

TEACHER AND STUDENT RESOURCE

Average Rates of Change



NC Math 1 • NC.M1.F-IF.6

Interpreting Functions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.F-IF.6 | NC Math 1 mathematics

This sample contains 20 complete selected pages from the 110-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 35-36, 65, 80-81

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 35

O02: sample PDF page(s) 18; original page(s) 36

Selected teacher prompts and keys: original page(s) 65, 80-81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

T1 Calculate and compare average rates using numerical data.

T2 Use graph endpoints and secants to calculate and explain average rates.

T3 Evaluate symbolic functions to calculate interval rates.

T4 Interpret signed rates, units, and the limits of endpoint evidence.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M04, M05, M18, M22, M27, O01, O02, O03, O04, O05, O06, O22, O27

T2: M06, M07, M08, M09, M10, M21, M25, M30, O07, O08, O09, O10, O11, O12, O24, O28, O30

T3: M11, M12, M13, M14, M15, M19, M24, M28, O13, O14, O15, O16, O17, O18, O23, O26

T4: M16, M17, M20, M23, M26, M29, O19, O20, O21, O25, O29

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Evaluate functions, subtract signed quantities, interpret coordinates, and divide rational numbers. Distinguish output units from input units.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

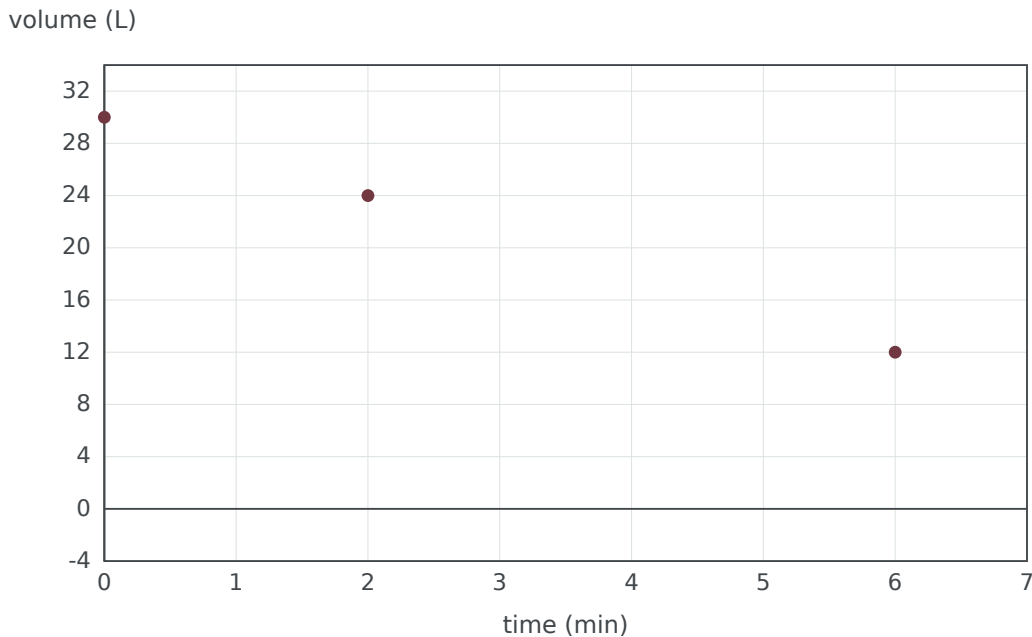
Review convention

Use exact fractions unless an approximation is explicitly requested. Average rate is $[f(b)-f(a)]/(b-a)$, where $a \neq b$. Graph segments shown as straight are piecewise-linear models; isolated table observations do not determine intermediate values.

TEACHER MODEL

A secant tells a net-change story

A tank has 30 L at minute 0, 24 L at minute 2, and 12 L at minute 6. Calculate the average rate over $[0,6]$ and compare the two shorter intervals.



Overall: $(12-30)/(6-0) = -18/6 = -3$ L/min. First interval: $(24-30)/2 = -3$; second: $(12-24)/4 = -3$. These observations lie on the same secant, $y = 30 - 3x$. The table alone does not prove that unobserved volume followed that line at every instant. Say: “Subtract ending minus starting output, then divide by the elapsed input. Keep the units with the quotient.”

