

TEACHER AND STUDENT RESOURCE

Rigid Motions and Nonrigid Transformations



NC Math 2 • NC.M2.G-CO.2

Congruence

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.G-CO.2 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Experiment with transformations in the plane.

- Represent transformations in the plane.
- Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations).
- Understand that rigid motions produce congruent figures while dilations produce similar figures.

T1 Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

T2 Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

T3 Distinguish congruence and similarity using uniform length ratios, angle preservation and nondegenerate-figure conditions.

T4 Justify classifications with general-coordinate arguments, purposeful experiments, counterexamples and explicit model assumptions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M03, M10, M15, M16, O16, M21, O21, M24, O24

T2: M02, O02, O03, M05, O05, M06, O06, M09, M13, O13, M20, O20, M26, O26

T3: M04, O04, M07, O07, M11, M12, O12, M17, O17, M19, O19, M22, M25, O25, M27, O27, M28, O28

T4: M08, O08, O09, O10, O11, M14, O14, O15, M18, O18, O22, M23, O23, M29, O29, M30, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Learning target: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Learning target: Distinguish congruence and similarity using uniform length ratios, angle preservation and nondegenerate-figure conditions.

Learning target: Justify classifications with general-coordinate arguments, purposeful experiments, counterexamples and explicit model assumptions.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Learning target: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Learning target: Distinguish congruence and similarity using uniform length ratios, angle preservation and nondegenerate-figure conditions.

Learning target: Justify classifications with general-coordinate arguments, purposeful experiments, counterexamples and explicit model assumptions.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Coordinate pairs, distance and midpoint formulas, slope-based perpendicularity or an equivalent right-angle test, transformations of points, ratio reasoning and basic triangle congruence/similarity. Dot products appear only as optional exact perpendicularity checks; slope or Pythagorean checks are equally valid.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All coordinate diagrams use equal unit scales. Origin-centered rotations specify angle and direction. All dilation scale factors in this packet are positive; an unstated center is never assumed. "Rigid" means distance and angle preservation for every plane figure, not only a tested subset. A similarity may have scale 1, so congruent figures are also similar. Unless explicitly discussed otherwise, triangles are nondegenerate and polygons are bounded. Sketches are models, not physical measuring templates.

TEACHER MODEL

Uniform scaling is not the same as rigidity

Compare $T(x, y) = (x+3, y+1)$, $D(x, y) = (2x, 2y)$, and $S(x, y) = (2x, y)$ on $O = (0, 0)$, $A = (2, 0)$, $B = (0, 1)$. What measurements separate the three maps?

T gives $(3, 1)$, $(5, 1)$, $(3, 2)$. Its side lengths remain $2, 1, \sqrt{5}$, so it is rigid and the triangles are congruent. D gives $(0, 0)$, $(4, 0)$, $(0, 2)$, with lengths $4, 2, 2\sqrt{5}$; every ratio is 2, so the triangles are similar but not congruent. S gives $(0, 0)$, $(4, 0)$, $(0, 1)$, with lengths $4, 1, \sqrt{17}$; ratios $2, 1, \sqrt{17}/\sqrt{5}$ differ, so these triangles are not similar. All three retain the right angle at the image of O . Checking only that angle would miss the difference. Rigidity requires preservation of every distance and angle, not a few convenient measurements.

VISUAL ANCHOR

Test distance and angle separately

A successful check on one special figure does not prove a plane-wide preservation claim. Use exact measurements and the stated transformation family.

Map type	Distance behavior	Angle behavior
Translation, reflection, rotation	Every distance unchanged	Every angle unchanged
Positive dilation k	Every distance multiplied by k	Every angle unchanged
Nonuniform axis stretch	Scale depends on direction	Some angles change
Identity dilation $k=1$	Every distance unchanged	Every angle unchanged

Rigid motions produce congruent figures. Positive dilations produce similar figures; for $k=1$ they also preserve size. A particular nonrigid map can send a special figure to a congruent image set, so distinguish a figure relationship from the behavior of the specified point correspondence.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Have learners calculate a complete image and at least two measurements before naming a category. Keep the word “every” visible when discussing rigidity.

Core task

Translate triangle $A=(0, 0)$, $B=(3, 0)$, $C=(0, 2)$ by $(2, 1)$. List and sketch both triangles. Compare AB with $A'B'$ and AC with $A'C'$. State whether the figures are congruent and explain why.

