

TEACHER AND STUDENT RESOURCE

Investigating the Centers of Triangles



NC Math 3 • NC.M3.G-CO.10

Congruence

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-CO.10 | NC Math 3 mathematics

This sample contains 25 complete selected pages from the 139-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-17, 35-38, 76-77, 105-106

Representative activities

M01: sample PDF page(s) 18; original page(s) 17

M02: sample PDF page(s) 18; original page(s) 17

O01: sample PDF page(s) 19-20; original page(s) 35-36

O02: sample PDF page(s) 21-22; original page(s) 37-38

Selected teacher prompts and keys: original page(s) 76-77, 105-106

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Verify experimentally properties of the centers of triangles (centroid, incenter, and circumcenter).

T1 Construct and distinguish medians, internal angle bisectors and side perpendicular bisectors; check three-line concurrency.

T2 Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

T3 Experimentally verify incenter equality of perpendicular side distances, interior location and incircle tangency.

T4 Experimentally verify circumcenter equality of vertex distances, circumcircle construction and location across triangle types.

T5 Design, compare and critique center investigations using varied triangles, appropriate measurements and honest limits on experimental evidence.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M08, M18, M25, O01, O03, O27

T2: M05, M09, M12, M17, M19, M24, M28, O05, O09, O13, O15, O19, O24, O28

T3: M02, M06, M10, M20, M29, O02, O06, O10, O12, O16, O20, O25

T4: M03, M07, M11, M16, M21, M26, O07, O11, O17, O21, O23, O26

T5: M04, M13, M14, M15, M22, M23, M27, M30, O04, O08, O14, O18, O22, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Build the right lines and measurements

Learning target: Distinguish the three construction families and the distances they test.

Learning target: Construct a candidate center, then check the independent third line.

Use M01–M08 and O01–O08 selectively. Preserve a diagnostic drawing before modeling. Perform one center construction fully rather than attempting every bank item in one lesson.

Session 2: Investigate, compare and challenge

Learning target: Test center properties across different shapes, contexts and representations.

Learning target: Use measurements and counterexamples without confusing experimental support with proof.

Choose from M09–M23 and O09–O23 across multiple lessons. Have each learner predict before a group construction; record both raw measurements and the adopted tolerance.

Session 3: Repair and independently verify

Learning target: Repair the specific confusion among medians, side distances and vertex distances.

Learning target: Produce a fresh construction and a correctly qualified conclusion.

Select M24–M27/O24–O27 by observed need, then use M28–M30/O28–O30 for independent transfer. Keep teacher keys covered until student evidence is collected.

PLANNING

Materials, access and pacing

Materials

Ruler marked in millimeters, straightedge, compass, protractor, equal-scale graph paper, plain or tracing paper, pencil and eraser. For the optional balancing investigation use uniform cardstock, scissors, a small clip and a narrow rounded support. Dynamic geometry software is optional, not required.

Prerequisites

Identify vertices and opposite sides; locate a midpoint; construct or recognize perpendicular lines and internal angle bisectors; use a ruler, compass and protractor; plot coordinates on an equal-scale grid; apply the distance formula or Pythagorean theorem to check measured lengths; interpret ordered ratios.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

Review convention

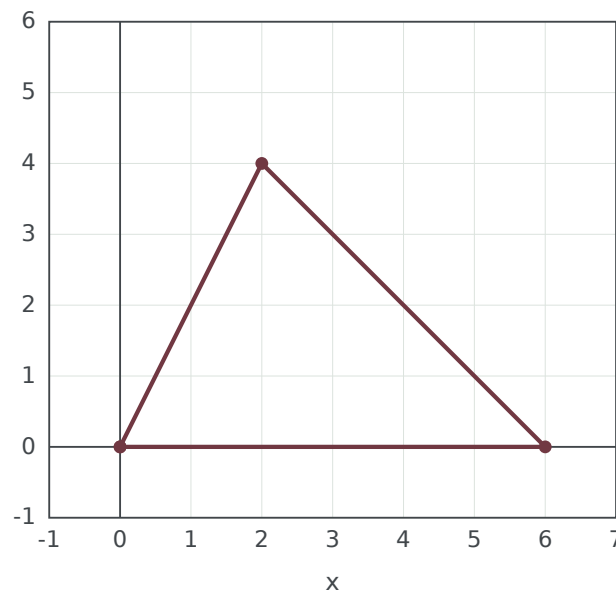
Use nondegenerate triangles. G denotes centroid, I incenter and U circumcenter. A median joins a vertex to the midpoint of the opposite side; an internal angle bisector splits a vertex angle equally; a side perpendicular bisector passes through its midpoint at 90° . Distance to a side means perpendicular distance to its supporting line; incenter tangent feet lie on the actual sides. A center being “inside” means strictly inside; “on” means on the boundary. Coordinate diagrams are schematic references, not physical-scale measuring templates: redraw with equal unit scales for measurement and circles. Measurement data and practical tolerances are instructional, not exact proofs. Keep raw readings; report approximate equality only within a declared tolerance. Worked exact coordinates check a construction but do not replace the requested experiment.

