

TEACHER AND STUDENT RESOURCE

Polynomial and Rational Inequalities



Precalculus • PC.A.1

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.A.1 | North Carolina Precalculus mathematics

This sample contains 21 complete selected pages from the 114-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 35-36, 66, 82-83

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 35

O02: sample PDF page(s) 19; original page(s) 36

Selected teacher prompts and keys: original page(s) 66, 82-83

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of solving inequalities that include rational and polynomial expressions in one variable.

T1 PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

T2 PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

PC.A.1.1 Implement algebraic (sign analysis) methods to solve rational and polynomial inequalities.

PC.A.1.2 Implement graphical methods to solve rational and polynomial inequalities.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M03, M05, M07, M09, M11, M13, M15, M17, M19, M21, M23, M25, M27, M29, O01, O03, O05, O07, O09, O11, O13, O15, O17, O19, O21, O23, O25, O27, O29

T2: M02, M04, M06, M08, M10, M12, M14, M16, M18, M20, M22, M24, M26, M28, M30, O02, O04, O06, O08, O10, O12, O14, O16, O18, O20, O22, O24, O26, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Learning target: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, straightedge, sign-chart organizers and the supplied graph pages. A graphing calculator may verify a sketch or enlarge a viewing window; retain the written domain and endpoint reasoning. All exact answers can be obtained without paid software.

Prerequisites

Factoring polynomials, interval notation, real-number sign rules and function graphs.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All variables are real unless a context restricts them. A square bracket includes a finite endpoint; a parenthesis excludes it. Infinity always uses parentheses. Denominator zeros are excluded even after cancellation. Braces describe isolated solutions. Graphs are shown in finite windows: use the stated function, every critical value and branch behavior to justify any unbounded interval. Dashed lines indicate a supplied asymptote or labeled comparison level, not part of the function.

TEACHER REFERENCE

Sign intervals are not just a list of roots

Put zero on one side

For $f(x) \leq g(x)$, analyze $h(x) = f(x) - g(x) \leq 0$. Graphically this means the graph of h lies on or below the horizontal axis. If using two graphs, identify where f lies on or below g .

Mark zeros and exclusions separately

Numerator zeros can make a rational expression equal to zero when its denominator is nonzero. Denominator zeros are forbidden, whether they produce vertical asymptotes or removable holes. Keep the original domain after canceling factors.

Test each interval, then decide endpoints

A continuous nonzero rational or polynomial expression has constant sign between consecutive critical values. Use factor signs or one test value per interval. Decide each boundary separately; a non-strict inequality does not include undefined values.

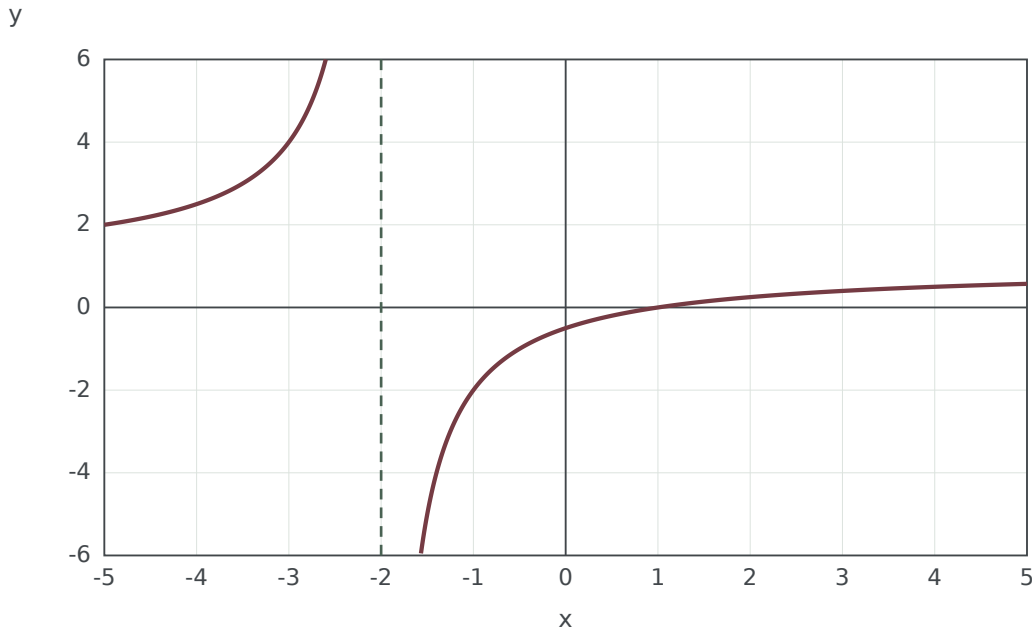
Make the graph check independent

Read where the graph is above, below or on the required level. Check touching points as well as crossings. Extend the window when a zero or pole lies outside it. Do not mistake the visible part of a curve for its entire domain.

