

TEACHER AND STUDENT RESOURCE

Proving and Solving with Plane Figures



NC Math 3 • NC.M3.G-CO.14

Congruence

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-CO.14 | NC Math 3 mathematics

This sample contains 24 complete selected pages from the 129-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-18, 33-34, 65-66, 94-95

Representative activities

M01: sample PDF page(s) 19; original page(s) 18

M02: sample PDF page(s) 19; original page(s) 18

O01: sample PDF page(s) 20; original page(s) 33

O02: sample PDF page(s) 21; original page(s) 34

Selected teacher prompts and keys: original page(s) 65-66, 94-95

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

T1 Use definitions, congruence, angle relationships, coordinate evidence and exact constructions to prove statements about triangles and lines.

T2 Establish and apply similarity, proportional segments and area scaling, keeping correspondence and geometric feasibility explicit.

T3 Derive and apply polygon angle, perimeter and area relationships through decomposition, auxiliary lines, counting and algebra.

T4 Use inclusive quadrilateral definitions and justified side, angle and diagonal properties to classify figures and prove conditional conclusions.

T5 Evaluate geometric implications and their converses, distinguish sufficient evidence from appearance, and construct exact counterexamples.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M04, M05, M06, M07, M14, M19, M26, M28, O03, O05, O06, O12, O16, O20, O22, O24, O25, O27

T2: M08, M09, M10, M15, M20, M29, O09, O10, O11, O29

T3: M02, M03, M11, M12, M16, M21, M22, M23, M24, M27, O02, O04, O07, O13, O15, O21, O30

T4: M13, M18, M25, M30, O08, O14, O17, O19, O26, O28

T5: M17, O01, O18, O23

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Establish premises and build a proof

Learning target: Read definitions and hypotheses before selecting a theorem.

Learning target: Connect an auxiliary line, correspondence or dissection to a complete conclusion.

Select M01–M08 and O01–O08 over diagnostic and modeling lessons. Start with a student claim before displaying a completed key. Include a construction and a synthetic proof, not only numeric answers.

Session 2: Connect representations and test implications

Learning target: Use similarity, coordinates and polygon decompositions with their correct domains.

Learning target: Construct counterexamples and justify both directions of an equivalence when requested.

Choose M09–M23 and O09–O23 across several lessons. Have every learner record an individual starting argument before group comparison. Mix a proportional model, a quadrilateral certificate, an area proof and a claim audit rather than treating the 60-item bank as one worksheet.

Session 3: Repair a precise gap and transfer independently

Learning target: Distinguish shared bases from similarity, equilateral from regular and correspondence from unordered side lists.

Learning target: Produce an independent proof or exact geometric design with a complete diagram.

Choose M24–M27/O24–O27 according to the observed misconception, then use fresh M28–M30/O28–O30 for transfer. Record neutral access supports separately from hints naming the needed theorem.

PLANNING

Materials, access and pacing

Materials

Straightedge, compass, pencil, eraser, equal-scale graph paper and tracing paper for rotations or reflections. Highlighters can track corresponding sides and angles. A calculator is optional for checking radical or trigonometric approximations after exact relationships are recorded. A protractor may support a sketch but cannot establish an exact theorem. Dynamic geometry software is optional, not required; every task can be completed with the stated paper tools.

Prerequisites

Use triangle angle sums, linear pairs and parallel-line angle relationships; apply SSS, SAS, ASA, AAS and corresponding parts; recognize similarity and transformations; use the Pythagorean theorem and its converse, midpoint and distance formulas, proportional reasoning and elementary algebra. Sine-rule SSA analysis is introduced explicitly in M14. O11 and M27 use $\text{area} = (1/2)ab \sin C$; the teacher may supply that prerequisite identity. Basic straightedge-and-compass routines, positive side lengths, equal-scale plotting and square-root simplification are required. Coordinate formulas verify diagrams; they do not replace a requested construction or proof.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