VISUAL ANCHOR

Keep change, rate and value separate

Use consistent subtraction order and attach units.

Quantity	Computation	Meaning
Output change	$f(b) - f(a)$	Net amount; output units
Input change	$b - a$	Elapsed interval; input units
Average rate	Output change $\div$ input change	Net output per input unit
Secant slope	Rise $\div$ run	Slope through interval endpoints

A zero average rate means equal endpoint outputs. It does not mean that nothing happened between them.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Calculate and compare average rates using numerical data.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Museum visitors: x is hours; y is visitors.

Input x	1	2	4
Output y	25	42	70

Calculate and interpret the average rate on [1,4]. Plot the observations and draw the secant through the two interval endpoints. Explain why the rate does not determine every intermediate change.

Teacher solution and discussion

Rate = $(70 - (25)) / (4 - 1) = 15$ visitors per hours. The net change is 45 visitors over 3 hours. Plot the three listed observations; join only the two outer endpoints for the secant. Interior changes need not equal the average. Teacher plot shows the observations without asserting linear behavior between observations.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Calculate and compare average rates using numerical data.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Battery charge: x is hours; y is percent.

Input x	1	3	5
Output y	86	70	38

Calculate and interpret the average rate on [1,5]. Plot the observations and draw the secant through the two interval endpoints. Explain why the rate does not determine every intermediate change.

Teacher solution and discussion

Rate = $(38 - (86)) / (5 - 1) = -12$ percentage points per hour. The net change is -48 percentage points over 4 hours. Plot the three listed observations; join only the two outer endpoints for the secant. Interior changes need not equal the average. Teacher plot shows the observations without asserting linear behavior between observations.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Use graph endpoints and secants to calculate and explain average rates.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Read the endpoint values from the piecewise-linear graph. Calculate the average rate on $[0,3]$ and interpret it in L per min. Mark the two endpoint coordinates in your explanation and compare the secant rate with at least one individual segment rate.

Teacher solution and discussion

Endpoints: $(0,24)$, $(3,6)$; average = $(6 - (24)) / (3 - (0)) = -6$ L/min. The first segment rate is -6; the second is -1. The interval rate uses only its endpoints; it can differ from segment rates.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Evaluate symbolic functions to calculate interval rates.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

For $g(t)=5 \cdot 2^t$, calculate the average rate of change on $[1,3]$. Make an endpoint table, show substitution, and explain what the sign tells you about the net output change. State what it does not tell you about intermediate outputs.

Teacher solution and discussion

Substitute: $5 \cdot 2^1 = 10$; $5 \cdot 2^3 = 40$. Endpoint table: $x=1$, output=10; $x=3$, output=40. Difference quotient $= (40 - (10)) / (3 - (1)) = 15$. The net output change is 30 over 2 input units. A positive/negative/zero average means the ending output is above/below/equal to the starting output; it does not establish monotonicity within the interval.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Interpret signed rates, units, and the limits of endpoint evidence.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

A delivery van has position 12 km at 1 h and -3 km at 4 h. A learner says its average speed was -5 km/h. Calculate the average rate of position change, explain the wording error, and say whether its average speed is determined.

Teacher solution and discussion

$(-3-12)/(4-1)=-5$ km/h is average velocity (signed position change). Speed is total distance divided by time and cannot be negative. The distance could be 15 km or more if the van reversed direction; average speed is at least 5 km/h but is not uniquely determined.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Use graph endpoints and secants to calculate and explain average rates.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Construct two different piecewise-linear graphs on $0 \leq x \leq 6$, both beginning at (0,2) and ending at (6,14). Make one always increasing and one that decreases somewhere. Calculate both average rates and explain what your examples establish.

Teacher solution and discussion

Example A: join (0,2),(3,8),(6,14). Example B: join (0,2),(2,10),(4,6),(6,14). Both rates are $(14-2)/6=2$. B decreases on $[2,4]$, whereas A increases throughout. Thus equal average rates do not force equal intermediate behavior. Two teacher graphs appear on separate labeled panels.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Use graph endpoints and secants to calculate and explain average rates.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

A piecewise-linear graph joins $(0,6)$, $(2,2)$, $(5,11)$. Calculate the average rates on $[0,2]$, $[2,5]$, and $[0,5]$. Draw the overall secant on the blank grid and explain how the whole-interval rate relates to the two segment rates.