TEACHER MODEL

Build a circumcenter and decide what the measurements establish

For $A(0,0)$, $B(6,0)$, $C(2,4)$, construct all three side perpendicular bisectors. Locate U , compare UA , UB and UC , and state the strongest justified experimental conclusion.

y



First mark the midpoint $(3,0)$ of AB ; its perpendicular bisector is $x=3$. Next mark $(1,2)$, the midpoint of AC ; AC has slope 2, so its perpendicular bisector is $y-2=-(1/2)(x-1)$. The intersection is $U=(3,1)$. Independently construct the BC bisector through $(4,2)$ with slope 1: $y=x-2$; it passes through U . Each radius is $\sqrt{10} \approx 3.16$ units. Keep ruler measurements rather than replacing them with the exact value; agree on an appropriate tolerance before comparing them. A circle centered at U with this radius passes through all three vertices. The drawing supports concurrency and equal vertex distances for this triangle. Repeat with right and obtuse triangles to test location changes; the experiment itself is not a general proof.

VISUAL ANCHOR

Three constructions, three different checks

Centroid G: vertex $\rightarrow$ opposite-side midpoint; check $AG:GD=2:1$.

Incenter I: internal angle bisectors; check equal PERPENDICULAR side distances.

Circumcenter U: side midpoint + right angle; check equal VERTEX distances.

Construct two lines to locate a candidate; construct the third to check concurrency.

Measurements support a conjecture. A verified counterexample can refute an “always” claim.

Start with the question being asked. A balance point, a tangent-circle center and a through-vertex-circle center are usually different points.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Construct and distinguish medians, internal angle bisectors and side perpendicular bisectors; check three-line concurrency.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask learners to point to the opposite side before drawing a median. In O01, preserve the first two-line construction and ask what still has not been checked. Use O02 to compare a perpendicular segment with two slanted ones; the shortest segment, not the most convenient endpoint, measures side distance. O04 sets an explicit standard for recording raw data and qualifying claims.

Core task

On graph paper, draw $A(0,0)$, $B(8,0)$, $C(2,6)$. Find and label D , the midpoint of BC , and E , the midpoint of AC . Draw medians AD and BE . Give D and E , estimate the medians' intersection G , and explain why drawing only these two lines does not yet experimentally check all-three-median concurrency.

Teacher solution and discussion

$D = ((8+2)/2, (0+6)/2) = (5,3)$ and $E = ((0+2)/2, (0+6)/2) = (1,3)$. Draw AD from $(0,0)$ to $(5,3)$ and BE from $(8,0)$ to $(1,3)$. AD has $y=3x/5$; BE has $y=-(3/7)(x-8)$. Equating gives $21x=120-15x$, so $x=10/3$ and $y=2$; a careful graph gives $G \approx (3.33, 2)$. Two nonparallel lines meeting does not establish a three-line claim. Construct $F=(4,0)$, the midpoint of AB , and draw CF to check whether this third median passes through G . It does: two-thirds of the way from $C(2,6)$ to $F(4,0)$ is $(10/3, 2)$.

Evidence and response

Require correct midpoint coordinates, two actual vertex-to-opposite-midpoint segments, an intersection estimate, and the need to construct the third median. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Use the model or O07 to show midpoint marks and perpendicular construction marks before naming U. Construct two bisectors, pause at the candidate center, then construct the third independently. For O05 distinguish vertex-part/midpoint-part from vertex-part/whole. For O06 carry out an actual angle-bisector construction, then draw perpendicular radii. Do not merely plot the coordinates supplied in the key and call that an experimental construction.

Core task

Use graph paper at 1 cm per coordinate unit to draw $A(0,6)$, $B(-6,0)$, $C(6,0)$. Construct all three side midpoints and medians. Label their common intersection G. Measure AG and GD, where D is the midpoint of BC, and report AG:GD and AG:AD. Check the division on a second median. State what this experiment verifies and what it does not prove.

