

TEACHER AND STUDENT RESOURCE

Graphs Digraphs and Ranking



DCS.GT.1

Graph Theory

Seven instructional sections. Thirty multiple-choice and thirty open or performance tasks, original mathematical visuals, generous workspaces, teaching scripts, and worked keys.

Free sample | Contents

DCS.GT.1 | NC Discrete Mathematics for Computer Science mathematics

This sample contains 20 complete selected pages from the 102-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 33-34, 63, 78-79

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 33

O02: sample PDF page(s) 18; original page(s) 34

Selected teacher prompts and keys: original page(s) 63, 78-79

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand graph theory to model relationships and solve problems.

DCS.GT.1.1 Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table.

DCS.GT.1.2 Test graphs and digraphs for Euler paths, Euler circuits, Hamiltonian paths, or Hamiltonian circuits.

DCS.GT.1.3 Interpret a complete digraph to determine rank.

The decimal codes are the official objectives. All are represented in the item bank. These materials support several sessions, not one 60-task lesson.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

DCS.GT.1.1: M01, M02, M03, M04, M05, M06, M22, M28, O01, O02, O03, O04, O05, O06, O07, O29

DCS.GT.1.2: M07, M08, M09, M10, M11, M12, M13, M14, M15, M16, M17, M18, M19, M20, M21, M29, M30, O08, O09, O10, O11, O12, O13, O14, O15, O16, O17, O18, O19, O20, O21, O22, O30

DCS.GT.1.3: M23, M24, M25, M26, M27, O23, O24, O25, O26, O27, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Build and interpret representations

Learning target: Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table. Test graphs and digraphs for Euler paths, Euler circuits, Hamiltonian paths, or Hamiltonian circuits.

Use selected Warm-up and I Do tasks; compare an initial prediction with the worked model.

Session 2 Apply and explain methods

Learning target: Interpret a complete digraph to determine rank.

Use We Do and collaborative tasks across the objectives. Require each learner to explain one mathematical choice.

Session 3 Transfer and collect independent evidence

Learning target: Apply the listed objectives with accurate representations, justified methods, and contextual checks.

Choose independent tasks covering each objective. Reteach an evidenced gap, then use a fresh exit task without answer-supplying cues.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, printed tasks, and optional scratch paper. A calculator may support arithmetic after the method is stated. Use graph paper or movable labels when useful without supplying the requested construction.

Prerequisites

Reading tables, counting vertex degrees, and following an ordered route.

Pacing

Select two to four tasks for a session. Preserve first independent evidence, discuss a model, and revisit with a fresh task. The full banks are reusable choices, not a timed requirement.

Neutral access

Read directions aloud without identifying a method or answer. Enlarge diagrams, allow dictated or typed reasoning, and provide accessible tables. Preserve all mathematical conditions.

Keep evidence interpretable

Record neutral supports separately from answer-supplying cues. Retain first responses. A selected distractor is not by itself a diagnosis.

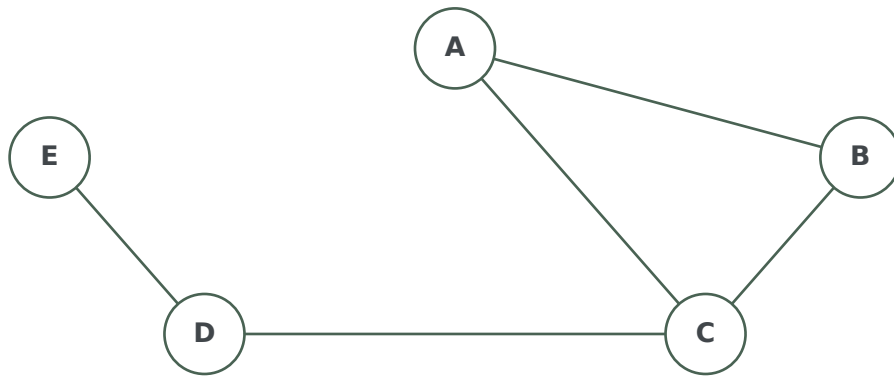
Conventions

Euler routes use every edge or arc once. Hamiltonian routes visit every vertex once. A circuit returns to its start. An open trail ends elsewhere. For ranking, a tournament has one directed result per pair; the specified wins-only rule may leave ties.