TEACHER MODEL

A pole is never a closed endpoint

Solve $(x-1)/(x+2) > 0$ and verify the result graphically.



The original domain excludes -2 ; the numerator zero is 1 . Factor signs give positive, negative, positive on $(-\infty, -2)$, $(-2, 1)$, $(1, \infty)$. The solution is $(-\infty, -2) \cup (1, \infty)$. The graph is above zero on those two intervals. The pole -2 is undefined, and the zero 1 fails the strict inequality. If $>$ were changed to $\geq$, only the zero 1 would become included.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Solve $(x+4)(x-1)>0$ using a sign chart. Include a test value in every interval, state the solution in interval notation, and explain both endpoint decisions.

Teacher solution and discussion

Critical values are -4 and 1 . At test values -5 , 0 and 2 , the factor signs are $(-, -)$, $(+, -)$ and $(+, +)$, so overall signs are $+$, $-$, $+$. The solution is $(-\infty, -4) \cup (1, \infty)$. Both finite endpoints make the product zero and are excluded by >0 .

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Solve $(x+1)^2(x-4) > 0$. Explain why the sign does not reverse at $x = -1$ and why the zero still matters to a strict inequality.

Teacher solution and discussion

The squared factor is positive except at -1 . Away from that zero, the product has the sign of $x-4$: negative for $x < 4$ and positive for $x > 4$. At -1 and 4 the product is zero. The solution is $(4, \infty)$. The even-multiplicity root -1 does not change the surrounding negative sign, but must still be identified as a zero.

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Solve $(x+2)^2(x-1) \geq 0$. Show why the answer contains an isolated point rather than one continuous interval between the roots.

Teacher solution and discussion

The squared factor is positive except at -2 ; away from -2 , the sign equals that of $x-1$. For $x < 1$ the product is negative except that it is zero at -2 . For $x \geq 1$ it is nonnegative. Hence the solution is $\{-2\} \cup [1, \infty)$. For example, $x=0$ lies between the roots but gives -4 , so the entire interval between them does not qualify.

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Use the graph of $f(x) = (x-1)^2/(x+2)$ to solve $f(x) > 0$. Explain both the break at $x = -2$ and the single missing point at $x = 1$ in the positive-side intervals.

Teacher solution and discussion

The graph is below zero for $x < -2$ and above zero for $x > -2$ except where it touches zero at $x = 1$. The solution is $(-2, 1) \cup (1, \infty)$. At -2 the function is undefined; at 1 it is defined but equals zero, which fails strict > 0 .

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Solve $(x-4)^2/[(x-4)(x+2)] \geq 0$ on the original domain. Explain why $x=4$ is not an included numerator zero after cancellation.

Teacher solution and discussion

The original domain excludes 4 and -2 . Away from them, the expression simplifies to $(x-4)/(x+2)$. The reduced expression is positive for $x < -2$ and $x > 4$, negative between the critical values, and would be zero at 4. But the original expression is undefined at 4. The solution is $(-\infty, -2) \cup (4, \infty)$.

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Use the graph of $f(x) = (x^2 - 1)/(x - 1)^2$ and the comparison level $y = 1$ to solve $f(x) \geq 1$. Explain why the horizontal asymptote $y = 1$ does not supply an equality solution.

Teacher solution and discussion

For $x \neq 1$, $f(x) = (x+1)/(x-1) = 1 + 2/(x-1)$. The left branch is below 1 and the right branch above 1, so the solution is $(1, \infty)$. The equation $f(x) = 1$ would require $2/(x-1) = 0$, which has no finite solution. Approaching a horizontal asymptote does not mean the graph reaches it.

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Sketch $f(x) = (x+1)/[(x-2)(x+3)]$, showing both vertical asymptotes and its zero. Use the sketch to solve $f(x) \geq 0$ and check the sign on all four intervals.

Teacher solution and discussion

The poles are -3 and 2 , and the zero is -1 . From left to right the signs are $-$, $+$, $-$, $+$. Thus the graph is nonnegative on $(-3, -1] \cup (2, \infty)$. Include the defined zero -1 but exclude both poles. Test values -4 , -2 , 0 , 3 give signs $-$, $+$, $-$, $+$ respectively.