Teacher solution and discussion

The side midpoints are $D=(0,0)$, $E=(3,3)$ on AC, and $F=(-3,3)$ on AB. Draw AD, BE and CF; they meet at $G=(0,2)$. Along AD, $AG=4$ cm, $GD=2$ cm and $AD=6$ cm, so $AG:GD=2:1$ and $AG:AD=2:3$. On BE, $BG=\sqrt{6^2+2^2}=\sqrt{40}\approx 6.32$ cm and $GE=\sqrt{3^2+1^2}=\sqrt{10}\approx 3.16$ cm; $BG:GE=2:1$. The third median also passes through G; its pieces are equal to the corresponding pieces on BE by symmetry. Actual ruler readings may differ slightly. This triangle supports concurrency and the 2:1 division, but its symmetry and finite measurement accuracy do not prove the properties for every triangle.

Evidence and response

Check all midpoint labels and three median lines; require two correctly oriented division checks and a qualified experimental conclusion. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O09 to calculate the three raw ratios before discussing tolerance; do not round readings to force agreement. O10 offers folding as another representation of angle bisection. In O11 keep AB fixed while changing the third vertex, and have learners predict inside/on/outside before constructing. In O12 and O13 ask which endpoint or object each distance reaches before doing arithmetic.

Core task

Three medians of a drawn triangle meet near G. D, E and F are the opposite-side midpoints for vertices A, B and C. The measured lengths in centimeters are AG=6.02, GD=3.01; BG=8.01, GE=4.02; CG=5.49, GF=2.75. Compute the three vertex-part/midpoint-part ratios to two decimal places. Using an agreed ratio tolerance of 0.03 from 2, evaluate the data and explain why slightly unequal calculated ratios need not mean the property failed.

Teacher solution and discussion

$AG/GD=6.02/3.01=2.00$. $BG/GE=8.01/4.02\approx 1.99$. $CG/GF=5.49/2.75\approx 2.00$ (unrounded, about 1.996). Each ratio differs from 2 by less than 0.03, so all three are consistent with the centroid's 2:1 division at the stated tolerance. Ruler resolution, line thickness, midpoint placement and the small region in which the medians seem to meet can cause slightly different readings. Report the observed values rather than replacing them with exact 2s. The measurements support the property for this drawing, not a universal proof.

Evidence and response

Compute each ordered ratio, apply the stated numerical tolerance and name a plausible measurement source without concealing the data. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T5: Design, compare and critique center investigations using varied triangles, appropriate measurements and honest limits on experimental evidence.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Give individual prediction time before assigning constructor, measurer, checker and reporter roles. O14 produces one shared diagram but each learner explains a different verification. For O15 keep the uniform-material condition explicit and record actual observations without inventing expected measurements. O16 and O17 require choosing the center from the purpose; accept a well-supported infeasibility conclusion rather than moving a center to satisfy a drawing preference.

Core task

A team is given triangle $A(0,0)$, $B(8,0)$, $C(0,6)$. Assign one construction to each of three members: medians, internal angle bisectors and side perpendicular bisectors. A fourth member checks construction marks. Make a shared labeled diagram showing G , I and U . For G check a median ratio, for I check all three perpendicular side distances, and for U check all three vertex distances. Write a short comparison explaining why the three answers are compatible rather than contradictory.

Teacher solution and discussion

The three constructions give $G=(8/3,2)$, $I=(2,2)$ and $U=(4,3)$. A shared diagram should mark the medians' side midpoints, equal-angle construction evidence, and the perpendicular bisectors' midpoint/right-angle conditions. For G , choose median from C to midpoint $F=(4,0)$ of AB : $CG=4\sqrt{13}/3 \approx 4.81$ and $GF=2\sqrt{13}/3 \approx 2.40$, ratio 2:1. For I , the distances to AB and AC are each 2; the perpendicular distance to BC , $3x+4y=24$, is $|6+8-24|/5=2$. For U , $UA=UB=UC=5$. G and I are strictly inside, whereas U lies on BC . The answers are compatible because each construction answers a different question: median division/balance, equal side distances, or equal vertex distances. None of these requires the three points to coincide.

Evidence and response

Assess the shared diagram and distinct measurement evidence, not only the three coordinates; ensure each learner can explain the comparison. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use a fresh item without showing the teacher diagram. O19 requires reconstruction plus a check, not only coordinate averaging. O20 assesses all three perpendicular side distances; O21 assesses midpoint and perpendicular conditions; O22 assesses what dilation changes and preserves; O23 tests a false statement with a zero side distance. Record neutral access supports separately from hints identifying the center or the next line.