TEACHER MODEL

Audit edges and vertices separately

For edges AB, BC, CA, CD, DE, find an Euler trail and test for a Hamiltonian circuit.



Degrees are 2,2,3,2,1. C and E are odd, so E-D-C-A-B-C is an Euler trail. A Hamiltonian circuit is impossible because E has degree 1. Repeating C in the Euler trail is allowed; that would not be allowed internally in a Hamiltonian route.

Think aloud: First I decide whether the task covers edges or vertices. Then I use the appropriate condition and verify a route or give a decisive obstruction.

REUSABLE REASONING GUIDE

Represent justify interpret

Givens and conditions

Model and calculation

Justification and independent check

Conclusion in context

WARM-UP

Teaching script

Core 001 | DCS.GT.1.1

Collect a short first response before discussion. Ask learners what is given and which condition controls the method.

Display and ask

Four labs A,B,C,D share cables AB, AC, BC, CD. Draw the graph, write its vertex-edge table, and build the adjacency matrix in A,B,C,D order.

Work and discuss

The edge table has rows (A,B),(A,C),(B,C),(C,D). Draw exactly those four undirected edges. Adjacency matrix= $[0,1,1,0;1,0,1,0;1,1,0,1;0,0,1,0]$. Symmetry and zero diagonal check the simple undirected representation.

Listen for evidence

Three mutually consistent representations and explicit vertex order.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M01 for contrast after discussion. Select another task from M01-M04 or O01-O04 to check transfer. Record support and the new evidence separately.

I DO

Teaching script

Core 005 | DCS.GT.1.1

Make a prediction before modeling. Think aloud through the core task, making each calculation and check visible.

Display and ask

A graph's degree sequence is $(3,3,2,2,2)$. Use handshaking to find the edge count. Construct one simple graph with these degrees and verify it.

Work and discuss

The degree sum 12 gives 6 edges. Example on A,B,C,D,E: AB, AC, AD, BC, BE, DE. Degrees are A3,B3,C2,D2,E2, as required. Other correct simple constructions are valid.

Listen for evidence

Edge count, valid construction, and all degree checks.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M05 for contrast after discussion. Select another task from M05-M08 or O05-O08 to check transfer. Record support and the new evidence separately.

WE DO

Teaching script

Core 009 | DCS.GT.1.2

Ask learners to propose the next step and have a partner verify it against the original conditions.

Display and ask

For graph AB, BC, CA, CD, DE, find an Euler trail and prove no Euler circuit exists. State every vertex degree and identify endpoints.

Work and discuss

Degrees A2,B2,C3,D2,E1. Exactly C,E are odd, so an open Euler trail exists and an Euler circuit does not. Route E-D-C-A-B-C uses ED, DC, CA, AB, BC exactly once, ending at C.

Listen for evidence

Parity test, endpoint identification, and complete edge audit.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M09 for contrast after discussion. Select another task from M09-M13 or O09-O13 to check transfer. Record support and the new evidence separately.

COLLABORATIVE PRACTICE

Teaching script

Core O14 | DCS.GT.1.2

Assign reader, modeler, checker, and explainer roles. Require a private first attempt before comparing approaches.

Display and ask

Give a connected graph that has an Euler circuit but no Hamiltonian circuit. Supply an Euler route and a structural reason ruling out Hamiltonian circuits.

Work and discuss

Two triangles O-A-B-O and O-C-D-O work. Route O-A-B-O-C-D-O is Euler; O has degree 4 and others 2. Removing O disconnects the graph. A cycle covering both components would have to revisit O, impossible for a Hamiltonian circuit.

Listen for evidence

A verified construction and a valid structural obstruction, not merely failure to find a tour.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M14 for contrast after discussion. Select another task from M14-M18 or O14-O18 to check transfer. Record support and the new evidence separately.

INDEPENDENT PRACTICE

Teaching script

Core O19 | DCS.GT.1.2

Provide only neutral access supports. Keep the first response and ask for justification without selecting a method.

Display and ask

Construct a strongly connected digraph with equal in-degree and out-degree at every vertex. Give a directed Euler circuit and explain why following an undirected version of its edges would be insufficient.