Evidence and response

Ask for the original domain, all numerator zeros and denominator zeros, and a sign on each interval. Even multiplicity may preserve the sign; non-strict inequalities may include isolated zeros. For graph methods, require above/below reasoning and correct open/closed endpoints. A sample point supports its interval only after all critical values are accounted for.

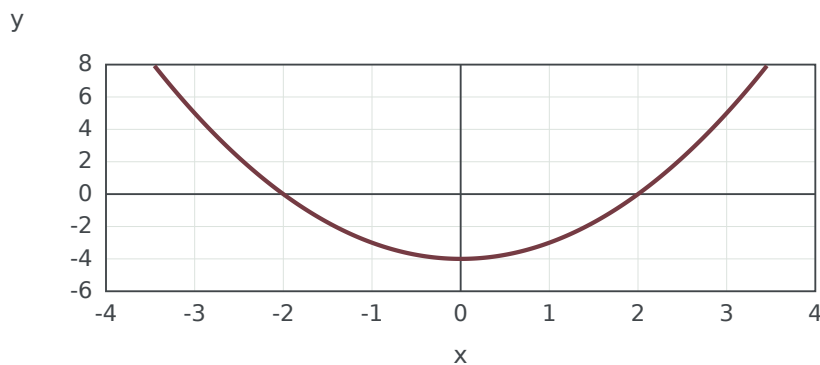
Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Solve $(x-2)(x+3)<0$.**

- A. $(-\infty, -3) \cup (2, \infty)$
- B. $[-3, 2]$
- C. $(-3, 2)$
- D. $\{-3, 2\}$

Reasoning: _____

M02 | Warm-up | T2**Use the graph of $f(x)=x^2-4$ to solve $f(x)\geq 0$.**

- A. $[-2, 2]$
- B. $(-\infty, -2] \cup [2, \infty)$
- C. $(-\infty, -2) \cup (2, \infty)$
- D. $\{-2, 2\}$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.

Solve $(x+4)(x-1)>0$ using a sign chart. Include a test value in every interval, state the solution in interval notation, and explain both endpoint decisions.

Interval / boundary	Test value or domain check	Factor signs / conclusion

Representation / calculation

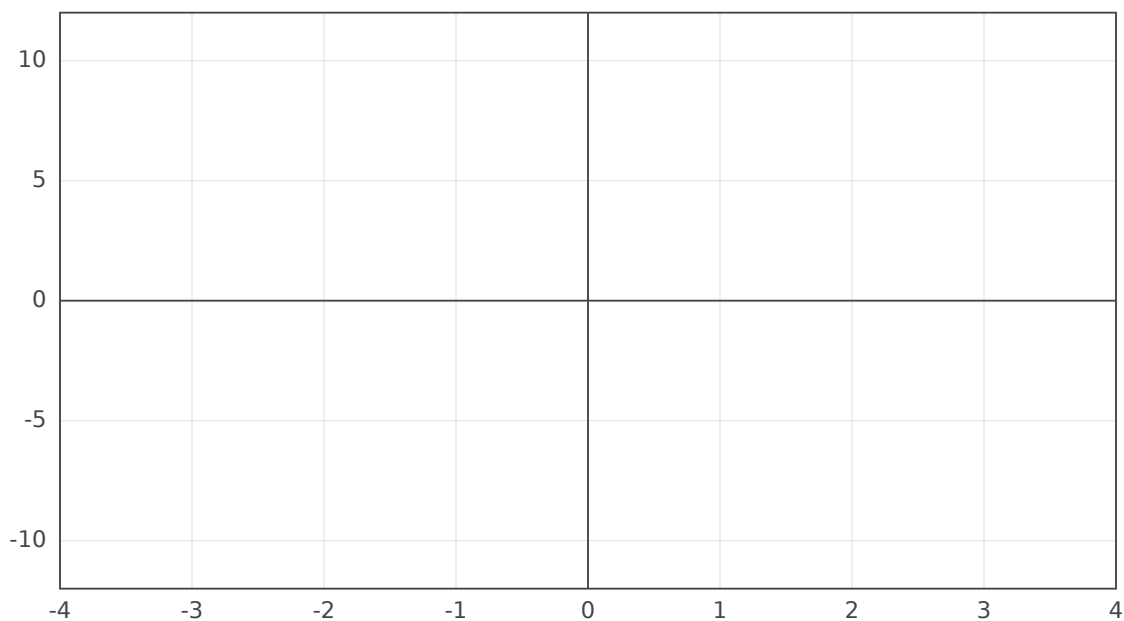
Explanation / check

WARM-UP

002 • Show your reasoning**T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.**

Sketch $p(x)=(x+3)x(x-2)$, showing its intercepts and sign on each interval. Use your graph to solve $p(x) \geq 0$, and check one input in each included interval.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.****Answer: C. $(-3, 2)$**

The factors have opposite signs only between -3 and 2 ; exclude the endpoints.

