

TEACHER AND STUDENT RESOURCE

Structure Reveals Equivalent Expressions



NC Math 3 • NC.M3.A-SSE.2

Seeing Structure in Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-SSE.2 | NC Math 3 mathematics

This sample contains 21 complete selected pages from the 94-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76-77

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76-77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use the structure of an expression to identify ways to write equivalent expressions.

T1 Identify common factors, square/cube identities and multivariable patterns.

T2 Choose equivalent forms that reveal a useful value, bound, zero or numerical shortcut.

T3 Justify equivalence, diagnose invalid transformations and preserve original domains.

T4 Treat a repeated expression as one entity when substituting, grouping or factoring.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M07, M08, M12, M18, M20, M21, M25, M26, O01, O02, O07, O10, O17, O20, O28

T2: M10, M13, M16, M23, M28, M30, O08, O12, O18, O23, O30

T3: M11, M14, M17, M27, M29, O03, O04, O09, O13, O14, O16, O19, O24, O25, O26

T4: M04, M05, M06, M09, M15, M19, M22, M24, O05, O06, O11, O15, O21, O22, O27, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Recognize and justify patterns

Learning target: Identify complete square/cube patterns and common factors.

Learning target: Verify identities by expansion and a stated domain.

Use selected Warm-up and I Do tasks. Require learners to name the structure before calculating.

Session 2: Use a structure for a purpose

Learning target: Choose a form that reveals a bound, zero or efficient computation.

Learning target: Treat binomials, powers and reciprocals as repeated entities.

Use selected We Do and Collaborative tasks. Compare two methods and critique invalid cancellation or overgeneralization.

Session 3: Transfer and repair

Learning target: Explain why the transformation is valid for the full allowed domain.

Learning target: Construct and verify equivalent forms independently.

Use Independent tasks, then select reteach by error type. Exit tasks ask for a clear structure, identity and check.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Distribute and combine like terms; use exponent rules, basic polynomial multiplication, real and complex square roots, and denominator restrictions. The complex-factor and quartic-identity tasks are selected extensions, not prerequisites for the initial tasks.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

An identity holds for every input in the stated common domain. A finite set of successful numerical checks is not a general proof. State the allowed coefficient set when asking for complete factorization. A simplified expression must retain original denominator or radical restrictions. Area pictures use positive lengths, while the related polynomial identities can hold on a broader real domain.

TEACHER MODEL

Keep the repeated group intact

Rewrite $(x+2)^2-6(x+2)+9$ without first expanding every term. Verify the result and explain what the new form reveals.

Let $u=x+2$. The expression is $u^2-6u+9=(u-3)^2$. Substitute back: $[(x+2)-3]^2=(x-1)^2$. Direct expansion gives $x^2+4x+4-6x-12+9=x^2-2x+1$, matching $(x-1)^2$. The square form shows a nonnegative value for every real x , with minimum 0 at $x=1$.

VISUAL ANCHOR

Recognize → replace → return → verify

$$(x+2)^2-6(x+2)+9$$

Repeated entity $u=x+2 \rightarrow u^2-6u+9$

Perfect square $(u-3)^2 \rightarrow (x-1)^2$

Expansion check: x^2-2x+1 ; minimum 0 at $x=1$.

The identity acts on the entire repeated quantity, not only on the letter inside it.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Identify common factors, square/cube identities and multivariable patterns.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Before factoring, have learners mark the operation between the terms and identify whole squared quantities. Use O01 to contrast three superficially similar expressions. Require a statement of the allowed coefficient set before declaring a quadratic unfactorable.

Core task

Sort x^2-16 , x^2+16 and $x^2-8x+16$ into difference of squares, sum of squares and perfect-square trinomial. Factor each as far as possible using real linear factors, and explain why their similar terms do not justify the same rule.

Teacher solution and discussion

x^2-16 is a difference of squares: $(x-4)(x+4)$. x^2+16 is a sum of squares and has no real linear factors because it is positive for every real x . $x^2-8x+16$ is $(x-4)^2$ because its middle term is $-2 \cdot x \cdot 4$. The signs and the presence of the middle term distinguish the patterns; matching only x^2 and 16 is insufficient.

Evidence and response

Classify all three structures, justify the real-factor limitation and identify the double-product term. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T4: Treat a repeated expression as one entity when substituting, grouping or factoring.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

For O05, physically box every occurrence of $3x+2$ and rename that whole box u . Factor in u , substitute back and expand the final product. Say why the substitution must be consistent in every term. Do not skip the return to x .

Core task

Rewrite $(3x+2)^2-5(3x+2)+6$ by using a substitution before expanding. Return to x , simplify the factors and verify with a direct expansion.