Core task

Two vertices of a triangle are $A(1,1)$ and $B(7,1)$, and its centroid is $G(3,3)$. Use the centroid-coordinate relationship to find the missing vertex C . Draw the resulting triangle, construct all three medians and verify that they pass through the given G . Check the 2:1 division on one median.

Teacher solution and discussion

The centroid is the coordinate average, so $(1+7+x_C)/3=3$ gives $x_C=1$, and $(1+1+y_C)/3=3$ gives $y_C=7$. Thus $C=(1,7)$. The side midpoints are $D=(4,4)$ on BC , $E=(1,4)$ on AC and $F=(4,1)$ on AB . AD lies on $y=x$ and contains G . The segment from B to E has displacement $(-6,3)$; two-thirds along it is $(7,1)+(-4,2)=(3,3)$. The segment from C to F has displacement $(3,-6)$; two-thirds along it is $(1,7)+(2,-4)=(3,3)$. On AD , $AG=\sqrt{2^2+2^2}=2\sqrt{2}$ and $GD=\sqrt{1^2+1^2}=\sqrt{2}$, so $AG:GD=2:1$. Construction and measurement should agree with these coordinates within drawing accuracy.

Evidence and response

Solve both coordinate equations, verify each of the three actual medians and show one correctly ordered part-to-part ratio. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose the representation that matches the error. O24 uses a measured strip to separate 2:1 from 1:1 and part-to-whole from part-to-part. O25 repairs measuring to an arbitrary side point. O26 shows that one perpendicular bisector is a locus, not a unique center. O27 relabels lines only after checking their defining features. Follow with a new unsupported construction, not a repeat of the same marked diagram.

Core task

Draw a 12 cm segment AM and regard it as a median from vertex A to opposite-side midpoint M. A learner marks H at 6 cm from A and calls it the centroid. Mark the correct position G, label AG and GM, and compute AG/GM , AG/AM and GM/AM . Explain the difference between the midpoint of a side and the centroid on a median.

Teacher solution and discussion

The centroid divides the median in a 2:1 ratio from the vertex. Split 12 cm into three equal 4 cm units. Put G 8 cm from A and 4 cm from M, so H at 6 cm is not correct. $AG/GM=8/4=2$; $AG/AM=8/12=2/3$; $GM/AM=4/12=1/3$. The side midpoint M divides the opposite side into two equal lengths and is an endpoint of this median. The centroid G is the common intersection of three medians and divides each median 2:1, not into equal halves. This segment model repairs the ratio placement but a full triangle with its other medians is needed to check concurrency.

Evidence and response

Require a measured segment with two labeled parts, all three ratios, and a distinction between the side midpoint, median midpoint and centroid. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Experimentally verify centroid median division, interior location and the balance interpretation for a uniform triangular lamina.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Collect a construction, measurement record and conclusion before discussing the key. O28 checks a fresh median experiment; O29 compares two circle goals in one triangle; O30 distinguishes a counterexample, supporting data and a property that was never measured. Do not require a formal universal proof for this standard, but do require students to recognize that experimental verification is not the same as one.

Core task

Draw triangle $A(0,0)$, $B(10,0)$, $C(0,10)$ on a convenient equal-scale grid. Construct all three medians and locate G . On the median from A , measure both parts and the whole, then report the two ratios AG/GD and AG/AD , where D is the midpoint of BC . State a centroid conjecture supported by your work, and identify one limitation of the experiment.

Teacher solution and discussion

The side midpoints are $D=(5,5)$, $E=(0,5)$ and $F=(5,0)$; the medians meet at $G=(10/3,10/3)\approx(3.33,3.33)$. On AD , $AG=10\sqrt{2}/3\approx4.71$ units, $GD=5\sqrt{2}/3\approx2.36$, and $AD=5\sqrt{2}\approx7.07$. Therefore $AG/GD=2$ and $AG/AD=2/3$; ruler measurements should approximate these values at the chosen scale. A supported conjecture is that all three medians are concurrent and their intersection lies two-thirds of the way from each vertex to the opposite midpoint. This drawing checks concurrency and the division on one median, not all shapes or exact universal equality; line thickness or this triangle's special symmetry is a legitimate limitation. Testing other medians and scalene triangles would strengthen the experimental evidence.

Evidence and response

Collect the unassisted construction, measured part and whole lengths, two distinct ratios and an appropriately scoped conjecture. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

In triangle ABC, D is the midpoint of BC. Which construction starts the search for the centroid?

