

TEACHER AND STUDENT RESOURCE

Counting and Constraining Polynomial Zeros



NC Math 3 • NC.M3.N-CN.9

The Complex Number System

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.N-CN.9 | NC Math 3 mathematics

This sample contains 21 complete selected pages from the 101-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 81

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

T1 Determine the actual degree and apply the Fundamental Theorem with its hypotheses.

T2 Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.

T3 Use real-coefficient conjugate pairs, equal multiplicities and parity.

T4 Test possible root inventories and construct polynomials satisfying stated constraints.

T5 Connect real graph evidence to zeros without overclaiming from an intercept set or viewing window.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M04, M09, M10, M19, M22, M24, O01, O04, O22

T2: M03, M16, M18, M25, M29, O02, O03, O15, O24, O25, O28

T3: M02, M05, M06, M15, M21, M26, O06, O10, O13, O16, O19, O26, O27

T4: M07, M08, M14, M17, M20, M23, M28, M30, O05, O07, O08, O09, O14, O17, O18, O21, O29

T5: M11, M12, M13, M27, O11, O12, O20, O23, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Count what the theorem counts

Learning target: Determine actual degree and distinguish multiplicity from distinct values.

Learning target: Identify the real-coefficient condition for conjugate pairs.

Use M01–M08 and O01–O08 selectively. Diagnose vocabulary before modeling a root-inventory table.

Session 2: Use constraints and challenge claims

Learning target: Apply degree, conjugate pairs and graph evidence together.

Learning target: Construct examples and counterexamples with verified degree.

Use selected M09–M23 and O09–O23 over multiple sessions. Require individual conjectures before group comparison.

Session 3: Repair and transfer

Learning target: Repair the specific misconception shown in prior work.

Learning target: Explain what an intercept set does and does not determine.

Choose reteach items M24–M27/O24–O27 by need; use M28–M30/O28–O30 for fresh independent evidence.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Identify the highest nonzero power, expand simple products, solve $x^2 = -a^2$ using $i^2 = -1$, read x-intercepts, and recognize that a real number is also a complex number.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Unless a task explicitly allows complex coefficients, its polynomial coefficients are real. A zero is an input z with $p(z)=0$. Count multiplicity only when asked or when applying the degree total; distinguish this from the number of distinct values. Conjugate-pair conclusions require real coefficients. The nonconstant Fundamental Theorem does not assign a finite root count to the zero polynomial.

TEACHER MODEL

Fill root slots without inventing information

A real degree-7 polynomial has zero 2 with multiplicity 3 and zero $-1+2i$ with multiplicity 1. Account for all forced roots, determine the possibilities for the remaining roots, and construct one example for each possibility.

Real coefficients force $-1-2i$ once. The known occurrences total $3+1+1=5$, leaving two slots. They can be two real occurrences, possibly equal, or one nonreal conjugate pair. A real-root completion is $(x-2)^3[(x+1)^2+4](x-5)^2$; a nonreal completion is $(x-2)^3[(x+1)^2+4](x^2+9)$. Both have degree 7. The information does not select one completion uniquely.

VISUAL ANCHOR

Keep three inventories separate

$p(x)=(x-2)^2(x^2+9)$: degree 4

Occurrences: 2, 2, 3i, -3i → total 4

Distinct complex zeros: {2,3i,-3i} → count 3

Real graph: only x-intercept (2,0) → count 1

A real-axis graph records distinct real zeros. It does not separately display multiplicity or nonreal zeros.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Determine the actual degree and apply the Fundamental Theorem with its hypotheses.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Have learners identify the highest nonzero power before counting roots. Ask them to state whether each number is a count of terms, distinct values or occurrences. Model the vocabulary only after preserving their first response. Use O01 to make the zero leading coefficient visible.

Core task

Determine the actual degree and total complex-root count of $p(x)=0x^7+4x^4-2$. Explain why the displayed 7 does not control the answer.

Teacher solution and discussion

The x^7 coefficient is zero, so that term contributes nothing. The simplified polynomial is $4x^4-2$, degree 4. It therefore has four complex zeros counted with multiplicity. A displayed exponent matters only if its coefficient is nonzero.

Evidence and response

Identify the zero coefficient, state degree 4, and include multiplicity in the count. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T4: Test possible root inventories and construct polynomials satisfying stated constraints.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Use a root-inventory table with columns for root, multiplicity and reason. For O05, write $2i$, then its forced conjugate $-2i$, then the given real root 1 . Multiply the conjugate factors before the real factor. Say explicitly that three distinct roots give a lower bound, while the constructed cubic proves the bound is attainable.

Core task

Construct a monic real polynomial of least degree with zeros 1 and $2i$. Show the forced root, give a real-coefficient factored form, and expand your result.

Teacher solution and discussion

Real coefficients force $-2i$. The three distinct zeros require degree at least 3. The product is $(x-1)(x-2i)(x+2i)=(x-1)(x^2+4)=x^3-x^2+4x-4$. It is monic, has real coefficients, and has exactly the three required root occurrences.

