

TEACHER AND STUDENT RESOURCE

Inverse Trigonometry, Triangles and Quadrant Reasoning



Precalculus • PC.F.3

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.F.3 | North Carolina Precalculus mathematics

This sample contains 21 complete selected pages from the 98-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 77-78

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 77-78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of trigonometry to solve problems involving all types of triangles.

T1 PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

T2 PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

T3 PC.F.3.3 Use the Pythagorean identity with quadrant information to determine trigonometric values.

PC.F.3.1 Implement a strategy to generate all solutions to an equation that involves inverse trigonometric functions.

PC.F.3.2 Implement the Law of Sines and the Law of Cosines to solve problems.

PC.F.3.3 Implement the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M04, M07, M10, M13, M16, M19, M22, M25, M28, O01, O04, O07, O10, O13, O16, O19, O22, O25, O28

T2: M02, M05, M08, M11, M14, M17, M20, M23, M26, M29, O02, O05, O08, O11, O14, O17, O20, O23, O26, O29

T3: M03, M06, M09, M12, M15, M18, M21, M24, M27, M30, O03, O06, O09, O12, O15, O18, O21, O24, O27, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Learning target: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

Learning target: PC.F.3.3 Use the Pythagorean identity with quadrant information to determine trigonometric values.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Scientific calculator with both radian and degree modes, pencils and straightedge. Use the mode specified by the question. Triangle diagrams are schematic; calculate from labeled data, not ruler measurements.

Prerequisites

Unit-circle values, reference angles, real domains, algebraic equations and the Pythagorean theorem.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Inverse-function equations use radians unless explicitly marked in degrees. $\arcsin$ has range $[-\pi/2, \pi/2]$, $\arccos$ has range $[0, \pi]$, and $\arctan$ has range $(-\pi/2, \pi/2)$. Triangle angles in this packet are labeled in degrees; side a is opposite A , b opposite B and c opposite C . Triangle angles must be positive and sum to 180° , and side lengths must satisfy the triangle inequality. Keep full precision until the requested rounding. Quadrants exclude their boundary axes. The identity $\sin^2\theta + \cos^2\theta = 1$ gives magnitudes; quadrant information selects signs.

TEACHER REFERENCE

Three checks prevent most trigonometric errors

Principal values versus all solutions

For $\sin x = q$ with $-1 \leq q \leq 1$, let $\alpha = \arcsin q$. The families $x = \alpha + 2k\pi$ and $x = \pi - \alpha + 2k\pi$ cover all real solutions; at $q = \pm 1$ the families overlap. For cosine use $x = \pm \arccos q + 2k\pi$. For tangent use $x = \arctan q + k\pi$. Restrict these families to a requested interval only after generating them.

Inverse expressions have ranges

An equation such as $\arcsin u = 2\pi/3$ has no real solution even though taking sine produces an algebraic candidate. For $\arcsin(\sin x)$, the output must lie in the principal arcsine interval. Check the original inverse expression rather than only a transformed equation.

Choosing a triangle law

The Law of Sines is $a/\sin A = b/\sin B = c/\sin C$. It is useful when an opposite side-angle pair is known. The Law of Cosines is $c^2 = a^2 + b^2 - 2ab \cos C$, useful for SAS or SSS data. In SSA data, a principal arcsine can hide a second valid triangle, but a supplementary candidate can also be impossible.

Signs and identity domains

From $\cos^2 \theta = 1 - \sin^2 \theta$, taking a square root gives $|\cos \theta| = \sqrt{1 - \sin^2 \theta}$. Do not silently choose the positive root. Dividing the Pythagorean identity by $\cos^2 \theta$ gives $1 + \tan^2 \theta = \sec^2 \theta$ only where $\cos \theta \neq 0$.

TEACHER MODEL

One inverse value may hide another triangle

For $A=30^\circ$, $a=7$ and $b=10$, how many triangles satisfy the data?

The Law of Sines gives $\sin B=5/7$. A calculator in degree mode returns $B\approx 45.6^\circ$, but the supplement 134.4° has the same sine. Both leave a positive third angle after subtracting $A=30^\circ$, giving $C\approx 104.4^\circ$ or 15.6° . Therefore there are two triangles. The height check $10\sin 30^\circ=5<7<10$ independently predicts the ambiguous case. Keep both configurations until the angle-sum and original side relations have been checked.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Solve $\sin x = 0.4$ for every real x . Express the complete set with $\alpha = \arcsin(0.4)$, then list all solutions on $0 \leq x < 2\pi$ to four decimals and check their quadrants.

Teacher solution and discussion

Let $\alpha = \arcsin(0.4) \approx 0.4115$. The complete set is $x = \alpha + 2k\pi$ or $x = \pi - \alpha + 2k\pi$ for integer k . In $[0, 2\pi)$, the values are approximately 0.4115 and 2.7301. Sine is positive in quadrants I and II, matching these solutions. A lone principal inverse value misses the second family.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

A triangle has $a=6$ cm, $b=9$ cm and included angle $C=40^\circ$. Solve for c and the remaining angles to one decimal place. Explain why starting with the Law of Sines is not possible before another value is found.