- A. Bisect angle A, without checking where the ray meets BC.
- B. Draw AD, and repeat from another vertex to the midpoint of its opposite side.
- C. Join D to the midpoint of AB.
- D. Draw the line through D perpendicular to BC, whether or not it passes through A.

Reasoning: _____

M02 | Warm-up | T3

A point P is inside a triangle. Which measurement gives the perpendicular distance from P to the supporting line containing one side of the triangle?

- A. The length from P to the midpoint of that side, even if the segment is slanted.
- B. The length of the perpendicular segment from P to that supporting line.
- C. The length from P to either endpoint of that side.
- D. Half the length of that side.

Reasoning: _____

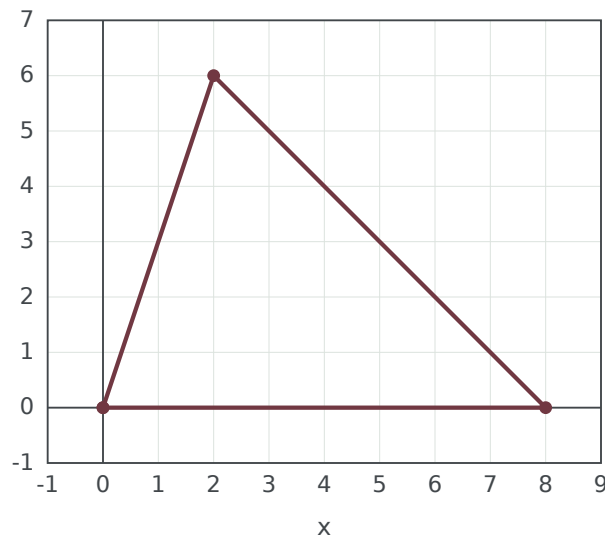
WARM-UP

001 • Show your reasoning

T1: Construct and distinguish medians, internal angle bisectors and side perpendicular bisectors; check three-line concurrency.

On graph paper, draw $A(0,0)$, $B(8,0)$, $C(2,6)$. Find and label D , the midpoint of BC , and E , the midpoint of AC . Draw medians AD and BE . Give D and E , estimate the medians' intersection G , and explain why drawing only these two lines does not yet experimentally check all-three-median concurrency.

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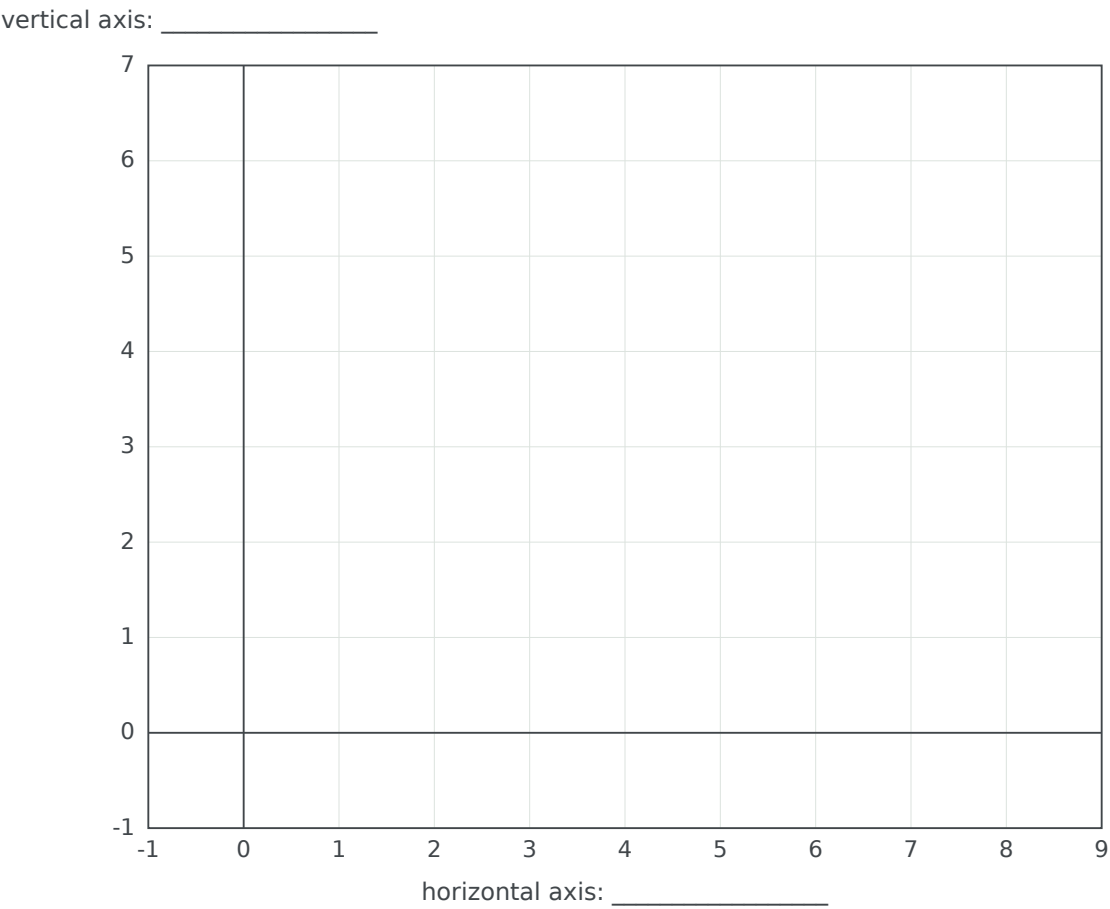
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Use the construction workspace on the following page. Additional graph paper may be used.