Review convention

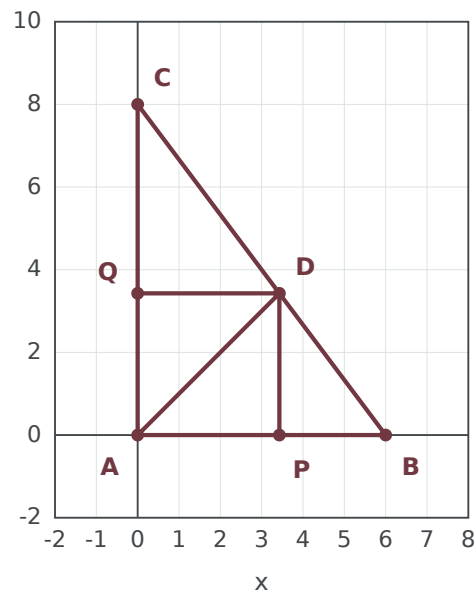
Use nondegenerate triangles and simple polygons; convexity is stated whenever an argument needs it. Vertices are named in boundary order. Isosceles means at least two equal sides, so equilateral triangles are included. A rhombus has four equal sides; a rectangle has four right angles; a square is both. A kite has two adjacent equal-side pairs, allowing a rhombus unless explicitly excluded. Trapezoid means exactly one parallel opposite pair in this packet. Congruence and similarity statements preserve vertex order. Distances and exact equalities are givens only when stated, not inferred from a picture. Measurements suggest conjectures, but proofs require definitions, established theorems and valid deductions. A construction must retain its defining marks and a completed diagram. All diagrams are schematic equal-scale references, not physical measuring templates. Contexts and data are original instructional examples. Squared distances, positive radicals and equivalent exact forms are acceptable; area and length retain different units.

TEACHER MODEL

Derive an angle-bisector ratio by comparing the same two areas

In a nondegenerate triangle ABC , AD is an internal angle bisector meeting BC at D . Why is $BD/DC=AB/AC$? Explain the geometric reason for the ratio rather than quoting an unexplained rule.

y



Drop perpendiculars DP and DQ to AB and AC . Right triangles APD and AQD have a shared hypotenuse AD and matching acute angles at A , so AAS gives $DP=DQ$. Call their common length r . The areas of ABD and ACD , using bases AB and AC , are $AB \cdot r/2$ and $AC \cdot r/2$, so their ratio is AB/AC . Now use bases BD and DC : both triangles have the same perpendicular altitude from A to BC , so their area ratio is BD/DC . Since these are the same two positive areas, equate the ratios. In the pictured 6-8-10 triangle the side parts are $30/7$ and $40/7$, not equal. An angle bisector becomes a median only when its adjacent side lengths are equal.

VISUAL ANCHOR

Name the premise, choose the bridge, certify the conclusion

Congruence: align the vertex order; supply every side or angle required by SSS, SAS, ASA or AAS.

Similarity: prove a criterion first; corresponding lengths scale by k and corresponding areas by k^2 .

Area comparison: a shared base uses a height ratio; a shared height uses a base ratio. Neither alone proves congruence.

Polygons: count the actual boundary; distinguish an interior segment, a reflex angle and a signed exterior turn.

Classification: test the complete definition; equal sides, equal diagonals and equal angles are different hypotheses.

Logic: a counterexample satisfies the hypothesis and fails the conclusion; a converse is a new claim.

VISUAL ANCHOR CONTINUED

Name the premise, choose the bridge, certify the conclusion

Construction: retain the arcs, midpoint or perpendicular evidence that makes the drawing mathematically constrained.

A complete answer ties each requested conclusion to a stated given, definition or established theorem. A plausible sketch and a correct final number do not replace the bridge.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T5: Evaluate geometric implications and their converses, distinguish sufficient evidence from appearance, and construct exact counterexamples.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask which facts are explicitly given before accepting a classification. M01 separates midpoint information from symmetry. In O01 preserve the inclusive isosceles convention while distinguishing a converse from a contrapositive. For O02 trace the rays at B and C before writing a triangle sum. In O03 require strict inequalities and test each endpoint. O04 uses an actual dissection to explain the area rule, not a memorized formula alone.

Core task

Use the inclusive definition that an isosceles triangle has at least two congruent sides. State the converse and contrapositive of “If a triangle is equilateral, then it is isosceles.” Decide whether each statement is true. Construct and label an exact counterexample to any false statement; verify its side lengths rather than relying on its appearance.