Teacher solution and discussion

$A'=(2, 1)$, $B'=(5, 1)$, $C'=(2, 3)$. $AB=A'B'=3$ and $AC=A'C'=2$. Every point has the same displacement $(2, 1)$. Translation preserves all distances and angles, so the triangles are congruent, not just two equal-length segments. Burgundy is the original; green is the image.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model the comparison table. Contrast an axis-aligned rectangle, whose right angles survive a stretch, with oblique perpendicular directions, whose angle changes.

Core task

Make a comparison table for translation $T(x, y) = (x+2, y-1)$, reflection $F(x, y) = (-x, y)$, dilation $D(x, y) = (3x, 3y)$, and stretch $S(x, y) = (2x, y)$. For each, say whether it preserves all distances, all angles, and congruence for every figure. Give a specific counterexample for each “no.”

Teacher solution and discussion

T and F preserve all distances and angles, so each always produces a congruent figure. D triples the distance from $(0, 0)$ to $(1, 0)$, so neither all distances nor congruence is preserved, but all angles are. S doubles that same horizontal distance. It also changes the angle between $(1, 1)$ and $(1, -1)$: originally 90° , while image vectors $(2, 1)$, $(2, -1)$ have dot product 3, so are not perpendicular. Thus S fails all three all-figure claims.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Justify classifications with general-coordinate arguments, purposeful experiments, counterexamples and explicit model assumptions.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Ask learners to cancel common coordinate shifts or square sign changes. Separate a general proof from an experiment on selected points.

Core task

Prove that reflection $F(x, y) = (4 - x, y)$, across $x = 2$, preserves distance between arbitrary points $P = (p, q)$ and $Q = (r, s)$. Find the fixed points and explain why a point need not be fixed for distances to be preserved.

Teacher solution and discussion

The image difference is $((4 - r) - (4 - p), s - q) = (p - r, s - q)$. Its squared length is $(p - r)^2 + (s - q)^2 = (r - p)^2 + (s - q)^2$, the original squared distance. Fixed points satisfy $4 - x = x$, so $x = 2$, with any y . For example $(0, 0)$ moves to $(4, 0)$, but its distances to other simultaneously reflected points are preserved. Preserving pairwise distances is not the same as leaving each point in place.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Justify classifications with general-coordinate arguments, purposeful experiments, counterexamples and explicit model assumptions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign rule builder, length checker, angle checker and counterexample checker. Require each proposed conclusion to state whether it is about one figure or the whole plane.

Core task

$A=(0, 0)$, $B=(3, 0)$, $C=(0, 4)$ are mapped to $A'=(0, 0)$, $B'=(3, 0)$, $C'=(0, 2)$. A student claims the triangles are congruent because $AB=A'B'$. Refute the claim with two additional side comparisons. Give a coordinate transformation that realizes all three listed correspondences.

Teacher solution and discussion

$AB=A'B'=3$, but $AC=4$ while $A'C'=2$. Also $BC=5$ while $B'C'=\sqrt{13}$. Therefore the triangles are not congruent. The map $(x, y) \rightarrow (x, y/2)$ realizes every pair; it is a vertical compression. Its length factors depend on direction, so it is not rigid and these triangles are not similar either.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Distinguish congruence and similarity using uniform length ratios, angle preservation and nondegenerate-figure conditions.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect exact computations and explanations before giving hints. Include nonorigin centers, compositions and the special case of a nonrigid map with a congruent image set.

Core task

A 2-by-5 rectangle is dilated by $\frac{3}{2}$. Compute the new side lengths, both perimeters and both areas. Explain why a change in area is consistent with preserved angles, and decide congruence and similarity.

Teacher solution and discussion

New sides are 3 and $\frac{15}{2}$. The original perimeter is $2(2+5)=14$, and image perimeter is $2(3+\frac{15}{2})=21$, a factor of $\frac{3}{2}$. Original area is 10; image area is $3 \cdot \frac{15}{2} = \frac{45}{2}$, a factor of $\frac{9}{4}$. Both rectangles have right angles. Dilation preserves angles while changing lengths and area, so the rectangles are similar but not congruent.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use one direction-sensitive point, one horizontal segment and one vertical segment. Repair the evidenced misconception, then test a fresh figure without reusing the cue.

Core task

A learner rotates triangle $A=(1, 0)$, $B=(3, 0)$, $C=(1, 2)$ clockwise by 90° but reports $A'=(0, 1)$, $B'=(0, 3)$, $C'=(-2, 1)$. Identify the error, give all corrected images and explain why unchanged lengths alone would not catch the direction error.