Work and discuss

Example arcs $A \rightarrow B, B \rightarrow C, C \rightarrow A$ form a directed cycle with $\text{in}=\text{out}=1$ at every vertex. $A \rightarrow B \rightarrow C \rightarrow A$ is Euler. $A \rightarrow C \rightarrow B \rightarrow A$ would use the same undirected pairs but reverse every arrow and is invalid. Other verified balanced strongly connected constructions are accepted.

Listen for evidence

Directed construction, balance, reachability, and a valid route.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M19 for contrast after discussion. Select another task from M19-M23 or O19-O23 to check transfer. Record support and the new evidence separately.

SMALL-GROUP RETEACH

Teaching script

Core O24 | DCS.GT.1.3

Use the earlier work to identify a specific gap. Model that connection, then remove the cue on a fresh item.

Display and ask

Teams A,B,C form the cycle $A \rightarrow B \rightarrow C \rightarrow A$. Add team D that loses to all three. Rank by wins only and explain which ties remain.

Work and discuss

A,B,C each gain a win over D in addition to one cycle win, giving 2 wins each. D has 0. Thus A,B,C tie for first and D is last. The pairwise cycle does not establish a wins-only order among the tied teams.

Listen for evidence

Correct scores and no invented tiebreaker.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M24 for contrast after discussion. Select another task from M24-M27 or O24-O27 to check transfer. Record support and the new evidence separately.

EXIT TICKET

Teaching script

Core O28 | DCS.GT.1.3

Collect a concise claim and supporting computation before showing the key. Use the evidence to select a next task.

Display and ask

A source defines arrows $\text{loser} \rightarrow \text{winner}$ instead of $\text{winner} \rightarrow \text{loser}$. Explain how to rank by wins under this convention, then rank the arcs $B \rightarrow A, C \rightarrow A, C \rightarrow B$ for teams A, B, C .

Work and discuss

Wins are in-degrees under this convention. A has 2 incoming arcs, $B1, C0$, so $A > B > C$. Using out-degree would reverse the result. The convention, not an assumed arrow meaning, controls interpretation.

Listen for evidence

Adapt ranking to the stated direction and calculate the correct degree type.

Probe and respond

Ask whether the condition concerns edges or vertices. Have learners cross off the appropriate objects while tracing a route. For digraphs, check arrow direction and in/out degree separately.

Next: use M28 for contrast after discussion. Select another task from M28-M30 or O28-O30 to check transfer. Record support and the new evidence separately.

MULTIPLE-CHOICE BANK

M01 / M02 Select and justify**M01 | Warm-up | DCS.GT.1.1**

A graph models exams that share students. What is a suitable vertex-edge interpretation?

- A. Vertices are days; every edge is a student.
- B. Vertices are exams; an edge joins exams that conflict.
- C. Every pair of exams must be joined.
- D. Vertices are exams; edges mean they can share a time.

Reasoning: _____

M02 | Warm-up | DCS.GT.1.1

An undirected graph has edges AB, AC, BC, CD. In vertex order A,B,C,D, what is adjacency row C?

- A. (1,1,0,1)
- B. (1,0,0,1)
- C. (1,1,1,1)
- D. (0,0,1,0)

Reasoning: _____

WARM-UP

001 Show your reasoning

DCS.GT.1.1 | Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table.

Four labs A,B,C,D share cables AB, AC, BC, CD. Draw the graph, write its vertex-edge table, and build the adjacency matrix in A,B,C,D order.

Representation and calculation

Explanation and check

WARM-UP

002 Show your reasoning

DCS.GT.1.1 | Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table.

For the adjacency matrix $[0,1,0,1;1,0,1,0;0,1,0,1;1,0,1,0]$ in A,B,C,D order, recover the edge table and draw the graph. Find every degree and verify handshaking.

Representation and calculation

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 | DCS.GT.1.1****Answer B: Vertices are exams; an edge joins exams that conflict.**

A graph should encode the relation being modeled, not unrelated quantities. Here an undirected edge means the two exams cannot share a time.

- A. This does not encode the stated exam-pair conflicts.
- B. Correct. The edge records the scheduling restriction between two exams.
- C. Only pairs that actually share students need conflict edges.
- D. This reverses the given conflict relation.

