

TEACHER AND STUDENT RESOURCE

Build and interpret two way frequency tables



NC Math 2 • NC.M2.S-CP.4

Conditional Probability and the Rules for Probability

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.S-CP.4 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 126-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 34-35, 64-65, 94-95

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 34

O02: sample PDF page(s) 19; original page(s) 35

Selected teacher prompts and keys: original page(s) 64-65, 94-95

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Represent data on two categorical variables by constructing a two-way frequency table of data. Interpret the two-way table as a sample space to calculate conditional, joint and marginal probabilities. Use the table to decide if events are independent.

T1 Construct and validate a two-way frequency table for two categorical variables.

T2 Treat a table as a sample space to calculate and interpret joint, marginal and conditional probabilities.

T3 Use table proportions to decide whether events are independent and qualify conclusions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M05, M09, M10, M14, M15, M20, M24, M28, M30, O01, O02, O03, O05, O09, O10, O11, O14, O15, O20, O24, O28, O30

T2: M04, M06, M07, M11, M12, M13, M18, M19, M21, M23, M26, M29, O04, O06, O07, O12, O13, O18, O19, O21, O23, O26, O29

T3: M08, M16, M17, M22, M25, M27, O08, O16, O17, O22, O25, O27

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Construct and validate a two-way frequency table for two categorical variables.

Learning target: Treat a table as a sample space to calculate and interpret joint, marginal and conditional probabilities.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Treat a table as a sample space to calculate and interpret joint, marginal and conditional probabilities.

Learning target: Use table proportions to decide whether events are independent and qualify conclusions.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Construct and validate a two-way frequency table for two categorical variables.

Learning target: Treat a table as a sample space to calculate and interpret joint, marginal and conditional probabilities.

Learning target: Use table proportions to decide whether events are independent and qualify conclusions.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Classify observations into categories, tally records, add counts, and simplify fractions. Distinguish a person or object from its recorded category labels.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Unless stated otherwise, select one recorded observation uniformly. Row/column categories partition the observations; margins summarize the same records. A joint probability uses the grand total; a conditional probability uses a positive conditioning-group total. A sample table models its recorded population directly; generalizing requires justified sampling.

TEACHER MODEL

Turn records into a probability model

A survey has 40 people: B row [6 in A, 4 not A], not B row [18 in A, 12 not A]. Build the margins, then identify joint, marginal and conditional probabilities and decide independence.

	A	Not A	Total
B	6	4	10
Not B	18	12	30
Total	24	16	40

Each record belongs to exactly one interior cell. Margins count the same records, not additional records.

Count each person in one interior cell. Margins are B 10, not B 30, A 24, not A 16, total 40. Joint $P(A \text{ and } B) = 6/40 = 3/20$. Marginal $P(A) = 24/40 = 3/5$. Conditional $P(A|B) = 6/10 = 3/5$. Equality of the last two means independence for a uniformly selected person from this population. Do not add margins to interior cells.

VISUAL ANCHOR

Three questions from one interior cell

A cell alone is a count. The chosen denominator determines the probability question.

	Packed	School	Total
Bus	12	8	20
Walk	4	16	20
Total	16	24	40

Each record belongs to exactly one interior cell. Margins count the same records, not additional records.

Joint bus and packed: 12/40. Marginal bus: 20/40. Conditional packed given bus: 12/20.
Reverse conditional bus given packed: 12/16. Compare $P(\text{packed}|\text{bus})=3/5$ with $P(\text{packed})=2/5$: the events are dependent in this table.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Sample space, events and known information**Representation and calculation****Justification or check****Conclusion in context**

WARM-UP

Teaching script

Core: 001 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Have learners identify one observation and its two category labels. Tally each record once, then verify the interior sum equals the source-record count.

Core task

Eight students have records (Bus, Yes), (Walk, No), (Bus, No), (Bus, Yes), (Walk, Yes), (Walk, No), (Bus, Yes), (Walk, Yes), where Yes means packed lunch. Construct a labeled two-way frequency table with all totals. Explain your tally check.

