

## TEACHER AND STUDENT RESOURCE

# Why Equation Replacement Works



## NC Math 1 • NC.M1.A-REI.5

### Equations, inequalities and mathematical reasoning

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

# Free sample | Contents

NC.M1.A-REI.5 | NC Math 1 mathematics

This sample contains 19 complete selected pages from the 92-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75

Using this sample

Read the preparation and access guidance before using a selected activity.  
Keep teacher keys and private preparation pages separate from student copies.  
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

## ALIGNMENT

## Standard and learning targets

Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.

**T1 Form a replacement equation by a linear combination.**

**T2 Prove both directions of solution-set preservation.**

**T3 Analyze lost information, identities and contradictions.**

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

## NAVIGATION

# Choose evidence deliberately

**Warm-up: M01-M04 and O01-O04**

**I Do: M05-M08 and O05-O08**

**We Do: M09-M13 and O09-O13**

**Collaborative practice: M14-M18 and O14-O18**

**Independent practice: M19-M23 and O19-O23**

**Small-group reteach: M24-M27 and O24-O27**

**Exit ticket: M28-M30 and O28-O30**

T1: M01, O01, M02, O02, M08, O08, M11, O11, M12, O12, M18, O18, M21, O21, M22, O22, M28, O28

T2: M03, O03, M04, O04, M09, O09, M13, O13, M14, O14, M19, M23, O23, M24, O24, M29, O29

T3: M05, O05, M06, O06, M07, O07, M10, O10, M15, O15, M16, O16, M17, O17, O19, M20, O20, M25, O25, M26, O26, M27, O27, M30, O30

**PLANNING**

# Materials, access and pacing

## Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

## Prerequisites

Signed-number arithmetic, distributive property, substitution, products and square roots. Diagnose these separately before scoring the targeted reasoning.

## Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

## Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

## Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

## Review convention

An equation operation acts on both sides. Retain the other equation. Prove both original-to-new and new-to-original directions; a common sample point alone does not prove equivalence.

## TEACHER MODEL

## Keep one equation; recover the other

**Compare  $\{x+y=6, 2x-y=3\}$  with  $\{x+y=6, 3x=9\}$ .**

Forward: any common solution of the original equations satisfies their sum  $3x=9$  and still satisfies  $x+y=6$ . Reverse: subtract retained  $x+y=6$  from  $3x=9$  to recover  $2x-y=3$ . Thus each system implies the other. Both have  $(3,3)$  as their sole common solution. The equation  $3x=9$  alone also admits  $(3,0)$ ; the retained equation is essential.

## VISUAL ANCHOR

## Track the complete solution set

Start

$$\mathbf{E_1: x + y = 6; E_2: 2x - y = 3}$$

↓ Keep  $E_1$ ; add  $E_1$  to  $E_2$ .

Result

$$\mathbf{E_1: x + y = 6; F: 3x = 9}$$

Start

$$\mathbf{E_1: x + y = 6; F: 3x = 9}$$

↓ Keep  $E_1$ ; subtract  $E_1$  from  $F$ .

Result

$$\mathbf{E_1: x + y = 6; E_2: 2x - y = 3}$$

The forward and reverse routes prove equality of the COMMON solution sets.  
 The individual second equations have different lines of solutions.  
 Both systems have (3,3); the single equation  $3x=9$  has extra points.

## REUSABLE REASONING GUIDE

# Represent, justify, interpret

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Givens, quantities and restrictions****Representation and calculation****Justification or check****Conclusion, domain and units**

## WARM-UP

# Teaching script

**Core: 001 • T1: Form a replacement equation by a linear combination.**

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

## Core task

For  $E_1: x+y=5$  and  $E_2: x-y=9$ , replace  $E_2$  by  $E_2+E_1$ . Prove both the forward and reverse solution implications.

## Teacher solution and discussion

New equation:  $2x=14$ ; retain  $x+y=5$ . A point satisfying both originals satisfies their sum. Conversely, a point satisfying the retained equation and the sum satisfies their difference:  $x-y=9$ . Thus the systems have exactly the same solutions.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

## Teaching script

**Core: 005 • T3: Analyze lost information, identities and contradictions.**

Optional bank: M05–M08; 005–008. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 005, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

### Core task

Use  $x+y=4$  and  $x-y=7$  to show that replacing the entire system by its sum can add solutions. Give an explicit extra point and explain the correct replacement procedure.

### Teacher solution and discussion

The sum is  $2x=11$ . The point  $((11)/2, 0)$  satisfies it, but  $x+y=4$  would require  $11=2 \cdot 4$ , which is false since  $7 \neq 4$ . Retain one original equation along with the sum; then subtract it to recover the other.

### Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

## WE DO

# Teaching script

**Core: 009 • T2: Prove both directions of solution-set preservation.**

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

## Core task

Explain why the multiplier 2 in  $E_2 + 2E_1$  could be replaced by any real number  $k$ , including 0 or a negative number. Identify the inverse replacement.

## Teacher solution and discussion

For any fixed real  $k$ , add  $k$  times both sides of  $E_1$  to  $E_2$ . Subtracting  $kE_1$  while retaining  $E_1$  recovers  $E_2$ . No division by  $k$  is needed, so  $k=0$  is allowed; negative  $k$  is ordinary subtraction. Both solution implications follow.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

## COLLABORATIVE PRACTICE

# Teaching script

**Core: O14 • T2: Prove both directions of solution-set preservation.**

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

## Core task

A learner says, “Adding 3 times  $E_1$  to  $E_2$  gives a true equation, so the systems are equivalent.” Complete the missing reasoning and explain the role of retaining  $E_1$ .