Evidence: Map both vertices and edges to the context.

M02 | DCS.GT.1.1**Answer A: (1,1,0,1)**

The row has 1 at each neighbor's column and 0 at C's own column.

- A. Correct. C connects to A,B,D and has no loop.
- B. This omits edge BC.
- C. This adds a loop at C.
- D. This records only a nonexistent loop.

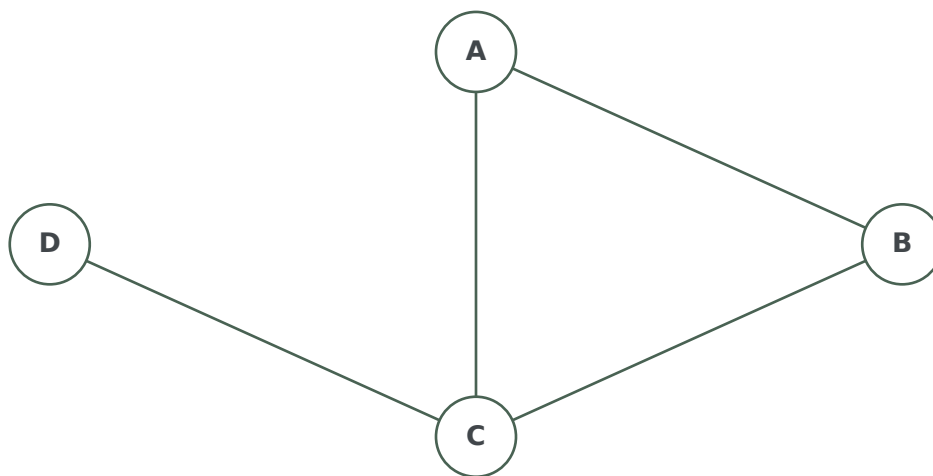
Evidence: Correct row order and loop convention.

WORKED TEACHER KEYS

001

001 | DCS.GT.1.1

The edge table has rows (A,B),(A,C),(B,C),(C,D). Draw exactly those four undirected edges. Adjacency matrix= $[0,1,1,0;1,0,1,0;1,1,0,1;0,0,1,0]$. Symmetry and zero diagonal check the simple undirected representation.



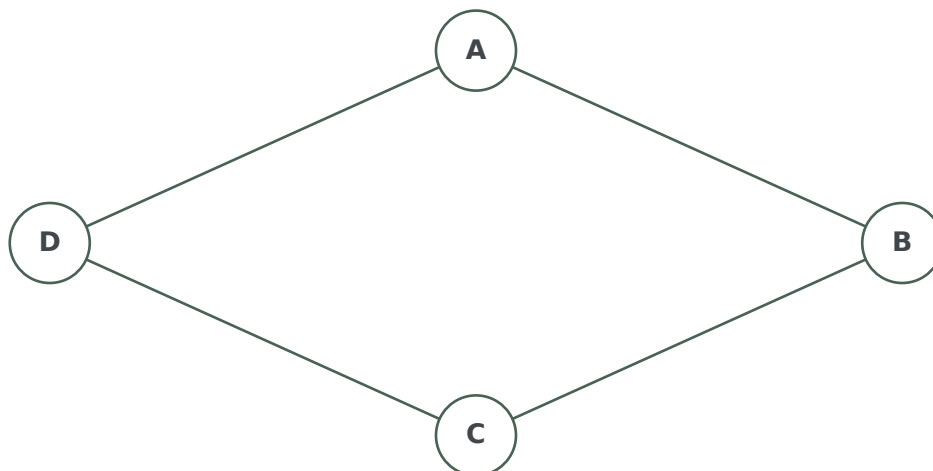
Evidence: Three mutually consistent representations and explicit vertex order.

WORKED TEACHER KEYS

002

002 | DCS.GT.1.1

Edges AB, AD, BC, CD form a 4-cycle. Every row sums to 2, so all degrees are 2. Degree sum $8=2 \cdot 4$ edges. Read each symmetric pair only once when constructing the edge table.



Evidence: Matrix-to-graph conversion, degree interpretation, and double-count check.