

TEACHER AND STUDENT RESOURCE

Logarithmic, Trigonometric and Parametric Equations



Precalculus • PC.A.2

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.A.2 | North Carolina Precalculus mathematics

This sample contains 20 complete selected pages from the 98-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 77

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions.

T1 PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

T2 PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

T3 PC.A.2.3 Solve trigonometric equations using inverse functions, double-angle formulas and Pythagorean identities.

T4 PC.A.2.4 Eliminate a parameter while preserving the actual Cartesian curve and its restrictions.

PC.A.2.1 Use properties of logarithms to rewrite expressions.

PC.A.2.2 Implement properties of exponentials and logarithms to solve equations.

PC.A.2.3 Implement properties of trigonometric functions to solve equations including

- inverse trigonometric functions,
- double angle formulas, and Pythagorean identities.

PC.A.2.4 Implement algebraic techniques to rewrite parametric equations in cartesian form by eliminating the parameter.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M05, M09, M13, M17, M21, M25, O01, O05, O09, O13, O17, O21, O25

T2: M02, M06, M10, M14, M18, M22, M26, O02, O06, O10, O14, O18, O22, O26

T3: M03, M07, M11, M15, M19, M23, M27, M29, M30, O03, O07, O11, O15, O19, O23, O27, O29, O30

T4: M04, M08, M12, M16, M20, M24, M28, O04, O08, O12, O16, O20, O24, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

Learning target: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

Learning target: PC.A.2.3 Solve trigonometric equations using inverse functions, double-angle formulas and Pythagorean identities.

Learning target: PC.A.2.4 Eliminate a parameter while preserving the actual Cartesian curve and its restrictions.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, a scientific calculator with natural logarithms and radian mode, straightedge and graph paper. Exact values are preferred; decimal approximations are labeled. The supplied teacher graphs and formulas support all tasks without paid software.

Prerequisites

Exponent rules, factoring quadratics, unit-circle values, interval notation and function inverses.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All variables are real. $\ln$ is the natural logarithm; a logarithm base b satisfies $b > 0$ and $b \neq 1$. Each real logarithm argument must be positive. Trigonometric angles are in radians unless explicitly stated otherwise. $\arcsin$ has range $[-\pi/2, \pi/2]$, $\arccos$ has range $[0, \pi]$, and $\arctan$ has range $(-\pi/2, \pi/2)$. General trigonometric solutions use integer k . Eliminating a parameter does not automatically preserve its domain, branch or orientation; state restrictions separately.

TEACHER REFERENCE

Equivalent expressions need equivalent domains

Logarithm properties

For positive a, c and a valid base b , $\log_b(ac) = \log_b a + \log_b c$ and $\log_b(a/c) = \log_b a - \log_b c$. A power may move outside a logarithm when the resulting expression retains the intended domain. For real $x \neq 0$, $\ln(x^2) = 2\ln|x|$; writing $2\ln x$ would lose negative inputs.

Equation substitutions

When substituting $u = e^x$, impose $u > 0$. When exponentiating a logarithmic equation, retain every original argument restriction. Factoring may create algebraic candidates that do not belong to the original equation.

Inverse trigonometric functions

An inverse function returns one principal value, not every periodic solution. Use symmetry and period to find other solutions, and check whether the requested interval includes or excludes its right endpoint.