Teacher solution and discussion

Rows Bus/Walk, columns Yes/No: [3, 1] and [2, 2]. Row totals are 4, 4; column totals 5, 3; total 8. Mark each source record once, and verify both margin sums equal 8.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Construct interior counts before margins. Say, “Margins summarize the same observations.” Name the requested group before choosing any probability denominator.

Core task

Twenty-four people are classified as members or guests and as tea or coffee drinkers, with exactly one drink per person. There are 14 members, 9 tea drinkers, and 5 tea-drinking members. Construct the table, then verify every stated total.

Teacher solution and discussion

Member row tea/coffee=[5, 9]. Guest tea=9-5=4, guest coffee=10-4=6. Row totals 14, 10; column totals 9, 15; grand total 24. Checks: 14 members, 9 tea, 5 both all match.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Use a blank table and have learners justify missing counts by row or column equations. Compare joint, marginal and conditional questions using one cell.

Core task

The partial table is shown. Fill every blank and give two independent arithmetic checks on the completed table.

Teacher solution and discussion

The X-and-not-A cell is $17-7=10$; Y-and-A is $19-7=12$; Y-and-not-A is $23-12=11$. Not-A total is 21; grand total is 40. Checks: $17+23=40$ and $19+21=40$, also $7+10+12+11=40$.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign source-record auditor, table constructor, probability interpreter and independence checker. Require partners to reconcile tables with the original data.

Core task

In a 120-record population, $P(B)=0.25$, $P(A|B)=0.40$, and 48 records are in A. Reconstruct all four cells and check the conditional and marginal information.

Teacher solution and discussion

$B \text{ total}=30$, $\text{overlap}=0.4 \times 30=12$. $B\text{-not-}A=18$; $\text{not } B\text{-and-}A=48-12=36$; $\text{neither}=120-12-18-36=54$. $P(A|B)=12/30=0.4$; $P(B)=30/120=0.25$; $A \text{ total}=48$.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Treat a table as a sample space to calculate and interpret joint, marginal and conditional probabilities.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect the table, labels, fractions and context separately. Do not supply missing counts or conditioning denominators; neutral enlargement and read-aloud support are appropriate.

Core task

Combine two sites into one table. Site 1 has B row [6, 4] and not B row [2, 8]. Site 2 has B row [18, 12] and not B row [14, 6]. Find the combined $P(A|B)$ and $P(A)$; do not average site percentages without weighting.

Teacher solution and discussion

Add corresponding cells: $B=[24, 16]$, not $B=[16, 14]$, total 70. $P(A|B)=24/40=3/5$; $P(A)=40/70=4/7$. The combined values use pooled counts; unequal site and group sizes generally require weighting. Here the events are dependent because $3/5 \neq 4/7$.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Return to concrete record cards. Repair one omission, then use a fresh table to distinguish a margin from an interior cell and a restricted denominator.

Core task

A 10-record survey has nine complete records giving B row [3, 2] and not B row [1, 3]. The tenth person is known to be in B but their A status is missing. Give both possible completed tables and explain why inventing a single cell is unjustified.

Teacher solution and discussion

If the missing person is in A: $B=[4, 2]$, not $B=[1, 3]$. If not A: $B=[3, 3]$, not $B=[1, 3]$. Both total 10 and have B total 6. The source does not identify which cell receives the record, so report uncertainty or obtain the missing response rather than guessing.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Construct and validate a two-way frequency table for two categorical variables.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Ask for a valid construction or critique, not only a computed fraction. Check whether the learner can explain the unit of analysis and avoid overlapping categories.

Core task

Collect a fictional eight-record dataset for two binary categorical variables of your choice. Construct the two-way table and label the observation unit. Include at least one record in every cell, and verify that no record is counted twice.

Teacher solution and discussion

One example is eight orders classified by pickup/delivery and tea/coffee: records 1-2 pickup-tea, record 3 pickup-coffee, records 4-5 delivery-tea, and records 6-8 delivery-coffee. Table=[2, 1;2, 3], margins 3, 5 and 4, 4, total 8. The unit is an order; unique record labels 1-8 can be assigned to exactly one cell each.