Teacher solution and discussion

The original statement is true because three equal sides include at least two equal sides. Its converse is “If a triangle is isosceles, then it is equilateral,” which is false. For example, take $A(0,3)$, $B(-2,0)$, $C(2,0)$. $AB=AC=\sqrt{13}$, whereas $BC=4$, so the triangle is isosceles but not equilateral. Its area is 6, so it is nondegenerate. The contrapositive is “If a triangle is not isosceles, then it is not equilateral.” It is true: having no equal side pair rules out having three equal sides. A false converse does not make the logically equivalent contrapositive false.

Evidence and response

State both transformations of the conditional accurately, evaluate them independently and supply a constructed, length-checked counterexample only to the false converse. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Use definitions, congruence, angle relationships, coordinate evidence and exact constructions to prove statements about triangles and lines.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model a proof as given facts, justified bridge and conclusion. For O05 draw the auxiliary parallel in the exterior angle and name both angle pairs. O06 proves the converse, so do not cite the converse or assume a midpoint. In O07 physically rotate tracing paper about the shared-side midpoint before generalizing the dissection. O08 needs two different SSS correspondences for the two diagonals. Keep every construction or auxiliary segment visible in the completed teacher model.

Core task

Prove the exterior-angle theorem without citing it. In any nondegenerate triangle ABC, extend BC beyond C to D. Through C construct a line parallel to AB. Choose ray CE on that line inside exterior angle ACD. Make a labeled diagram, identify the two parallel-line angle equalities, and prove $m(\angle ACD) = m(\angle CAB) + m(\angle ABC)$.

Teacher solution and discussion

Draw CE parallel to AB with CE directed toward the A side of the construction, so it lies inside angle ACD. With AC as transversal, angle ECA equals angle CAB by alternate interior angles. With BC and its extension CD as transversal, angle DCE equals angle ABC by corresponding angles. Ray CE splits the exterior angle into adjacent angles, so $m(\angle ACD) = m(\angle DCE) + m(\angle ECA) = m(\angle ABC) + m(\angle CAB)$. This uses parallel-line properties and angle addition, not the theorem being proved. The completed teacher diagram shows one orientation; rotations or reflections of it preserve the argument.

Evidence and response

Add the specified auxiliary parallel, name both valid angle pairs and complete the angle-addition chain without circularly invoking the exterior-angle theorem. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Establish and apply similarity, proportional segments and area scaling, keeping correspondence and geometric feasibility explicit.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

In M15 pause before any proportion: the side lengths fail triangle existence. In O09 justify AA before enlarging the survey model. O10 compares the same two areas using different bases; label each perpendicular height so side distance is not confused with vertex distance. O11 deliberately withholds parallelism, so using similarity at the outset is circular. O12 checks both a rigid-motion mapping and exact side correspondence. For O13 distinguish n interior-point triangles from $n-2$ vertex-fan triangles and account for the central full turn.

Core task

A survey models an inaccessible segment BC as one side of triangle ABC. Points D and E lie on AB and AC, respectively, and a survey establishes DE parallel to BC. Measurements on accessible segments are $AD=12$ m, $DB=8$ m, $AE=15$ m and $DE=9$ m. Draw a complete labeled model, prove the relevant triangles similar, and determine BC, AC and EC. Explain why a visual impression that DE is nearly parallel would not be an exact substitute for the stated guarantee.

Teacher solution and discussion

Parallelism gives $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$, while angle A is shared, so ADE is similar to ABC by AA. $AB = AD + DB = 20$ m, giving enlargement factor $AB/AD = 20/12 = 5/3$. Therefore $BC = DE(5/3) = 15$ m and $AC = AE(5/3) = 25$ m. Since E lies on AC, $EC = AC - AE = 25 - 15 = 10$ m. The completed triangle can be drawn with $A(0,0)$, $B(20,0)$, $C(20,15)$, $D(12,0)$, $E(12,9)$. These coordinates are one consistent realization, not an extra premise. Approximate parallelism would yield approximate similarity, with measurement uncertainty, rather than these exact deductions.