Teacher solution and discussion

The given data have no known opposite side-angle pair, so first use $c^2=6^2+9^2-2\cdot6\cdot9\cos40^\circ$. This gives $c\approx5.9$ cm (unrounded 5.853819...). Then $\cos A=(9^2+c^2-6^2)/(2\cdot9\cdot c)$ gives $A\approx41.2^\circ$, and $B\approx98.8^\circ$. Using arccos directly determines the triangle angle without a supplementary sine candidate. If sine were used instead, a supplementary A could leave a positive B but would not match the given side $b=9$; positivity alone would not be enough to accept it. The largest side $b=9$ is opposite the obtuse largest angle.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: PC.F.3.3 Use the Pythagorean identity with quadrant information to determine trigonometric values.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Given $\tan\theta = -5/12$ and θ in quadrant IV, derive $\cos\theta$ and $\sin\theta$ using $1 + \tan^2\theta = \sec^2\theta$. Explain why the quadrant is still needed even after squaring.

Teacher solution and discussion

$1 + 25/144 = 169/144 = \sec^2\theta$, so $\cos^2\theta = 144/169$. Quadrant IV selects $\cos\theta = 12/13$. Then $\sin\theta = \tan\theta \cos\theta = (-5/12)(12/13) = -5/13$. Squaring determines only magnitudes and erases signs; quadrant II would have the same tangent but the opposite sine/cosine signs.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

For SSA data $A=30^\circ$, $a=5$ and $b=10$, solve the triangle and explain why there is only one nondegenerate configuration even though inverse sine often produces supplementary candidates.

Teacher solution and discussion

$\sin B = 10 \sin 30^\circ / 5 = 1$, so $B = 90^\circ$. Its supplement is also 90° , not a distinct second angle. Then $C = 60^\circ$ and $c = 5 \sin 60^\circ / \sin 30^\circ = 5\sqrt{3} \approx 8.7$. This is the altitude boundary $a = b \sin A$, where the two potential configurations coincide in one right triangle.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Solve $\cos x = 0.2$ for every real x and list the solutions on $[-\pi, \pi]$ to four decimals. Use a one-cycle graph to explain how the chosen interval changes which representatives are listed.

Teacher solution and discussion

Let $\alpha = \arccos(0.2) \approx 1.3694$. The complete solution is $x = \pm\alpha + 2k\pi$ for integer k . On $[-\pi, \pi]$, the representatives are -1.3694 and 1.3694 , not α and $2\pi - \alpha$. The latter positive representative lies beyond π ; subtracting 2π gives $-\alpha$. The two endpoints have cosine -1 and are not solutions.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: PC.F.3.3 Use the Pythagorean identity with quadrant information to determine trigonometric values.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

A learner proposes $\sin\theta = -2/3$ and $\cos\theta = -\sqrt{5}/3$ for a quadrant III angle. Determine whether the data are consistent, compute tangent, and explain whether the quadrant alone could validate the proposed magnitudes.

Teacher solution and discussion

The signs match quadrant III, and the squared sum is $4/9 + 5/9 = 1$, so the pair is consistent. Tangent is $(-2/3)/(-\sqrt{5}/3) = 2/\sqrt{5} = 2\sqrt{5}/5$. Matching quadrant signs alone would not validate arbitrary magnitudes; the unit identity must also hold. Here both conditions are satisfied.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Solve $2 \arctan x = \arccos(3/5)$ over the real numbers. Use the principal ranges to select a sign and check the original inverse equation, rather than retaining every root obtained after applying cosine.

Teacher solution and discussion

Let $\theta = \arctan x$. The right side is an acute angle, so $0 < 2\theta < \pi/2$ and $x > 0$. With $\tan \theta = x$, the identity $\cos(2\theta) = (1 - x^2)/(1 + x^2)$ follows from $\cos^2 \theta - \sin^2 \theta$ and $1 + \tan^2 \theta = \sec^2 \theta$. Thus $(1 - x^2)/(1 + x^2) = 3/5$, giving $5 - 5x^2 = 3 + 3x^2$ and $x^2 = 1/4$. Select $x = 1/2$ because $x > 0$. Its θ is acute with 2θ acute and cosine $3/5$, so $2 \arctan(1/2) = \arccos(3/5)$. The root $-1/2$ has the wrong sign for the original equation.