Teacher solution and discussion

Segment rates: $(2-6)/2=-2$ and $(11-2)/3=3$. Whole rate: $(11-6)/5=1$. The secant is $y=x+6$ from $(0,6)$ to $(5,11)$. Weighted check: $[2(-2)+3(3)]/5=1$. It lies between -2 and 3 .

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify

M01 | Warm-up | T1

Water volume: find the average rate of change from 0 to 4 min. Report output units per input unit.

Input x	0	2	4
Output y	36	29	20

x measures time (min); y measures volume (L).

- A. -4
- B. 9
- C. 4
- D. -16

Reasoning: _____

M02 | Warm-up | T1

Trail elevation: find the average rate of change from 2 to 6 km. Report output units per input unit.

Input x	2	3	6
Output y	140	155	188

x measures distance (km); y measures elevation (m).

- A. -12
- B. 35
- C. 48
- D. 12

Reasoning: _____

WARM-UP

001 • Show your reasoning

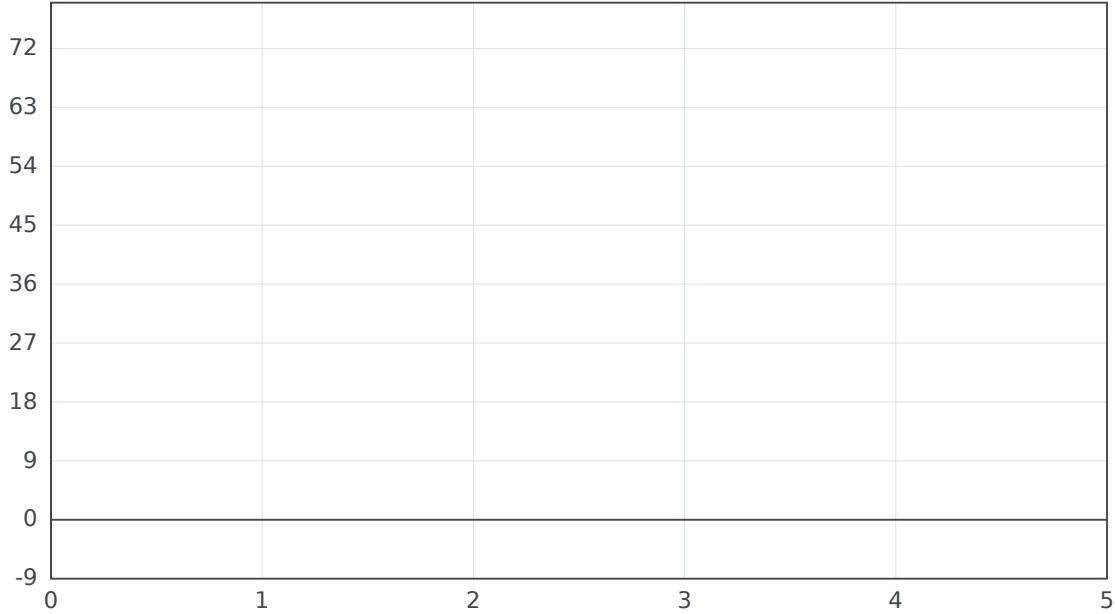
T1: Calculate and compare average rates using numerical data.

Museum visitors: x is hours; y is visitors.

Input x	1	2	4
Output y	25	42	70

Calculate and interpret the average rate on [1,4]. Plot the observations and draw the secant through the two interval endpoints. Explain why the rate does not determine every intermediate change.