Evidence and response

Justify the conjugate, prove minimal degree with a root count, and verify the real expansion. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Test possible root inventories and construct polynomials satisfying stated constraints.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Start with O09: distinguish a minimum degree from an exact degree by checking both constructions. Then ask learners to propose a conclusion from a graph, then ask what degree or completeness assumptions it needs. In O11, the given degree 3 makes the three real roots complete. Contrast O12: a cropped screen does not establish the absence of real zeros. Have learners name exactly which evidence changed the conclusion.

Core task

A nonzero real polynomial satisfies $p(-2)=p(0)=p(3)=0$ and $p(1)=4$. Find its least possible degree and construct one example. Then construct a higher-degree example satisfying the same four conditions.

Teacher solution and discussion

Three distinct zeros require degree at least 3. Let $p(x)=k(x+2)x(x-3)$. At $x=1$ this is $-6k$, so $k=-2/3$. The cubic $p(x)=(-2/3)(x+2)x(x-3)$ works. A degree-5 example is $q(x)=p(x)(x^2+1)/2$: the first three zeros persist and $q(1)=p(1) \cdot 2/2=4$. Hence these data determine a minimum, not an exact degree.

Evidence and response

Use all four conditions, solve the scale factor, and verify that the higher-degree construction preserves $p(1)$. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Test possible root inventories and construct polynomials satisfying stated constraints.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign inventory keeper, constructor, hypothesis checker and skeptic. Use O14 as the core design: each learner first states a minimum degree, then the group checks touch/cross multiplicities and the forced nonreal pair. Require a polynomial that attains the proposed minimum, not only a slot count.

Core task

Design a monic real polynomial of least degree with exactly two distinct real zeros: it touches the axis at 0, crosses at 1, and also has at least one nonreal zero. Give the polynomial and justify every part of the design.

Teacher solution and discussion

Touching at 0 requires even multiplicity, minimally 2; crossing at 1 requires odd multiplicity, minimally 1. A nonreal root forces a conjugate pair, requiring at least two more slots. Minimum degree is 5. The polynomial $x^2(x-1)(x^2+1)$ works: its zeros are 0 twice, 1 once, i and $-i$. Near 0 the squared factor gives no sign change, while the simple factor $x-1$ changes sign at 1. The quadratic has no real zeros, so no other real intercepts occur.

Evidence and response

Justify the lower bound, produce real coefficients, and verify graph behavior and absence of additional real roots. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Use real-coefficient conjugate pairs, equal multiplicities and parity.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Keep the first unsupported response. For O19, ask for both a root table and separate distinct counts. Do not supply the conjugate or point to a multiplicity. Accept an equivalent real factored polynomial when every root and exponent is correct. Record access supports separately from mathematical cues.

Core task

Complete a root inventory for a real degree-8 polynomial. Known roots are -2 with multiplicity 3, 0 with multiplicity 1 and $1+i$ with multiplicity 2. List every forced root and multiplicity, decide whether any slots remain, and give distinct-real and distinct-nonreal counts.

Teacher solution and discussion

The conjugate $1-i$ is forced with multiplicity 2. The complete inventory is $-2:3$, $0:1$, $1+i:2$, $1-i:2$. Total $3+1+2+2=8$, so no slots remain. There are two distinct real zeros and two distinct nonreal zeros, with four real and four nonreal occurrences. One matching polynomial is $(x+2)^3 x[(x-1)^2+1]^2$.

Evidence and response

Match conjugate multiplicities, exhaust all eight slots, and distinguish distinct counts from occurrence totals. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Select a task based on the actual error. O24 separates real from nonreal without excluding real numbers from the complex set. O25 repairs distinct-versus-multiplicity labels. O26 checks a conjugate product with a discriminant, and O27 represents conjugation as reflection across the real axis. Recheck transfer with a different task, not the same answer recalled.

Core task

Sort the numbers 0, -3, $1+0i$, $2i$, $-2i$, $4+i$ and $4-i$ into real and nonreal groups. Count all of them as complex numbers. Could they be the seven simple zeros of a real degree-7 polynomial? Explain without expanding a seventh-degree expression.

Teacher solution and discussion

Real: 0, -3, $1+0i$. Nonreal: $2i$, $-2i$, $4+i$, $4-i$. All seven are complex numbers and are distinct. They can be the simple zeros of a real degree-7 polynomial because the four nonreal values form two conjugate pairs. One real factored form is $x(x+3)(x-1)(x^2+4)[(x-4)^2+1]$, whose degree is $1+1+1+2+2=7$.

Evidence and response

Classify $1+0i$ as real, use inclusive complex language, and pair the four nonreal roots correctly. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask for a compact inventory and one sentence explaining what it proves. O28 integrates factor degrees with multiplicity. O29 asks whether slots remain. O30 tests whether the learner can construct two graphs with the same intercept set but different unseen root inventories. Collect work before displaying either construction.