Evidence and response

Ask whether an inverse result is one principal value or a complete solution set. In triangle work, label opposite pairs before choosing a law, test supplementary sine candidates and reject nonpositive third angles. In identity work, keep both square-root signs until the stated quadrant selects one. Require an original-equation or geometry check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****What is the principal value $\arcsin(\sqrt{3}/2)$?**

- A. $2\pi/3$
- B. $\pi/3$
- C. $-\pi/3$
- D. $\pi/6$

Reasoning: _____

M02 | Warm-up | T2**A triangle has $a=7$, $b=8$ and included angle $C=60^\circ$. What is c ?**

- A. $\sqrt{57}$
- B. $\sqrt{113}$
- C. 15
- D. 1

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Solve $\sin x = 0.4$ for every real x . Express the complete set with $\alpha = \arcsin(0.4)$, then list all solutions on $0 \leq x < 2\pi$ to four decimals and check their quadrants.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

A triangle has side lengths $a=5$, $b=7$ and $c=8$. Draw and label the opposite side-angle pairs, solve for all three angles to the nearest tenth of a degree, and verify their sum and size ordering.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.

Answer: B. $\pi/3$

The angle $\pi/3$ lies in $[-\pi/2, \pi/2]$ and has sine $\sqrt{3}/2$, so it is the inverse-function value.

Check each alternative

- A. This has the same sine but lies outside the principal arcsine range.
- B. Correct. The angle $\pi/3$ lies in $[-\pi/2, \pi/2]$ and has sine $\sqrt{3}/2$, so it is the inverse-function value.
- C. Its sine is negative.
- D. Its sine is $1/2$.

M02 • T2: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

Answer: A. $\sqrt{57}$

$$c^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cos 60^\circ = 49 + 64 - 56 = 57.$$

Check each alternative

- A. Correct. $c^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cos 60^\circ = 49 + 64 - 56 = 57$.
- B. This does not subtract the full cosine term $2ab \cos C = 56$.
- C. This adds the two sides rather than using the included angle.
- D. This subtracts the side lengths, which would correspond to a degenerate configuration.

WORKED TEACHER KEYS

001**001 • T1: PC.F.3.1 Generate complete solution sets using inverse trigonometric functions and their principal ranges.**

Let $\alpha = \arcsin(0.4) \approx 0.4115$. The complete set is $x = \alpha + 2k\pi$ or $x = \pi - \alpha + 2k\pi$ for integer k . In $[0, 2\pi)$, the values are approximately 0.4115 and 2.7301. Sine is positive in quadrants I and II, matching these solutions. A lone principal inverse value misses the second family.

Evidence to assess

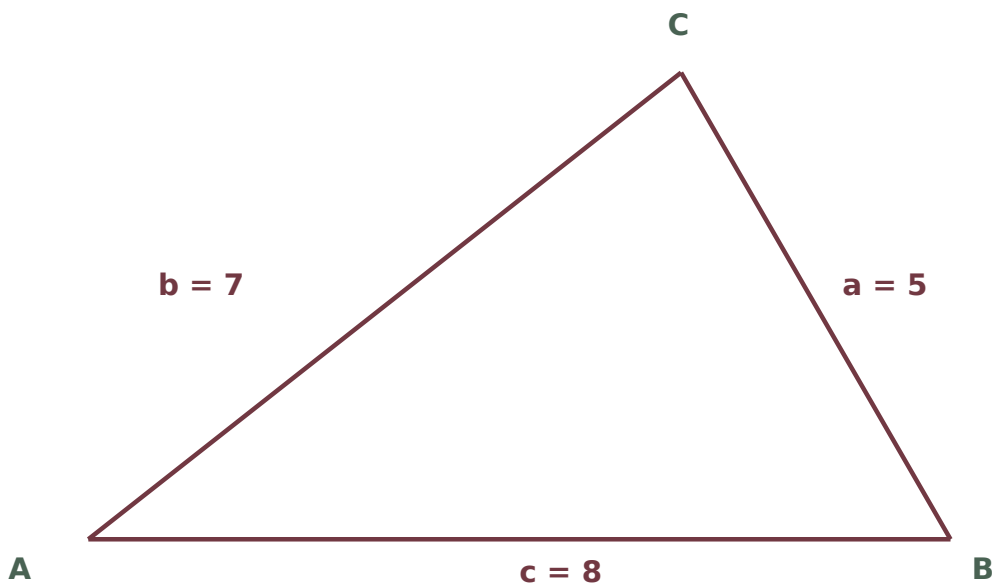
Full evidence: Both all-real families, integer parameter, both rounded interval values and sign/quadrant reasoning. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002

002 • T2: PC.F.3.2 Apply the Laws of Sines and Cosines, including ambiguous and impossible triangle cases.

$\cos A = (7^2 + 8^2 - 5^2) / (2 \cdot 7 \cdot 8) = 11/14$, so $A \approx 38.2^\circ$. $\cos B = (5^2 + 8^2 - 7^2) / (2 \cdot 5 \cdot 8) = 1/2$, so $B = 60.0^\circ$. Then $C = 180^\circ - A - B \approx 81.8^\circ$. The unrounded angles sum to 180° , and the largest angle C is opposite the largest side 8, while the smallest A is opposite 5. The triangle inequality holds.



Triangle schematic; use stated measures, not measurements from the page.

Evidence to assess

Full evidence: An opposite-pair diagram, correct cosine equations, all three angles, angle-sum check and side/angle ordering. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.