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

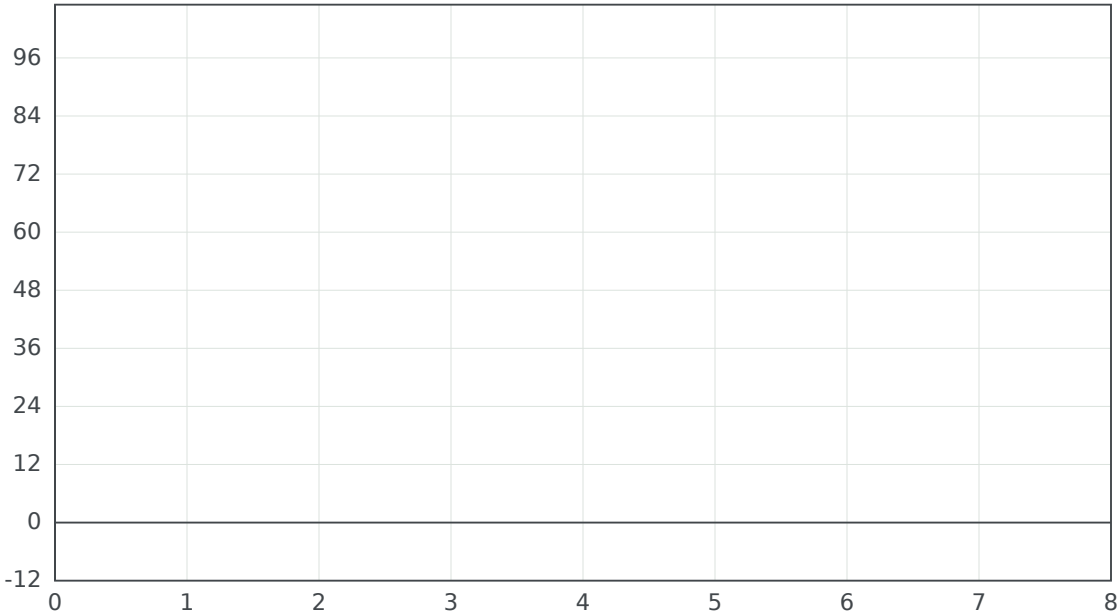
T1: Calculate and compare average rates using numerical data.

Ice mass: x is minutes; y is grams.

Input x	2	4	7
Output y	95	84	60

Calculate and interpret the average rate on [2,7]. Plot the observations and draw the secant through the two interval endpoints. Explain why the rate does not determine every intermediate change.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Calculate and compare average rates using numerical data.****Answer: A. -4**

The endpoints are (0, 36) and (4, 20). Average rate = $(20 - 36)/(4 - 0) = -16/4 = -4$. This is the net output change per input unit over this interval, not necessarily a constant rate within it.

Check each alternative

- A. Correct: divide the signed output change by the input change.
- B. This uses the starting output rather than the output change.
- C. This reverses only one subtraction, changing the sign.
- D. This is the output change only, not change per input unit.

M02 • T1: Calculate and compare average rates using numerical data.**Answer: D. 12**

The endpoints are (2, 140) and (6, 188). Average rate = $(188 - 140)/(6 - 2) = 48/4 = 12$. This is the net output change per input unit over this interval, not necessarily a constant rate within it.

Check each alternative

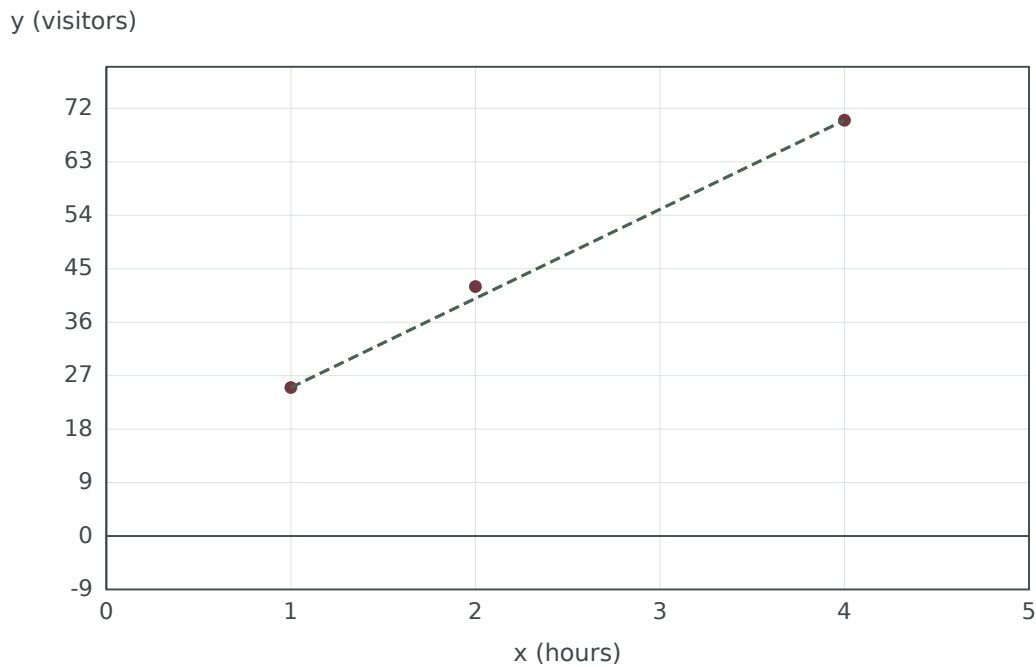
- A. This reverses only one subtraction, changing the sign.
- B. This uses the starting output rather than the output change.
- C. This is the output change only, not change per input unit.
- D. Correct: divide the signed output change by the input change.