Evidence and response

Build the nested-triangle model, establish AA from the given parallelism, calculate all three requested lengths and distinguish an exact survey premise from appearance. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Use inclusive quadrilateral definitions and justified side, angle and diagonal properties to classify figures and prove conditional conclusions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign premise-checker, constructor, proof-writer and skeptic roles after individual attempts. O14 requires a necessary-and-sufficient parameter condition, not one special kite. O15 has a numerical area/perimeter coincidence that should prompt a unit check. In O16 keep the construction arcs and prove the altitude rather than merely drawing it upright. O17 must not import parallelogram properties into the original arbitrary quadrilateral. O18 requires a true angle-matched counterexample and a corresponding-side repair, not a vague request for more information.

Core task

Consider $A(0,a)$, $B(-b,0)$, $C(0,-c)$, $D(b,0)$, where a,b,c are positive. Prove that ABCD is a convex kite and that AC is the perpendicular bisector of BD. Determine exactly when BD also bisects AC and when the kite is a rhombus. Construct the example $a=5$, $b=3$, $c=2$ to show which bisection fails in general.

Teacher solution and discussion

$AB^2=AD^2=a^2+b^2$, and $CB^2=CD^2=c^2+b^2$, so there are two congruent adjacent side pairs. Positive a,b,c place one vertex on each coordinate-axis ray in boundary order, making a convex, nondegenerate kite. AC lies on $x=0$ and BD on $y=0$, hence they are perpendicular. BD has midpoint $(0,0)$, so AC bisects it. AC has midpoint $(0,(a-c)/2)$, which lies on BD exactly when $a=c$. All four sides are equal exactly when $a^2+b^2=c^2+b^2$; positivity reduces this to $a=c$. Thus the second bisection and rhombus conditions are equivalent in this family. With $a=5,b=3,c=2$, the diagonal crossing $E=(0,0)$ gives $AE=5$ and $EC=2$, so BD does not bisect AC. Side pairs are $\sqrt{34}$ and $\sqrt{13}$, also confirming the nonsquare, nonrhombus kite.

Evidence and response

Prove the coordinate family properties for all positive parameters, establish necessity and sufficiency of $a=c$ and construct the unequal-half counterexample. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Use inclusive quadrilateral definitions and justified side, angle and diagonal properties to classify figures and prove conditional conclusions.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 demands both parallel-pair checks before classifying the exact trapezoid. O20 requires the geometric domain before choosing an algebraic root. O21 proves a global maximum by an inequality and an attainable equality case. O22 determines C but not A and B individually; require two valid pairs. In O23 ask learners to articulate both directions of the equivalence and use side multisets to reject congruence under every possible correspondence. Do not reveal the decisive theorem before collecting independent reasoning.

Core task

Classify the quadrilateral with consecutive vertices $A(0,0)$, $B(10,0)$, $C(7,4)$, $D(3,4)$, using the convention that a trapezoid has exactly one pair of parallel sides. Supply an exact certificate: both opposite-side direction checks, the two leg lengths, both diagonal lengths, the area and the perimeter. Draw the completed figure and explain why equal diagonals do not make this figure a rectangle.

Teacher solution and discussion

AB and CD are horizontal, so AB is parallel to CD . AD has slope $4/3$ and BC has slope $-4/3$; these nonparallel legs show there is exactly one parallel opposite pair. $AD=BC=\sqrt{3^2+4^2}=5$, so $ABCD$ is an isosceles trapezoid. The diagonals have squared lengths $AC^2=7^2+4^2=65$ and $BD^2=(-7)^2+4^2=65$, so both are $\sqrt{65}$. The bases have lengths 10 and 4 with perpendicular height 4, giving $\text{area}=(10+4)\cdot 4/2=28$ square units. $\text{Perimeter}=10+5+4+5=24$ units. It is not a rectangle: the second opposite pair is not parallel, and AB is not perpendicular to AD . Equal diagonals alone do not supply either missing condition.

Evidence and response

Build a complete exact classification certificate, check both parallel pairs rather than one and reject the rectangle conclusion despite genuine diagonal equality. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Use definitions, congruence, angle relationships, coordinate evidence and exact constructions to prove statements about triangles and lines.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose the demonstrated gap. O24 replaces a visual vertical-angle claim with common supplements. O25 makes the radius restriction and off-line intersections part of the perpendicular construction proof. O26 separates equilateral from equiangular with a verified rhombus. O27 recovers vertex order from incident side pairs before writing any corresponding angle. Pair each repair with a fresh unsupported attempt; copied completed diagrams do not by themselves demonstrate transfer.

