

TEACHER AND STUDENT RESOURCE

The Imaginary Unit and Complex Form



NC Math 2 • NC.M2.N-CN.1

The Complex Number System

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.N-CN.1 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.

T1 Use $i^2=-1$ to explain and simplify powers involving i .

T2 Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

T3 Distinguish principal roots from the two solutions of a negative-square equation.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M02, O02, M06, O06, M09, O09, M13, O13, M15, O15, M16, O16, M19, M21, O21, M22, O22, M26, O26, M28, O28

T2: M01, O01, M04, O04, M05, O05, M07, O07, M08, O08, O10, M11, O11, M12, O12, M14, O14, O19, M23, O23, M24, O24, M25, O25, M29, O29

T3: M03, O03, M10, M17, O17, M18, O18, M20, O20, M27, O27, M30, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Use $i^2=-1$ to explain and simplify powers involving i .

Learning target: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Learning target: Distinguish principal roots from the two solutions of a negative-square equation.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Use $i^2=-1$ to explain and simplify powers involving i .

Learning target: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Learning target: Distinguish principal roots from the two solutions of a negative-square equation.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Real-number arithmetic, integer powers, square roots, and the fact that every real square is nonnegative.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

The imaginary part is the real coefficient b in $a+bi$; the imaginary term is bi . Here $\sqrt{-a}=i\sqrt{a}$ for positive real a denotes the principal complex square root. Solving $x^2=-a$ requires both signs.

TEACHER MODEL

A new number preserves the algebra

Explain why $3i$ is not real, write it in $a+bi$ form, and calculate its square.

The number i has $i^2=-1$, which no real number can satisfy. If $3i$ were real, dividing by real nonzero 3 would make i real, a contradiction. Its standard form is $0+3i$, with real coefficients $a=0$ and $b=3$. Squaring gives $(3i)^2=9i^2=-9$.

VISUAL ANCHOR

Every real number is also complex

The standard form $a+bi$ uses two real coefficients. A zero coefficient is allowed.

Number	a	b
$5 + 2i$	5	2
$-3i$	0	-3
7	7	0
$\sqrt{2} - (1/2)i$	$\sqrt{2}$	-1/2
0	0	0

The powers of i cycle: $i, -1, -i, 1$. This follows from $i^2=-1$ and $i^4=1$, not a separate definition. Real numbers occur exactly when $b=0$.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask students to name a and b separately from the term bi . Use zero coefficients explicitly so real and pure imaginary numbers fit the same definition.

Core task

Write $z=6-(7/2)i$ in the form $a+bi$, naming a , b , and the imaginary term. Explain why b itself is real.

Teacher solution and discussion

$a=6$, $b=-7/2$, and the imaginary term is $(-7/2)i$. The coefficient b is an ordinary real rational number; multiplying it by i forms the imaginary term.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Start from $i^2=-1$ and show every algebraic replacement. Square the real coefficient as well as i , and distinguish a principal square-root expression from solving an equation with both signs.

Core task

Compare $-6i$ and -6 . Give a and b for each and explain why the two numbers are not equal.

Teacher solution and discussion

$-6i=0+(-6)i$ has $a=0, b=-6$. The real number $-6=(-6)+0i$ has $a=-6, b=0$. Equality requires matching both real and imaginary components; these do not match.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: Use $i^2 = -1$ to explain and simplify powers involving i .

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Have learners propose and verify both coefficient equations when complex numbers are equal. Use the defining property to justify each step of a power cycle rather than reciting it without evidence.

Core task

A learner claims that $(-6i)^2 = -(6i^2)$. Correct the calculation and identify both the coefficient-square issue and the role of i^2 .

Teacher solution and discussion

$(-6i)^2 = (-6)^2 i^2 = 36(-1) = -36$. The entire coefficient -6 must be squared, giving 36 ; i^2 is then replaced by -1 . One cannot square only the i or move the square into an unrelated minus sign.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign partners to construct examples and check every coefficient restriction. Compare real, nonreal, and pure imaginary cases; challenge the false claim that real numbers are not complex.

Core task

Find real p and q so $(2p+1)+(3q-4)i=-5+2i$. Verify by reconstructing the entire number. Explain why the real and imaginary equations may be solved separately.

Teacher solution and discussion

$2p+1=-5$ gives $p=-3$. $3q-4=2$ gives $q=2$. Substitution yields $-5+2i$. Complex equality requires the real parts to match and the imaginary coefficients to match, so the two equations are independent conditions.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Preserve the first response. Ask for standard form with real coefficients and checks of proposed roots. Keep contextual restrictions separate from the algebraic complex solution set.