WARM-UP WORKSPACE

001 • Construction and evidence

Continue the task from the preceding page. Label constructed points, record measurements, and justify your conclusion.



Explanation and check

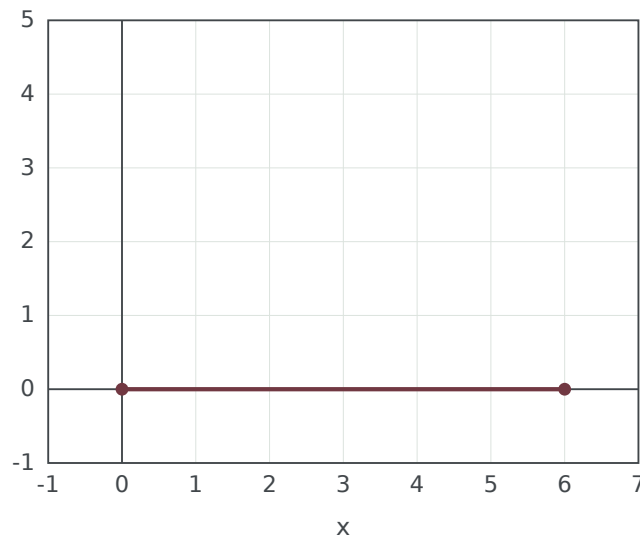
WARM-UP

002 • Show your reasoning

T3: Experimentally verify incenter equality of perpendicular side distances, interior location and incircle tangency.

Draw the segment from $Q(0,0)$ to $R(6,0)$ and the point $P(2,3)$. Mark F , the perpendicular foot from P to QR . Find PF , PQ and PR . Explain which is the distance from P to the side QR and why measuring to an arbitrary point on a side can spoil an incenter experiment.

y



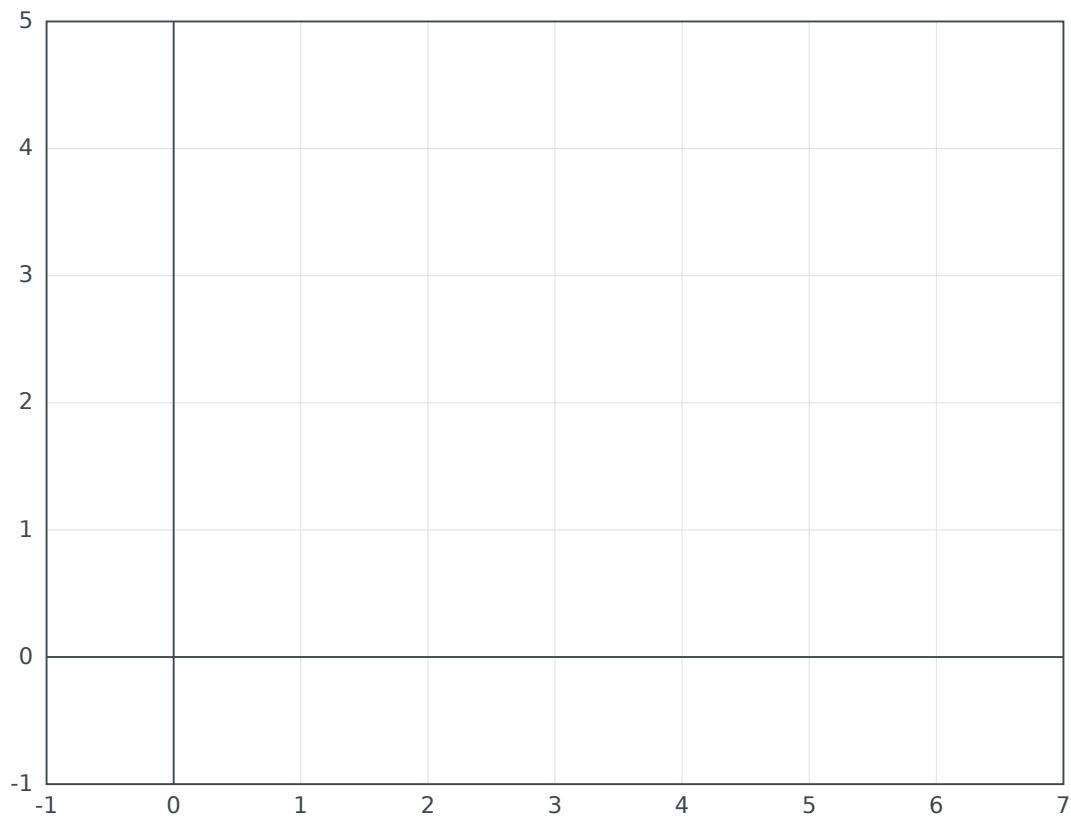
Use the construction workspace on the following page. Additional graph paper may be used.