Core task

For $p(x)=(x^2+4)^3(x-5)$, give the degree, the complete zero inventory with multiplicities, the number of distinct complex zeros and the number of real x -intercepts. Use the answers as a consistency check.

Teacher solution and discussion

The degree is $2 \cdot 3 + 1 = 7$. Since $x^2+4=(x-2i)(x+2i)$, the roots are $2i$ with multiplicity 3, $-2i$ with multiplicity 3 and 5 with multiplicity 1. The multiplicities sum to 7. There are three distinct complex zeros but only one real x -intercept, $(5,0)$. Six occurrences are nonreal and one is real.

Evidence and response

Connect factor degrees, complex factorization and graph count, and show that the occurrence total equals seven. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

For $p(x)=7x^6-2x^2+9$, how many complex zeros are guaranteed when multiplicities are counted?

- A. 6
- B. 2
- C. 3
- D. Cannot be determined without factoring

Reasoning: _____

M02 | Warm-up | T3

Which number of nonreal zeros is impossible for a degree-5 polynomial with real coefficients, counting multiplicity?

- A. 3
- B. 2
- C. 0
- D. 4

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Determine the actual degree and apply the Fundamental Theorem with its hypotheses.

Determine the actual degree and total complex-root count of $p(x)=0x^7+4x^4-2$. Explain why the displayed 7 does not control the answer.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.

A root inventory lists -1 three times, 2 twice and 5 once. Make a multiplicity table, state the least possible degree, and write one monic polynomial realizing exactly this inventory.

Distinct zero	Multiplicity	Linear factor

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Determine the actual degree and apply the Fundamental Theorem with its hypotheses.

Answer: A. 6

The leading coefficient 7 is nonzero, so p has degree 6. The Fundamental Theorem of Algebra gives exactly six complex zeros counted with multiplicity; it does not say that all six are real or distinct.

Check each alternative

- A. Correct. The leading coefficient 7 is nonzero, so p has degree 6. The Fundamental Theorem of Algebra gives exactly six complex zeros counted with multiplicity; it does not say that all six are real or distinct.
- B. The smallest displayed nonconstant exponent is not the degree.
- C. Three is the number of nonzero terms, not the degree.
- D. Factoring finds the zeros, but the degree already determines their total count.

Evidence to assess

Identify degree 6 from the highest nonzero power and explicitly include multiplicity in the root count.

WORKED TEACHER KEYS

M02 / M03**M02 • T3: Use real-coefficient conjugate pairs, equal multiplicities and parity.****Answer: A. 3**

Nonreal zeros of a real-coefficient polynomial occur in conjugate pairs with equal multiplicity. Their total count must be even. Three is odd, while 0, 2 and 4 all fit degree 5.

Check each alternative

- A. Correct. Nonreal zeros of a real-coefficient polynomial occur in conjugate pairs with equal multiplicity. Their total count must be even. Three is odd, while 0, 2 and 4 all fit degree 5.
- B. One conjugate pair and three real zeros are possible.
- C. All five zeros may be real, as for x^5 .
- D. Two conjugate pairs and one real zero are possible.

Evidence to assess

Use both real coefficients and pair counting, not merely the fact that the degree is odd.

M03 • T2: Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.**Answer: B. Two distinct zeros; five zeros counted with multiplicity.**

The zeros are 4 with multiplicity 3 and -2 with multiplicity 2. There are two distinct inputs, but $3+2=5$ root occurrences, matching the degree.

Check each alternative

- A. The three copies of 4 and two copies of -2 are not five different numbers.
- B. Correct. The zeros are 4 with multiplicity 3 and -2 with multiplicity 2. There are two distinct inputs, but $3+2=5$ root occurrences, matching the degree.
- C. Only $x=4$ and $x=-2$ make a factor zero.
- D. This ignores the exponents on the factors.

Evidence to assess

Name both zeros, attach the correct multiplicity to each, and keep the two counts separate.

WORKED TEACHER KEYS

001 / 002**001 • T1: Determine the actual degree and apply the Fundamental Theorem with its hypotheses.**

The x^7 coefficient is zero, so that term contributes nothing. The simplified polynomial is $4x^4 - 2$, degree 4. It therefore has four complex zeros counted with multiplicity. A displayed exponent matters only if its coefficient is nonzero.

Evidence to assess

Identify the zero coefficient, state degree 4, and include multiplicity in the count.

002 • T2: Distinguish distinct zeros, multiplicity, real zeros and nonreal complex zeros.

The table is -1: multiplicity 3; 2: multiplicity 2; 5: multiplicity 1. Total multiplicity is $3+2+1=6$, so the least degree is 6. The monic polynomial $(x+1)^3(x-2)^2(x-5)$ realizes the inventory. There are three distinct real zeros but six occurrences.

Evidence to assess

Match each factor's sign and exponent to the root table, and distinguish three distinct zeros from degree six.