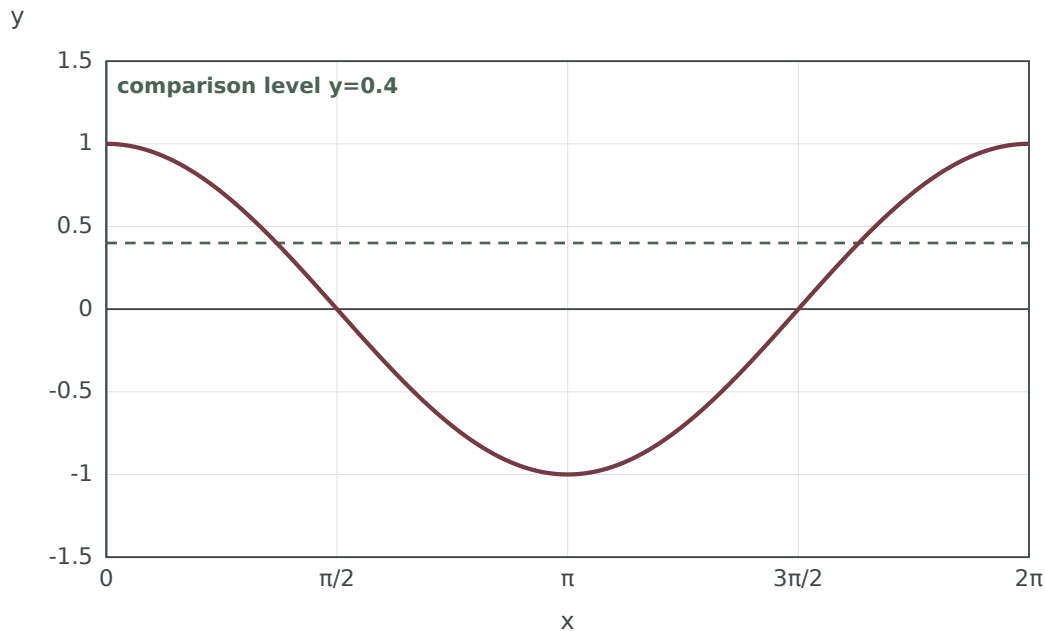
Parameter elimination

Solve for the parameter when possible, substitute, and then transfer all restrictions. Squaring a relation can add a branch. An equation for a full line, circle or parabola may describe more points than the original parameter interval permits.

TEACHER MODEL

One principal inverse value is not a full solution set

Solve $\cos x = 0.4$ over all real x , then restrict the answer to $[0, 2\pi)$.



Let $\alpha = \arccos(0.4)$, approximately 1.1593. The principal inverse supplies α , while cosine symmetry supplies $-\alpha$; periodicity gives $x = 2\pi k \pm \alpha$ for every integer k . On $[0, 2\pi)$, the answers are α and $2\pi - \alpha$, approximately 1.1593 and 5.1239. The two intersections with $y = 0.4$ support the interval answer. State the interval before listing values, and do not round until the final step.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

For $x > 0$ and $y > 0$, expand $\ln(5x^3/\sqrt{y})$ fully. Name the property used at each step and verify your result at $x=2$, $y=16$ without rounding.

Teacher solution and discussion

$\ln(5x^3/\sqrt{y}) = \ln 5 + \ln(x^3) - \ln(y^{1/2}) = \ln 5 + 3 \ln x - (1/2)\ln y$. Product and quotient properties separate the factors, then the power property moves exponents. At $x=2$, $y=16$, the original argument is $5 \cdot 8/4 = 10$, so the value is $\ln 10$. The expansion is $\ln 5 + 3\ln 2 - 2\ln 2 = \ln 10$.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Condense $(1/2)\ln(x-1)+\ln(x+2)$ into a single logarithm, state the original real domain, and show that the condensed expression has that same domain.

Teacher solution and discussion

The original domain requires $x-1>0$ and $x+2>0$, so $x>1$. Condensing gives $\ln[(x+2)\sqrt{x-1}]$. For $x>1$ both factors are positive. Conversely, the square root requires $x\geq 1$, but at $x=1$ the logarithm argument is zero, leaving exactly $x>1$.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Rewrite $\ln[(x-2)/(x+1)]$ as a difference of real logarithms without losing the negative-input branch. State the original domain and explain why $\ln(x-2) - \ln(x+1)$ is incomplete.

Teacher solution and discussion

The quotient must be positive, so the original domain is $x < -1$ or $x > 2$. On that domain, $\ln[(x-2)/(x+1)] = \ln|x-2| - \ln|x+1|$. The version without absolute values is defined only for $x > 2$ and loses $x < -1$. Keep the original domain even with absolute values; the rewritten formula by itself would also be defined on $(-1, 2)$, where the original logarithm is not.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Solve $16^{(x-1)} = 8^{(2x+1)}$ by rewriting both sides with base 2. Check the resulting exponents at your solution.

Teacher solution and discussion

$16 = 2^4$ and $8 = 2^3$, so $2^{(4x-4)} = 2^{(6x+3)}$. Equate exponents: $4x-4 = 6x+3$, giving $x = -7/2$. At this input both exponents equal -18 , so both sides are $2^{(-18)}$.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: PC.A.2.3 Solve trigonometric equations using inverse functions, double-angle formulas and Pythagorean identities.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Solve $\cos x = 0.4$ for every real x using inverse notation. Also list the two solutions in $[0, 2\pi)$ to four decimal places and show them on one cosine cycle.

Teacher solution and discussion

Let $\alpha = \arccos(0.4) \approx 1.1593$, with α in $(0, \pi/2)$. Every real solution is $x = 2\pi k \pm \alpha$ for integer k . In $[0, 2\pi)$, the two values are approximately 1.1593 and 5.1239. The graph meets the level 0.4 once on the decreasing first half-cycle and once on the increasing second half-cycle.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: PC.A.2.4 Eliminate a parameter while preserving the actual Cartesian curve and its restrictions.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Eliminate t from $x=t^2-1$ and $y=t^2+2$, where t is any real number. Sketch the exact Cartesian set and explain why most points on it are reached by two parameter values.