Core task

Two distinct lines AB and CD intersect at O , with A,O,B in that order and C,O,D in that order. A student says, “Angle AOC and angle BOD are congruent because they look like mirror images.” Replace that claim with a proof using linear pairs and subtraction. Construct a labeled crossing and also explain why the adjacent angles need not be congruent.

Teacher solution and discussion

Angles AOC and COB form a linear pair, so their measures sum to 180 degrees. Angles BOD and COB also form a linear pair, because OC and OD are opposite rays, so their measures also sum to 180 degrees. Subtract the common measure of COB from both equations to obtain $AOC = BOD$. This proves vertical-angle congruence without trusting the drawing. Adjacent angles are supplementary, not necessarily equal: for example, a 70-degree angle has a 110-degree neighbor. They become congruent only when both are 90 degrees. The labeled key is one generic crossing; its exact drawn angles are not assumed to be the 70/110 example.

Evidence and response

Replace visual evidence with two named linear-pair equations and a common-subtraction argument, and distinguish vertical equality from adjacent supplementation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Use inclusive quadrilateral definitions and justified side, angle and diagonal properties to classify figures and prove conditional conclusions.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 proves the one guaranteed kite-angle pair and disproves the stronger opposite-pair claim. O29 integrates a constructed square, AA similarity, feasibility and a line-incidence check. O30 requires all requested measures and a boundary argument for the shared edge. Collect completed diagrams, theorem reasons and exact expressions before showing keys. Score the diagram, the justified mathematical bridge and the conclusion separately so a correct number does not conceal a missing proof.

Core task

A convex kite ABCD has $AB=AD$ and $BC=CD$. Prove that its opposite angles at B and D are congruent using one diagonal and a correctly ordered triangle-congruence statement. Then construct one kite for which the angles at A and C are not congruent; verify that failure with exact side or triangle facts rather than a protractor reading.

Teacher solution and discussion

Draw AC. Triangles ABC and ADC have $AB=AD$, $BC=DC$ and shared AC, so SSS gives triangle ABC congruent to triangle ADC with $B \leftrightarrow D$. Hence angle ABC = angle ADC. For a counterexample to equality of the other opposite angles, use $A(0,5)$, $B(-3,0)$, $C(0,-2)$, $D(3,0)$. The kite conditions hold because $AB=AD=\sqrt{34}$ and $CB=CD=\sqrt{13}$. Triangle ABD is isosceles with vertex angle A, while CBD is isosceles with vertex angle C, and both have base $BD=6$. If their vertex angles were equal, their base angles would also match, so the triangles would be similar by AA. Their corresponding bases are equal and positive, forcing scale 1 and $AB=CB$, contradicting $\sqrt{34} \neq \sqrt{13}$. Thus angles A and C differ even though B and D are congruent.

Evidence and response

Prove the correct opposite-angle pair by SSS and construct an exact counterexample to the unproved other pair with a nonmeasurement argument. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

In a nondegenerate triangle ABC, point D lies on segment BC and $BD=DC$. Which conclusion follows from the definition, without assuming any additional symmetry?

- A. $AB=AC$.
- B. AD is perpendicular to BC.
- C. AD bisects angle BAC.
- D. AD is a median of triangle ABC.

Reasoning: _____

M02 | Warm-up | T3

In triangle PQR, side QR is extended beyond R to S. The two remote interior angles at P and Q measure 37 degrees and 68 degrees. What is $m(\angle PRS)$?