WARM-UP WORKSPACE

002 • Construction and evidence

Continue the task from the preceding page. Label constructed points, record measurements, and justify your conclusion.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01

M01 • T1: Construct and distinguish medians, internal angle bisectors and side perpendicular bisectors; check three-line concurrency.

Answer: B. Draw AD, and repeat from another vertex to the midpoint of its opposite side.

A median joins a vertex to the midpoint of the opposite side. Intersecting two medians locates the candidate centroid; constructing the third median gives an experimental concurrency check.

Check each alternative

- A. That begins an incenter construction; an angle bisector need not reach the midpoint.
- B. Correct. A median joins a vertex to the midpoint of the opposite side. Intersecting two medians locates the candidate centroid; constructing the third median gives an experimental concurrency check.
- C. This is a midsegment, not a median from a vertex.
- D. That is a perpendicular bisector, used to construct a circumcenter, not a median in general.

Evidence to assess

Identify both endpoints of a median and distinguish locating a candidate from checking the third median.

WORKED TEACHER KEYS

M02

M02 • T3: Experimentally verify incenter equality of perpendicular side distances, interior location and incircle tangency.

Answer: B. The length of the perpendicular segment from P to that supporting line.

The distance from a point to a line is the length of its perpendicular segment to that line. For an arbitrary interior point, the perpendicular foot can lie on a side extension. Incenter investigations compare the three supporting-line distances; for the incenter itself, all three tangent feet lie on the actual sides.

Check each alternative

- A. The midpoint is not necessarily the perpendicular foot.
- B. Correct. The distance from a point to a line is the length of its perpendicular segment to that line. For an arbitrary interior point, the perpendicular foot can lie on a side extension. Incenter investigations compare the three supporting-line distances; for the incenter itself, all three tangent feet lie on the actual sides.
- C. This measures distance to a vertex, not distance to the side.
- D. Half a side length does not depend on P and is not the point-to-side distance.

Evidence to assess

Name the perpendicular requirement, rather than only saying to measure to the side.

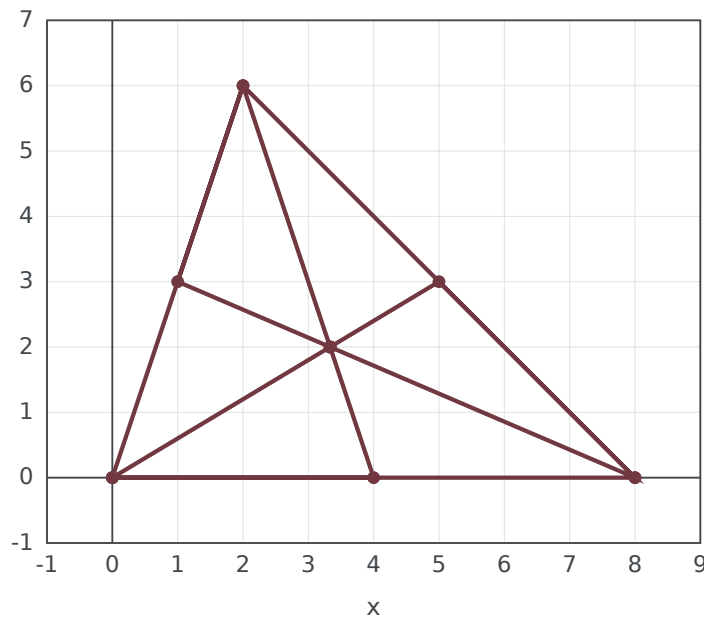
WORKED TEACHER KEYS

001

001 • T1: Construct and distinguish medians, internal angle bisectors and side perpendicular bisectors; check three-line concurrency.