Teacher solution and discussion

Subtracting the equations gives $y=x+3$. Since $t^2 \geq 0$, $x \geq -1$, so the set is the ray on that line starting at $(-1, 2)$, with its endpoint included. For $x > -1$, the parameter values $t = \pm\sqrt{x+1}$ give the same point; the endpoint corresponds only to $t=0$.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: PC.A.2.4 Eliminate a parameter while preserving the actual Cartesian curve and its restrictions.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Eliminate t from $x=t+1$ and $y=1/(t-2)$, with $0 \leq t \leq 4$ and $t \neq 2$. State the complete Cartesian domain, including which finite endpoints are included, and check one endpoint from the original equations.

Teacher solution and discussion

$t=x-1$, so $y=1/(x-3)$. The interval becomes $1 \leq x \leq 5$ with $x \neq 3$, or $[1, 3) \cup (3, 5]$. Both endpoints are included: at $t=0$, $x=1$ and $y=-1/2$; at $t=4$, $x=5$ and $y=1/2$. The excluded $t=2$ corresponds to the vertical asymptote input $x=3$.

Evidence and response

Ask which domain was stated before the transformation and which candidate values were checked afterward. For trigonometric equations, preserve every factor case and account for the requested interval or all periodic families. For parametric equations, check that each Cartesian point can arise from an allowed parameter, not merely that substitution produces a necessary equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****For $x > 0$ and $y > 0$, expand $\ln(3x^2/y)$.**

- A. $\ln 3 + \ln x - 2 \ln y$
- B. $\ln(3 + x^2 - y)$
- C. $\ln 3 + 2 \ln x - \ln y$
- D. $3 \ln x + 2 \ln y$

Reasoning: _____

M02 | Warm-up | T2**Solve $e^{(2x-1)}=7$.**

- A. $x = (1 + \ln 7)/2$
- B. $x = \ln 7/2 - 1$
- C. $x = (1 + 7)/2$
- D. $x = \ln(7/2) + 1$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.

For $x > 0$ and $y > 0$, expand $\ln(5x^3/\sqrt{y})$ fully. Name the property used at each step and verify your result at $x=2$, $y=16$ without rounding.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

Solve $5^{(x+1)}=12$. Give an exact logarithmic form and a decimal approximation to four decimal places, then describe a check in the original equation.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.****Answer: C. $\ln 3 + 2 \ln x - \ln y$** Apply product, power and quotient properties: $\ln 3 + \ln(x^2) - \ln y = \ln 3 + 2 \ln x - \ln y$.**Check each alternative**

- A. This assigns the exponent to the wrong variable.
- B. A logarithm of a product or quotient does not become the logarithm of a sum or difference.
- C. Correct. Apply product, power and quotient properties: $\ln 3 + \ln(x^2) - \ln y = \ln 3 + 2 \ln x - \ln y$.
- D. This loses the constant factor and reverses the quotient contribution.

M02 • T2: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.**Answer: A. $x = (1 + \ln 7)/2$** Taking $\ln$ gives $2x - 1 = \ln 7$, so $x = (1 + \ln 7)/2$.**Check each alternative**

- A. Correct. Taking $\ln$ gives $2x - 1 = \ln 7$, so $x = (1 + \ln 7)/2$.
- B. After $2x - 1 = \ln 7$, add 1 before dividing by 2.
- C. Taking a logarithm of the right side gives $\ln 7$, not 7.
- D. The exponent equation is linear in x ; dividing the logarithm is not the same as dividing its argument.

WORKED TEACHER KEYS

001 / 002**001 • T1: PC.A.2.1 Rewrite logarithmic expressions with valid properties and domains.**

$\ln(5x^3/\sqrt{y}) = \ln 5 + \ln(x^3) - \ln(y^{1/2}) = \ln 5 + 3 \ln x - (1/2)\ln y$. Product and quotient properties separate the factors, then the power property moves exponents. At $x=2$, $y=16$, the original argument is $5 \cdot 8/4 = 10$, so the value is $\ln 10$. The expansion is $\ln 5 + 3\ln 2 - 2\ln 2 = \ln 10$.

Full evidence: Correct expanded coefficients and signs, named properties, and an exact substitution check. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

002 • T2: PC.A.2.2 Solve exponential and logarithmic equations and check restrictions.

Taking $\ln$ gives $(x+1)\ln 5 = \ln 12$. Thus $x = \ln 12 / \ln 5 - 1 \approx 0.5440$. With the exact expression, $5^{(x+1)} = e^{[(\ln 12 / \ln 5)\ln 5]} = e^{\ln 12} = 12$. A rounded substitution should be close to 12 rather than exactly equal.

Full evidence: Exact value, a four-decimal approximation and an original-equation verification that distinguishes rounding. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.