion between intercept and extremum. Rebuild that step with a fresh quadratic and check transfer.

Core task

Complete the square for $3x^2+12x$. Explain how a quadratic with zero original constant can still have a nonzero minimum.

Teacher solution and discussion

$3x^2+12x=3(x^2+4x)=3[(x+2)^2-4]=3(x+2)^2-12$. The minimum is -12 at $x=-2$. The original constant 0 gives the value at $x=0$, not the minimum; the linear term shifts the vertex.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Interpret contextual or restricted-domain extrema and repair equivalence errors.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Use a new model, a parameter condition, or a learner-created quadratic. Require the completion process, extremum and location, domain check when relevant, and an expansion verification.

Core task

A cost model $C(x) = 3x^2 - 18x + 40$ dollars uses an input length x in meters, with $0 \leq x \leq 8$. Complete the square, interpret the smallest model cost, and explain why the domain does not invalidate the vertex conclusion.

Teacher solution and discussion

$C = 3(x^2 - 6x) + 40 = 3[(x - 3)^2 - 9] + 40 = 3(x - 3)^2 + 13$. The minimum model cost is \$13 at input length $x = 3$ m, and 3 lies inside $[0, 8]$. The positive leading coefficient makes every allowed squared contribution nonnegative.

Evidence and response

Look for an equivalence-preserving compensation, especially when a leading factor is not 1. Ask what number the square contributes at its zero and how the sign of the outside coefficient bounds the whole expression. Keep output value and input location separate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Which constant makes x^2+6x+c a perfect square?**

- A. 36
- B. 3
- C. 6
- D. 9

Reasoning: _____

M02 | Warm-up | T1**Which expression equals x^2+6x+5 ?**

- A. $(x+3)^2+5$
- B. $(x+3)^2+4$
- C. $(x+6)^2-31$
- D. $(x+3)^2-4$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Find c so $x^2+10x+c$ is a perfect square. Give the factored form and explain why the cross term is $10x$, not $5x$.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Complete the square in x^2+8x+3 . Show where the compensating subtraction appears, then expand your answer to verify the original expression.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Answer: D. 9

A square $(x+h)^2$ expands to $x^2+2hx+h^2$. Matching $2h=6$ gives $h=3$ and $c=h^2=9$.

Check each alternative

- A. This squares the full linear coefficient rather than its half.
- B. This is half the linear coefficient, but it still must be squared.
- C. This copies the linear coefficient rather than squaring its half.
- D. A square $(x+h)^2$ expands to $x^2+2hx+h^2$. Matching $2h=6$ gives $h=3$ and $c=h^2=9$.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02

M02 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Answer: D. $(x+3)^2-4$

Add and subtract 9: $x^2+6x+5=(x^2+6x+9)-9+5=(x+3)^2-4$.

Check each alternative

- A. This expansion has constant 14 rather than 5.
- B. This expansion has constant 13 rather than 5.
- C. This expansion has linear coefficient 12 rather than 6.
- D. Add and subtract 9: $x^2+6x+5=(x^2+6x+9)-9+5=(x+3)^2-4$.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.

Half of 10 is 5, so $c=25$ and $x^2+10x+25=(x+5)^2$. Expansion gives $x^2+5x+5x+25$; the two cross terms total $10x$. The half-coefficient must be squared to supply the corner term.

Evidence to assess

Require $c=25$, the correct squared binomial, and a cross-term explanation.

WORKED TEACHER KEYS

002**002 • T1: Rewrite quadratic expressions with nonzero integer leading coefficients by completing the square.**

Add and subtract 16: $x^2+8x+3=(x^2+8x+16)-16+3=(x+4)^2-13$. Expanding gives $x^2+8x+16-13=x^2+8x+3$. The subtraction preserves the original value after adding the square term.

Evidence to assess

Assess an equivalence chain, compensation by 16, and a correct expansion check.