Teacher solution and discussion

The listed points use $(-y, x)$, the counterclockwise rule. Clockwise rotation uses $(y, -x)$, giving $A'=(0, -1)$, $B'=(0, -3)$, $C'=(2, -1)$. Both rotations preserve all lengths, so length checking confirms rigidity but cannot distinguish the intended direction. A direction-sensitive point such as $(1, 0)$ is needed.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Distinguish congruence and similarity using uniform length ratios, angle preservation and nondegenerate-figure conditions.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Ask for a complete transformation classification with measurements and a statement of what the evidence cannot establish. Do not treat equal angles as proof of equal size.

Core task

Apply $F(x, y) = (-2y, 2x)$ to $A = (1, 0)$, $B = (3, 0)$, $C = (1, 1)$. Give all images and sketch both triangles. Decompose F into familiar transformations. Compute all original and image side lengths, then decide whether any rigid motion can match these triangles.

Teacher solution and discussion

$A' = (0, 2)$, $B' = (0, 6)$, $C' = (-2, 2)$. F is a 90° counterclockwise rotation about the origin followed by dilation with $k=2$. Original sides are $AB=2$, $AC=1$, $BC=\sqrt{5}$; image sides are 4, 2, $2\sqrt{5}$. The image perimeter is twice the positive original perimeter, so no rigid motion can match the two triangles, even under different vertex labeling. They are similar with scale 2.

Evidence and response

Assess the rule and complete correspondence, measurement evidence, global versus special-case reasoning, and conclusion separately. A correct final category is insufficient if the student tested only one convenient side or angle. Ask which quantified claim is proved or refuted. Accept exact slope, distance, Pythagorean or dot-product checks when valid; do not require a particular representation after the mathematics is established.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A translation moves every point 4 units left and 2 units up. What is the image of $P=(3, -5)$?

- A. $(-1, -3)$
- B. $(-4, 2)$
- C. $(-1, -7)$
- D. $(7, -7)$

Reasoning: _____

M02 | Warm-up | T2

Which property distinguishes a rigid motion from dilation with scale factor 3?

- A. A rigid motion always preserves orientation; the dilation reverses it.
- B. A rigid motion changes area; the dilation preserves area.
- C. A rigid motion preserves every distance; the dilation triples every nonzero distance.
- D. A rigid motion changes all angles; the dilation preserves them.

Reasoning: _____

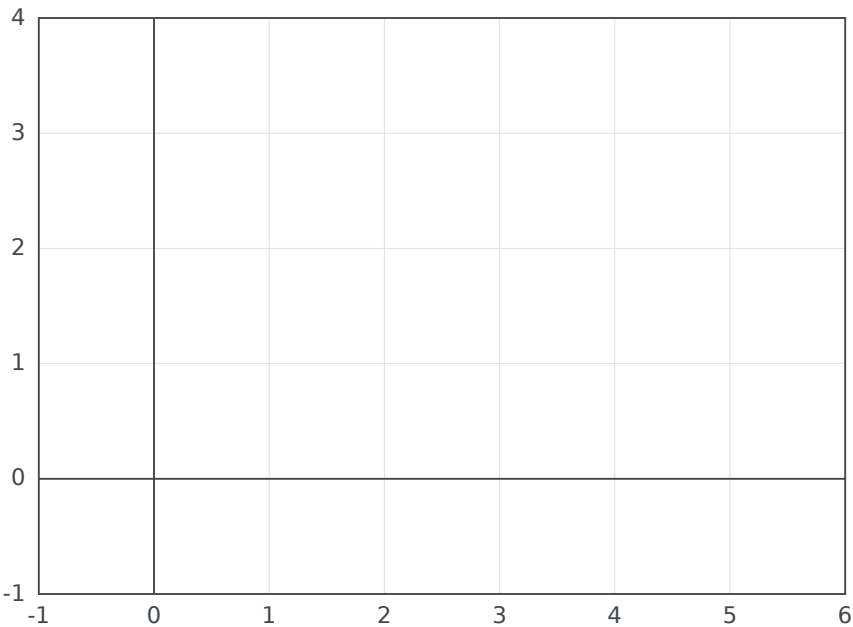
WARM-UP

001 • Show your reasoning

T1: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Translate triangle $A=(0, 0)$, $B=(3, 0)$, $C=(0, 2)$ by $(2, 1)$. List and sketch both triangles. Compare AB with $A'B'$ and AC with $A'C'$. State whether the figures are congruent and explain why.

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

T2: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Reflect $A=(-2, 1)$ and $B=(2, 4)$ across the y-axis. Find both images and the image of midpoint M . Compute AB and $A'B'$, then explain why reversing orientation does not mean changing shape.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

Answer: A. (-1, -3)

The rule is $(x, y) \rightarrow (x-4, y+2)$, so $P' = (3-4, -5+2) = (-1, -3)$.