- A. 31 degrees
- B. 75 degrees
- C. 105 degrees
- D. 143 degrees

Reasoning: _____

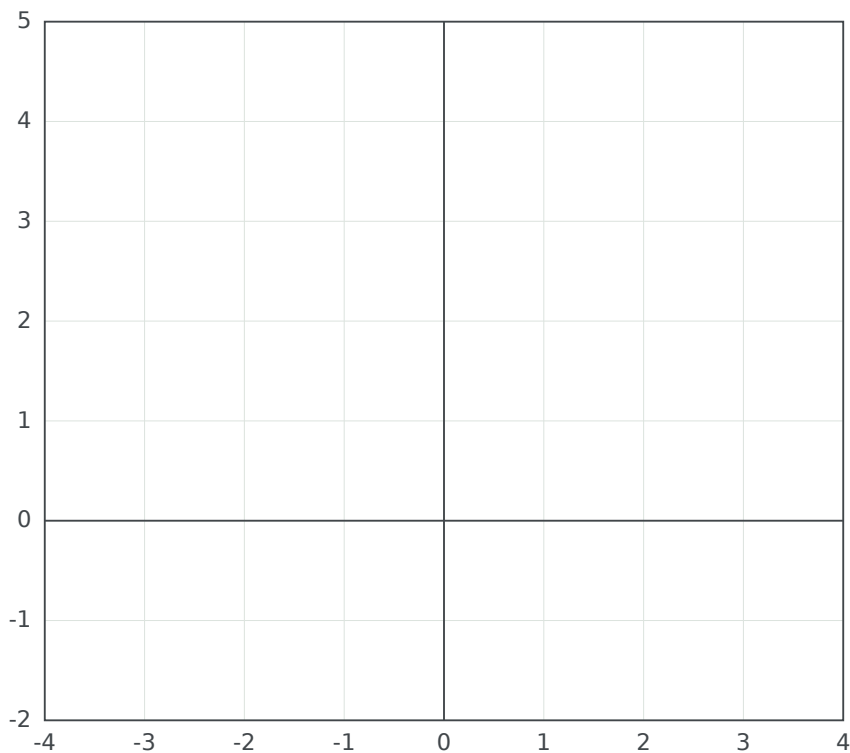
WARM-UP

001 • Show your reasoning

T5: Evaluate geometric implications and their converses, distinguish sufficient evidence from appearance, and construct exact counterexamples.

Use the inclusive definition that an isosceles triangle has at least two congruent sides. State the converse and contrapositive of “If a triangle is equilateral, then it is isosceles.” Decide whether each statement is true. Construct and label an exact counterexample to any false statement; verify its side lengths rather than relying on its appearance.

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

T3: Derive and apply polygon angle, perimeter and area relationships through decomposition, auxiliary lines, counting and algebra.

In triangle ABC, angle A=50 degrees and angle B=60 degrees. Extend AB beyond B to D. The internal bisector of exterior angle CBD meets ray AC beyond C at E. Determine angle CBD, angle CBE and angle BEC. Explain why angle BCE is not the original interior angle BCA.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Use definitions, congruence, angle relationships, coordinate evidence and exact constructions to prove statements about triangles and lines.

Answer: D. AD is a median of triangle ABC.

D is the midpoint of BC because it lies on that segment and divides it into equal lengths. A segment from a vertex to the midpoint of the opposite side is a median, so AD is a median. No angle or side symmetry was given.

Check each alternative

- A. D can be the midpoint of BC in any triangle, including a scalene triangle.
- B. An arbitrary median need not be an altitude; the equal halves alone do not give a right angle.
- C. A median need not bisect the opposite vertex angle in a scalene triangle.
- D. Correct. D is the midpoint of BC because it lies on that segment and divides it into equal lengths. A segment from a vertex to the midpoint of the opposite side is a median, so AD is a median. No angle or side symmetry was given.

Evidence to assess

Use both betweenness and equal lengths to identify a midpoint, then apply the median definition without adding symmetry.

WORKED TEACHER KEYS

M02

M02 • T3: Derive and apply polygon angle, perimeter and area relationships through decomposition, auxiliary lines, counting and algebra.

Answer: C. 105 degrees

The triangle angle sum gives angle $PRQ = 180 - 37 - 68 = 75$ degrees. Rays RQ and RS are opposite, so PRQ and PRS form a linear pair. Thus $PRS = 180 - 75 = 105$ degrees, equal to $37 + 68$.

Check each alternative

- A. Subtracting the remote angles is not the exterior-angle relationship.
- B. Seventy-five degrees is the interior angle PRQ , not the exterior angle PRS .
- C. Correct. The triangle angle sum gives angle $PRQ = 180 - 37 - 68 = 75$ degrees. Rays RQ and RS are opposite, so PRQ and PRS form a linear pair. Thus $PRS = 180 - 75 = 105$ degrees, equal to $37 + 68$.
- D. This is the supplement of 37 only; the second remote angle must also be included.