## Teacher solution and discussion

Truth at original solutions proves only one direction. Retaining  $E_1$  lets us subtract  $3E_1$  from the new second equation to recover  $E_2$ . Thus every new solution also satisfies the original pair. Without the retained relation, that recovery is not justified.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

## INDEPENDENT PRACTICE

# Teaching script

**Core: O19 • T3: Analyze lost information, identities and contradictions.**

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

## Core task

Construct two different linear equations with common solution (2,1). Replace BOTH equations by their sum. Give an extra solution of the new system, then explain why this differs from replacing only one equation.

## Teacher solution and discussion

Example:  $E_1$  is  $x+y=3$  and  $E_2$  is  $x-y=1$ . Both hold at (2,1). Replacing both by their sum gives  $2x=4$  twice, which also admits (2,0). That point fails the original equations. No original equation remains to subtract from the sum, so recovery is impossible. Accept other pairs with a demonstrated extra solution.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

## SMALL-GROUP RETEACH

# Teaching script

**Core: O24 • T2: Prove both directions of solution-set preservation.**

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

## Core task

A learner says, “Adding 4 times  $E_1$  to  $E_2$  gives a true equation, so the systems are equivalent.” Complete the missing reasoning and explain the role of retaining  $E_1$ .

## Teacher solution and discussion

Truth at original solutions proves only one direction. Retaining  $E_1$  lets us subtract  $4E_1$  from the new second equation to recover  $E_2$ . Thus every new solution also satisfies the original pair. Without the retained relation, that recovery is not justified.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

## EXIT TICKET

# Teaching script

**Core: 028 • T1: Form a replacement equation by a linear combination.**

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

## Core task

Retain  $x+2y=6$ ; replace  $4x-y=9$  by its difference with the retained equation. Show and justify both directions.

## Teacher solution and discussion

New:  $(3)x-3y=3$ . Any original solution satisfies the difference. Any new common solution, added to retained  $x+2y=6$ , gives  $4x-y=9$ ; hence it satisfies the original system.

## Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

## MULTIPLE-CHOICE BANK

**M01 / M02 • Select and justify****M01 | Warm-up | T1**

**E<sub>1</sub>:  $x + y = 4$ ; E<sub>2</sub>:  $x - y = 7$ . Replace E<sub>2</sub> with  $E_2 + E_1$ , retaining E<sub>1</sub>. What is the new second equation?**

- A.  $2x = 11$
- B.  $2y = 11$
- C.  $2x = 3$
- D.  $2x + 2y = 11$

Reasoning: \_\_\_\_\_

**M02 | Warm-up | T1**

**Keeping E<sub>1</sub>:  $x + y = 4$ , replace E<sub>2</sub>:  $2x - y = 7$  by  $E_2 + 2E_1$ . Which equation results?**

- A.  $4x + 3y = 15$
- B.  $4x + 1y = 15$
- C.  $4x + 1y = 11$
- D.  $2x - y = 15$

Reasoning: \_\_\_\_\_

## WARM-UP

**001 • Show your reasoning**

**T1: Form a replacement equation by a linear combination.**

**For  $E_1: x+y=5$  and  $E_2: x-y=9$ , replace  $E_2$  by  $E_2+E_1$ . Prove both the forward and reverse solution implications.**

**Representation / calculation**

**Explanation / check**

## WARM-UP

**002 • Show your reasoning**

**T1: Form a replacement equation by a linear combination.**

**Retain  $x-y=4$ . Replace  $2x+y=7$  with that equation plus 2 times the retained equation. Derive the result and prove reversibility.**

**Representation / calculation**

**Explanation / check**

## WORKED TEACHER KEYS

**M01 / M02****M01 • T1: Form a replacement equation by a linear combination.****Answer: A.  $2x = 11$** 

Add corresponding sides:  $(x-y)+(x+y)=7+4$ , so  $2x=11$ . Subtracting retained  $E_1$  recovers  $E_2$ .

**Check each alternative**

- A. Add corresponding sides:  $(x-y)+(x+y)=7+4$ , so  $2x=11$ . Subtracting retained  $E_1$  recovers  $E_2$ .
- B. The  $y$  terms cancel, not  $x$ .
- C. The right sides must be added, not subtracted.
- D. Opposite  $y$  coefficients sum to zero.

**M02 • T1: Form a replacement equation by a linear combination.****Answer: B.  $4x + 1y = 15$** 

$2E_1$  is  $2x+2y=8$ . Add  $E_2$ :  $(4)x+(1)y=15$ . Reverse by subtracting  $2E_1$ .

**Check each alternative**

- A. The  $y$  coefficient is  $-1$  plus the multiplier.
- B.  $2E_1$  is  $2x+2y=8$ . Add  $E_2$ :  $(4)x+(1)y=15$ . Reverse by subtracting  $2E_1$ .
- C. The right side of  $E_1$  must also be multiplied.
- D. Both coefficients must be added to those of  $E_2$ .

## WORKED TEACHER KEYS

**001 / 002****001 • T1: Form a replacement equation by a linear combination.**

New equation:  $2x=14$ ; retain  $x+y=5$ . A point satisfying both originals satisfies their sum. Conversely, a point satisfying the retained equation and the sum satisfies their difference:  $x-y=9$ . Thus the systems have exactly the same solutions.

**002 • T1: Form a replacement equation by a linear combination.**

New:  $(4)x+(1-2)y=15$ , or  $4x - 1y=15$ . Original solutions satisfy this sum. Subtract  $2(x-y=4)$  from the new equation to recover  $2x+y=7$ ; this proves the reverse implication.