Core task

Construct a nonreal complex number whose real part is irrational and imaginary coefficient is a negative noninteger rational. Explain why each coefficient meets the definition and why the complete number is not real.

Teacher solution and discussion

One example is $\sqrt{3}-(2/5)i$. The real part $\sqrt{3}$ is irrational but real; the imaginary coefficient $-2/5$ is negative, rational, and not an integer. Since that coefficient is nonzero, the complete number is nonreal. Other examples satisfying every condition are valid.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Identify whether the difficulty is a missing zero coefficient, the sign $i^2=-1$, a coefficient that was not squared, or a missing second root. Teach that point explicitly, then collect an unrehearsed response.

Core task

Construct two nonreal complex numbers with different real parts whose sum is 10. Write the coefficient pairs and verify that both original numbers are nonreal but the sum is real.

Teacher solution and discussion

One pair is $3+2i$ and $7-2i$, with coefficient pairs $(3,2)$ and $(7,-2)$. Both imaginary coefficients are nonzero, so each is nonreal. Their sum is $(3+7)+(2-2)i=10$. The different real parts 3 and 7 meet the additional condition.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: 028 • T1: Use $i^2 = -1$ to explain and simplify powers involving i .

Optional bank: M28–M30; 028–030. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 028, including its visual when present. Use a mixed power, a coordinate representation of coefficients, and an explanation of negative-square solutions. Look for connected reasoning from the definition, not only remembered answers.

Core task

Simplify $i^{48} + i^{49} + i^{50} + i^{51}$ and explain the result using both the four-power cycle and pairing additive opposites.

Teacher solution and discussion

The powers are $1, i, -1, -i$, so the sum is 0. Because 48 is divisible by 4, the next four values form one full cycle. Pair 1 with -1 and i with $-i$ to see both real and imaginary components cancel.

Evidence and response

Ask the learner to name the real coefficients rather than treating bi as b . Check that i^2 is replaced by -1 after all coefficients have been squared. Separate one principal square-root value from both solutions of an equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T2****For $z=4-2i$, what are the real part and imaginary coefficient?**

- A. -2 and 4
- B. 4 and $-2i$
- C. 4 and 2
- D. 4 and -2

Reasoning: _____

M02 | Warm-up | T1**Use the defining property $i^2=-1$ to evaluate i^2 .**

- A. -1
- B. $-i$
- C. i
- D. 1

Reasoning: _____

WARM-UP

001 • Show your reasoning

T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

Write $z=6-(7/2)i$ in the form $a+bi$, naming a , b , and the imaginary term. Explain why b itself is real.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Use $i^2 = -1$ to explain and simplify powers involving i .

Explain why i cannot be a real number. Then evaluate $(6i)^2$ using the defining property, showing the sign.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01**M01 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.****Answer: D. 4 and -2**

In $a+bi$, a and b are real numbers. Here $a=4$ and $b=-2$; the imaginary term is $-2i$, but its real coefficient is -2 .

Check each alternative

- A. This reverses the real and imaginary components.
- B. The requested coefficient b is real; bi is the entire imaginary term.
- C. The minus sign is part of the coefficient b .
- D. In $a+bi$, a and b are real numbers. Here $a=4$ and $b=-2$; the imaginary term is $-2i$, but its real coefficient is -2 .

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02**M02 • T1: Use $i^2=-1$ to explain and simplify powers involving i .****Answer: A. -1** $i^2=(i^2)^1=(-1)^1=-1$. This follows directly from the defining property $i^2=-1$.**Check each alternative**A. $i^2=(i^2)^1=(-1)^1=-1$. This follows directly from the defining property $i^2=-1$.B. The power reduces to -1, not -i; apply $i^2=-1$ in pairs.C. The power reduces to -1, not i; apply $i^2=-1$ in pairs.D. The power reduces to -1, not 1; apply $i^2=-1$ in pairs.**Evidence to assess**

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T2: Write real and nonreal complex numbers in $a+bi$ form with real coefficients.

$a=6$, $b=-7/2$, and the imaginary term is $(-7/2)i$. The coefficient b is an ordinary real rational number; multiplying it by i forms the imaginary term.

Evidence to assess

Require $a=6$ and $b=-7/2$, distinguish the coefficient b from the term bi , and recognize that the rational coefficient is real.

WORKED TEACHER KEYS

002**002 • T1: Use $i^2=-1$ to explain and simplify powers involving i .**

Every real square is nonnegative, but $i^2=-1$. Thus i is not real. $(6i)^2=36i^2=-36$. The negative sign follows from i^2 , not from the positive coefficient.

Evidence to assess

Require a reason that i is not real and the calculation $(6i)^2=36i^2=-36$. Do not require unrelated coefficient or root comparisons.