Evidence to assess

Distinguish the exterior angle from the adjacent interior angle and justify the answer using a linear pair and triangle sum.

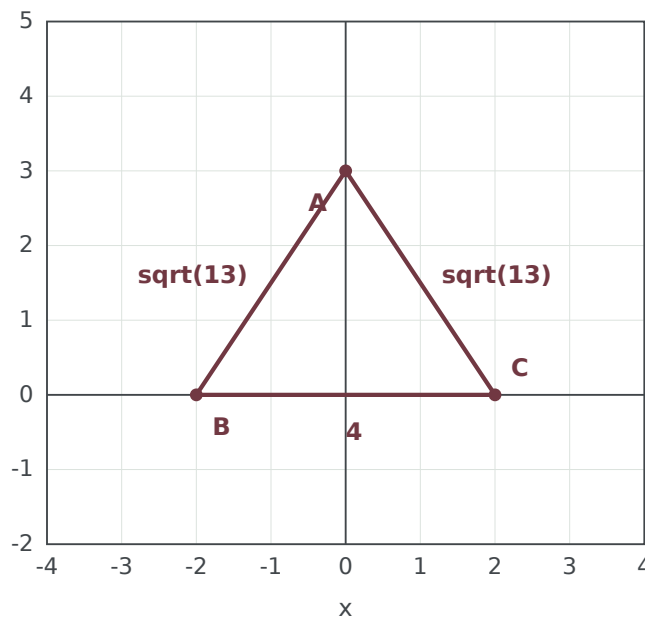
WORKED TEACHER KEYS

001

001 • T5: Evaluate geometric implications and their converses, distinguish sufficient evidence from appearance, and construct exact counterexamples.

The original statement is true because three equal sides include at least two equal sides. Its converse is “If a triangle is isosceles, then it is equilateral,” which is false. For example, take $A(0,3)$, $B(-2,0)$, $C(2,0)$. $AB=AC=\sqrt{13}$, whereas $BC=4$, so the triangle is isosceles but not equilateral. Its area is 6, so it is nondegenerate. The contrapositive is “If a triangle is not isosceles, then it is not equilateral.” It is true: having no equal side pair rules out having three equal sides. A false converse does not make the logically equivalent contrapositive false.

y

**Evidence to assess**

State both transformations of the conditional accurately, evaluate them independently and supply a constructed, length-checked counterexample only to the false converse.

WORKED TEACHER KEYS

002 / 003**002 • T3: Derive and apply polygon angle, perimeter and area relationships through decomposition, auxiliary lines, counting and algebra.**

The original angle C is $180 - 50 - 60 = 70$ degrees. Since BA and BD are opposite rays, exterior angle $CBD = 180 - 60 = 120$ degrees. Its internal bisector gives $CBE = 60$ degrees. E is beyond C on ray AC, so CA and CE are opposite rays. Thus BCE is the supplement of BCA: $180 - 70 = 110$ degrees, not 70. In triangle BCE, angle $BEC = 180 - 60 - 110 = 10$ degrees. All named ray directions matter: using the old angle C would incorrectly give 50 degrees at E.

Evidence to assess

Follow both extensions, obtain each requested angle and explain the changed ray at C before applying the second triangle sum.

003 • T1: Use definitions, congruence, angle relationships, coordinate evidence and exact constructions to prove statements about triangles and lines.

Positive side lengths require $x > 0$ (which also makes $x+1$ positive). The three strict triangle inequalities are $(x+1)+2x > 10$, giving $x > 3$; $(x+1)+10 > 2x$, giving $x < 11$; and $2x+10 > x+1$, giving $x > -9$. Combining all requirements gives $3 < x < 11$. At $x=5$ the sides are 6,10,10, and all three inequalities hold. At $x=3$ the sides are 4,6,10 with $4+6=10$; at $x=11$ they are 12,22,10 with $12+10=22$. Both endpoint figures collapse to a line and have zero area, so neither endpoint is included.

Evidence to assess

Intersect positivity with all strict triangle inequalities, demonstrate existence at an interior value and identify the distinct degeneracy at each endpoint.