$D = ((8+2)/2, (0+6)/2) = (5, 3)$ and $E = ((0+2)/2, (0+6)/2) = (1, 3)$. Draw AD from $(0, 0)$ to $(5, 3)$ and BE from $(8, 0)$ to $(1, 3)$. AD has $y = 3x/5$; BE has $y = -(3/7)(x-8)$. Equating gives $21x = 120 - 15x$, so $x = 10/3$ and $y = 2$; a careful graph gives $G \approx (3.33, 2)$. Two nonparallel lines meeting does not establish a three-line claim. Construct $F = (4, 0)$, the midpoint of AB, and draw CF to check whether this third median passes through G. It does: two-thirds of the way from $C(2, 6)$ to $F(4, 0)$ is $(10/3, 2)$.

y

**Evidence to assess**

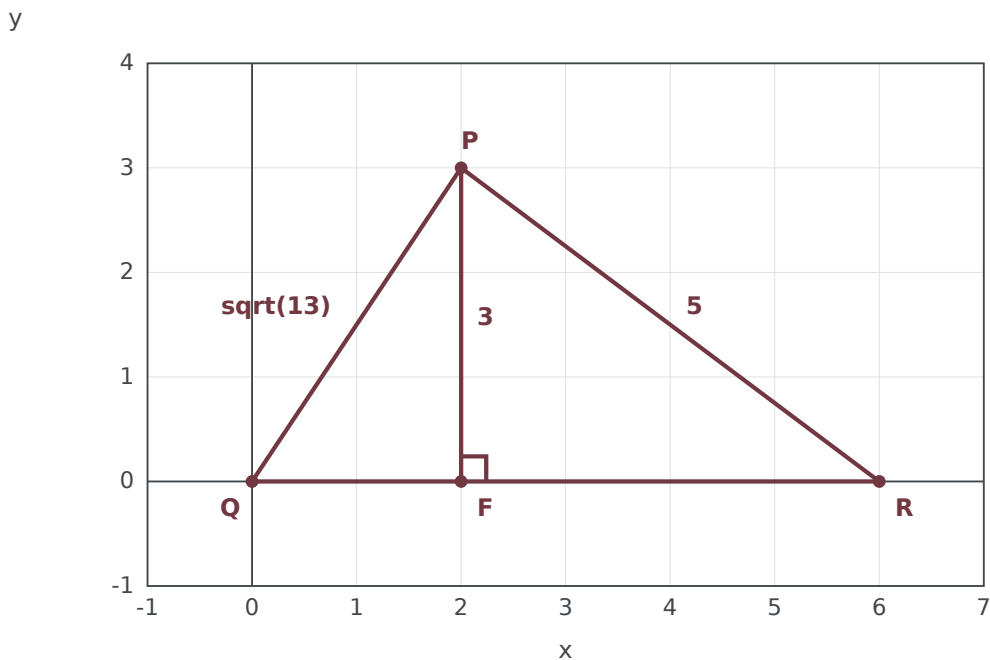
Require correct midpoint coordinates, two actual vertex-to-opposite-midpoint segments, an intersection estimate, and the need to construct the third median.

WORKED TEACHER KEYS

002

002 • T3: Experimentally verify incenter equality of perpendicular side distances, interior location and incircle tangency.

QR lies on $y=0$, so the perpendicular from P is the vertical line $x=2$ and $F=(2,0)$. $PF=3$, $PQ=\sqrt{2^2+3^2}=\sqrt{13}\approx 3.61$, and $PR=\sqrt{4^2+3^2}=5$. The point-to-side distance is $PF=3$, the perpendicular length. Segments to Q or R include horizontal displacement and are longer. At an incenter the three perpendicular side distances agree; arbitrary slanted segments to selected points generally do not, even when the construction is correct.

**Evidence to assess**

Mark a right angle at F, compute all three lengths, and distinguish point-to-line distance from point-to-point distance.