WORKED TEACHER KEYS

001

001 • T1: Calculate and compare average rates using numerical data.

Rate = $(70 - (25)) / (4 - 1) = 15$ visitors per hours. The net change is 45 visitors over 3 hours. Plot the three listed observations; join only the two outer endpoints for the secant. Interior changes need not equal the average. Teacher plot shows the observations without asserting linear behavior between observations.

**Evidence to assess**

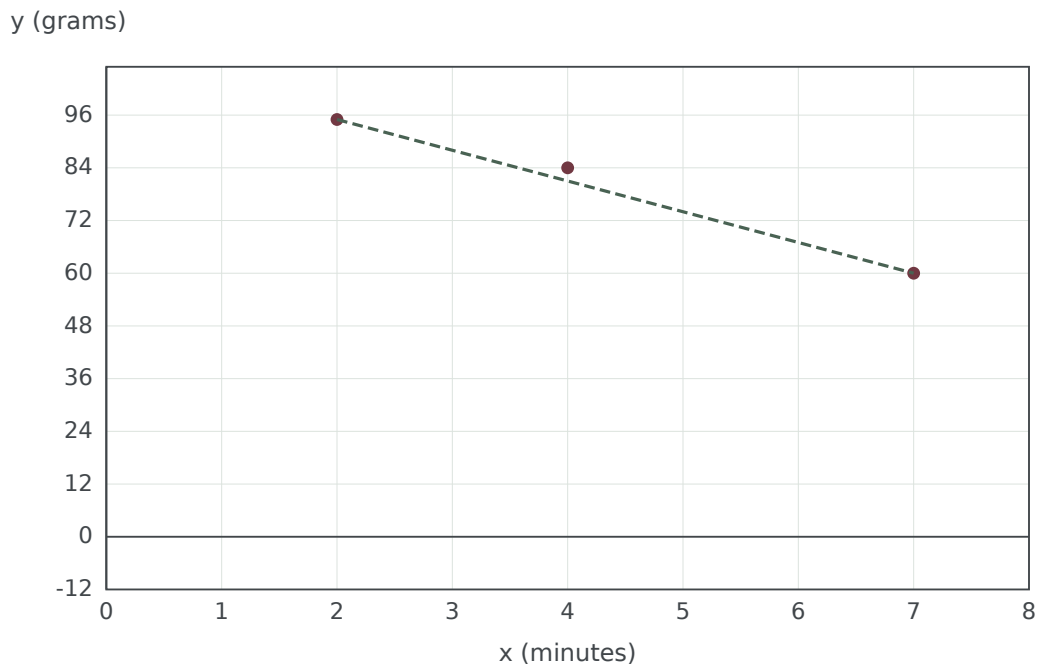
Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

002

002 • T1: Calculate and compare average rates using numerical data.

Rate = $(60 - 95)/(7 - 2) = -7$ grams per minutes. The net change is -35 grams over 5 minutes. Plot the three listed observations; join only the two outer endpoints for the secant. Interior changes need not equal the average. Teacher plot shows the observations without asserting linear behavior between observations.

**Evidence to assess**

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.