Check each alternative

- A. The product is positive, not negative, outside the roots.
- B. Strict inequality excludes both zeros.
- C. Correct. The factors have opposite signs only between -3 and 2 ; exclude the endpoints.
- D. Those values make the expression zero, not negative.

M02 • T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.**Answer: B. $(-\infty, -2] \cup [2, \infty)$**

The upward parabola is on or above the horizontal axis for $x \leq -2$ or $x \geq 2$.

Check each alternative

- A. The graph is at or below zero between the intercepts, not at or above.
- B. Correct. The upward parabola is on or above the horizontal axis for $x \leq -2$ or $x \geq 2$.
- C. The zeros satisfy ≥ 0 and must be included.
- D. The graph is also positive on the two outer intervals.

WORKED TEACHER KEYS

001**001 • T1: PC.A.1.1 Solve polynomial and rational inequalities by algebraic sign analysis.**

Critical values are -4 and 1 . At test values -5 , 0 and 2 , the factor signs are $(-, -)$, $(+, -)$ and $(+, +)$, so overall signs are $+$, $-$, $+$. The solution is $(-\infty, -4) \cup (1, \infty)$. Both finite endpoints make the product zero and are excluded by >0 .

Evidence to assess

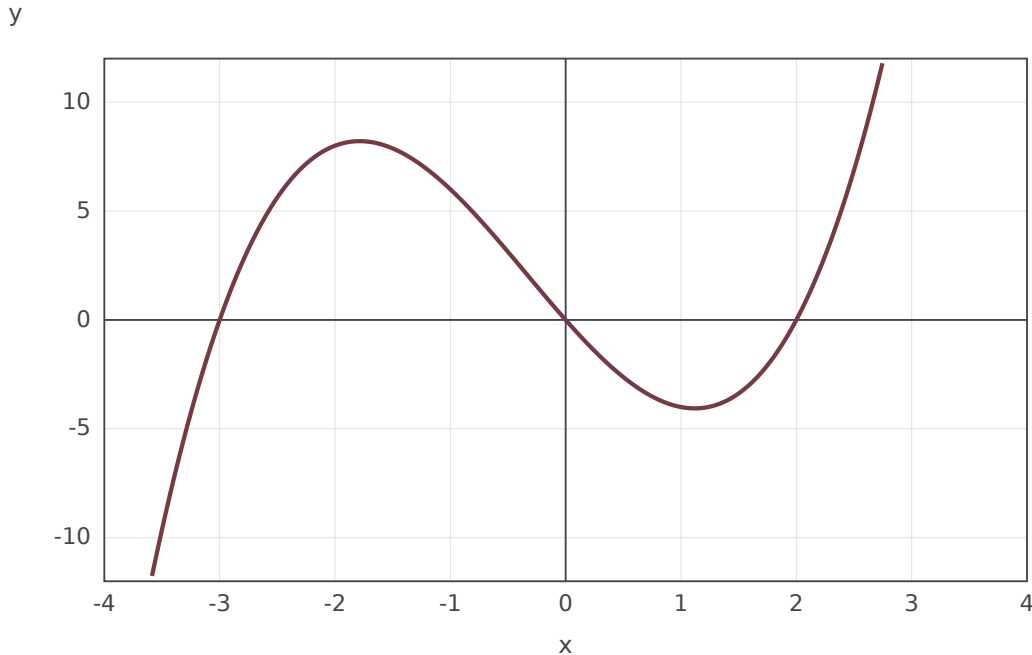
Full evidence: Two roots, all three interval signs with checks, correct union and strict-endpoint reasoning. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002

002 • T2: PC.A.1.2 Use graphs to solve and verify polynomial and rational inequalities.

The simple zeros are -3 , 0 and 2 . From left to right the signs are $-$, $+$, $-$, $+$. The graph is on or above zero on $[-3, 0] \cup [2, \infty)$. Checks: $p(-1)=2 \cdot (-1) \cdot (-3)=6$ and $p(3)=6 \cdot 3 \cdot 1=18$. All three zeros are included because equality is allowed.

**Evidence to assess**

Full evidence: A cubic sketch with three crossings, the correct two intervals, endpoint inclusion and two positive-value checks. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.