Check each alternative

- A. The rule is $(x, y) \rightarrow (x-4, y+2)$, so $P' = (3-4, -5+2) = (-1, -3)$.
- B. This is the displacement vector, not the image of P.
- C. The horizontal change is correct but the vertical change should be +2.
- D. This reverses both parts of the translation.

Evidence to assess

Require the stated vector applied to both coordinates, not the vector itself as the image.

WORKED TEACHER KEYS

M02

M02 • T2: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

Answer: C. A rigid motion preserves every distance; the dilation triples every nonzero distance.

Translations, reflections and rotations preserve distance and angle measure. A positive dilation preserves angles but multiplies distances by 3, so it is not rigid.

Check each alternative

- A. Reflections reverse orientation, and a positive dilation does not.
- B. Rigid motions preserve area, while dilation by 3 multiplies it by 9.
- C. Translations, reflections and rotations preserve distance and angle measure. A positive dilation preserves angles but multiplies distances by 3, so it is not rigid.
- D. Rigid motions also preserve angle measures.

Evidence to assess

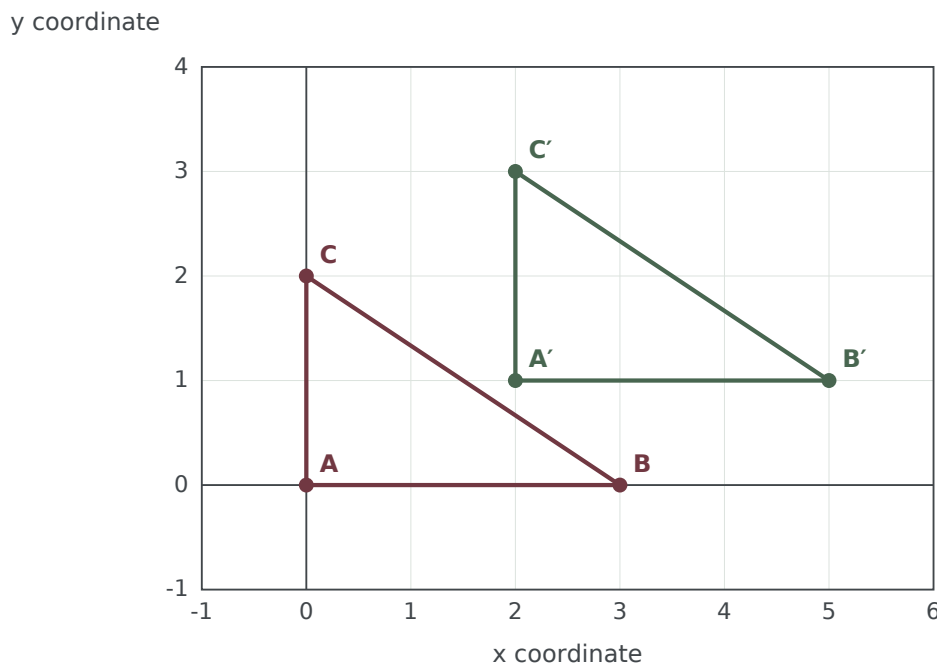
Require distance preservation as the deciding property while recognizing that both preserve angles.

WORKED TEACHER KEYS

001

001 • T1: Represent translations, reflections, rotations, dilations and stretches with coordinate rules, images and labeled equal-scale diagrams.

$A'=(2, 1)$, $B'=(5, 1)$, $C'=(2, 3)$. $AB=A'B'=3$ and $AC=A'C'=2$. Every point has the same displacement $(2, 1)$. Translation preserves all distances and angles, so the triangles are congruent, not just two equal-length segments. Burgundy is the original; green is the image.



Evidence to assess

Require all three images and labeled sketches, both length comparisons and the rigid-motion reason for congruence.

WORKED TEACHER KEYS

002

002 • T2: Compare preservation of distances and angle measures, including fixed-point and special-figure traps.

$A'=(2, 1)$, $B'=(-2, 4)$. $M=(0, 5/2)$ lies on the mirror and stays fixed. $AB=\sqrt{4^2+3^2}=5$; $A'B'=\sqrt{((-4)^2+3^2)}=5$. A reflection reverses clockwise/counterclockwise order for a noncollinear figure but preserves its distances and angles. Thus orientation reversal is compatible with congruence.

Point	Reflected image
$A=(-2, 1)$	$A'=(2, 1)$
$B=(2, 4)$	$B'=(-2, 4)$
$M=(0, 5/2)$	$M'=(0, 5/2)$

Evidence to assess

Require both images, fixed midpoint, two distance calculations and a distinction between orientation and shape.