Evidence and response

Require a traceable table with correct labels and margins, then separately assess the chosen numerator, denominator, context and independence comparison. Ask learners to explain errors with record-level or fraction-level evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Which pair of recorded variables is suitable for a two-way categorical frequency table without first creating numerical intervals?

- A. Travel mode and lunch choice
- B. Exact height and exact mass
- C. Exact trip time and exact distance
- D. Exact temperature and exact rainfall amount

Reasoning: _____

M02 | Warm-up | T1

Four records are (walk, hot), (bus, cold), (walk, hot), (walk, cold). What is the frequency in the walk-and-hot cell?

- A. 2
- B. 4
- C. 3
- D. 1

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Construct and validate a two-way frequency table for two categorical variables.

Eight students have records (Bus, Yes), (Walk, No), (Bus, No), (Bus, Yes), (Walk, Yes), (Walk, No), (Bus, Yes), (Walk, Yes), where Yes means packed lunch. Construct a labeled two-way frequency table with all totals. Explain your tally check.

Mode	Yes	No	Total

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Construct and validate a two-way frequency table for two categorical variables.

A library has 50 books: 30 fiction and 20 nonfiction. Eighteen fiction books and 7 nonfiction books are paperback; all others are hardcover. Construct the complete table. Identify the observation unit and both categorical variables.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01**M01 • T1: Construct and validate a two-way frequency table for two categorical variables.****Answer: A. Travel mode and lunch choice**

Travel mode and lunch choice place observations in named categories, so each person can be assigned to one row and one column.

Check each alternative

- A. Travel mode and lunch choice place observations in named categories, so each person can be assigned to one row and one column.
- B. These are quantitative measurements; meaningful categories would need to be defined first.
- C. These are quantitative measurements rather than the requested categorical labels.
- D. Exact temperature and amount are numerical measurements, not the requested categories.

Evidence to assess

Require identifying two categorical variables and one observation unit.

WORKED TEACHER KEYS

M02**M02 • T1: Construct and validate a two-way frequency table for two categorical variables.****Answer: A. 2**

Two records have both walk and hot. Repeated category pairs count separate people, so both records contribute.

Check each alternative

A. Two records have both walk and hot. Repeated category pairs count separate people, so both records contribute.

B. Four is the grand total, not the joint cell.

C. Three is the total number walking, including a cold lunch.

D. This counts the repeated pair only once rather than counting records.

Evidence to assess

Require counting records satisfying both categories.

WORKED TEACHER KEYS

001

001 • T1: Construct and validate a two-way frequency table for two categorical variables.

Rows Bus/Walk, columns Yes/No: [3, 1] and [2, 2]. Row totals are 4, 4; column totals 5, 3; total 8. Mark each source record once, and verify both margin sums equal 8.

	Yes	No	Total
Bus	3	1	4
Walk	2	2	4
Total	5	3	8

Each record belongs to exactly one interior cell. Margins count the same records, not additional records.

Evidence to assess

Item evidence: correct row and column labels; four counts 3, 1, 2, 2; all margins; record-level tally check. Award one point per requirement, up to 4; accept equivalent fractions and tables with transposed axes.

WORKED TEACHER KEYS

002

002 • T1: Construct and validate a two-way frequency table for two categorical variables.

Rows fiction/nonfiction, columns paperback/hardcover: [18, 12], [7, 13]. Column totals are 25, 25; row totals 30, 20; grand total 50. The observation unit is one book; variables are genre category and binding type.

	Paperback	Hardcover	Total
Fiction	18	12	30
Nonfiction	7	13	20
Total	25	25	50

Each record belongs to exactly one interior cell. Margins count the same records, not additional records.

Evidence to assess

Item evidence: four interior counts; all totals; book as observation unit; both variables. Award one point per requirement, up to 4; accept equivalent fractions and tables with transposed axes.