Teacher solution and discussion

Set $u=3x+2$. Then $u^2-5u+6=(u-2)(u-3)$. Returning to x gives $(3x)(3x-1)$. Its expansion is $9x^2-3x$. Directly, $(3x+2)^2-5(3x+2)+6=9x^2+12x+4-15x-10+6=9x^2-3x$. Both routes agree.

Evidence and response

Name and use the same repeated entity throughout, then verify the translated factors. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Justify equivalence, diagnose invalid transformations and preserve original domains.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O09 to separate two questions: do formulas agree where both exist, and do the naturally defined functions have the same domain? Record $x=3$ as excluded before canceling. Then ask which identity a different task needs rather than assuming every expression is a difference of squares.

Core task

Compare $F(x)=(x^2-x-6)/(x-3)$ with $G(x)=x+2$. Factor and simplify F , state where their values agree, and explain why they are not the same function if each is assigned its natural real domain. Check $x=0$ and $x=3$.

Teacher solution and discussion

Factor $x^2-x-6=(x-3)(x+2)$. Thus $F(x)=x+2$ for $x \neq 3$. At $x=0$ both equal 2. At $x=3$, F is undefined because its original denominator is 0, while $G(3)=5$. Their formulas agree on F 's domain, but their natural domains differ, so they are not identical functions with those domains.

Evidence and response

Keep the excluded input and distinguish equivalent values on a domain from equality of naturally defined functions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Justify equivalence, diagnose invalid transformations and preserve original domains.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Give one partner a proposed identity, another a symbolic-check role, and another a counterexample role. In O14 the two successful test inputs are special equality cases, so the group must factor the difference before making a universal claim. Record the scope of the conclusion explicitly.

Core task

Two expressions, x^2+x and $2x$, agree at $x=0$ and $x=1$. A learner concludes they are equivalent for every real x . Refute the conclusion with another input, then factor their difference to determine exactly when they agree.

Teacher solution and discussion

At $x=2$, $x^2+x=6$ while $2x=4$, so they are not identically equal. Their difference is $x^2+x-2x=x^2-x=x(x-1)$. It equals 0 exactly when $x=0$ or $x=1$. Those two checked inputs are precisely the special equality cases, not evidence of a universal identity.

Evidence and response

Give a counterexample and use a factored difference to characterize all equality cases. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Justify equivalence, diagnose invalid transformations and preserve original domains.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O19 to assess whether the learner retains both denominator exclusions after a higher-power cancellation. Preserve the first response without pointing to $x=-1$. Collect the factorization, allowed domain and final expression as separate evidence.

Core task

Simplify $(x^4-1)/(x^2-1)$ while preserving all real-domain restrictions. Give a factorization showing the cancellation, and explain why a solution that lists only $x \neq 1$ is incomplete.

Teacher solution and discussion

Factor $x^4-1=(x^2-1)(x^2+1)$. The original denominator $x^2-1=(x-1)(x+1)$ is zero at both $x=1$ and $x=-1$. For $x \neq \pm 1$, cancellation gives x^2+1 . The restriction $x \neq 1$ alone misses the second denominator zero at -1 ; neither excluded input is restored by simplification.

Evidence and response

Factor both the numerator structure and denominator zeros, retaining both exclusions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: Justify equivalence, diagnose invalid transformations and preserve original domains.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

For O24, box the whole product before applying the negative sign. O25 compares a numerator with a shared factor against one without it; require words explaining cancellation, not only crossed-out symbols. O26 distinguishes similarly named identities, and O27 uses opposite binomials deliberately.

Core task

Expand $-(x-2)(x+3)$. A learner obtains $-x^2+x-6$. Find the error and provide a numerical check that distinguishes the two expressions.

Teacher solution and discussion

First $(x-2)(x+3)=x^2+x-6$. The leading negative multiplies every term, yielding $-x^2-x+6$. The learner applied -1 to x^2 but did not reverse the signs of $+x$ and -6 . At $x=0$, the original is $-(-2)(3)=6$, the correct expansion is 6, and the proposed $-x^2+x-6$ gives -6 .

Evidence and response

Treat the entire product as the entity multiplied by -1 and check a simple input. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Identify common factors, square/cube identities and multivariable patterns.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask learners to identify the structure, perform one exact transformation and justify it. O28 combines two factoring stages, O29 tests transfer to an exponential entity, and O30 connects an identity to parity and an area model. Equivalent forms and valid alternate proofs are accepted.

Core task

Factor $6x^2-24$ completely over the integers. Explain why removing the common numerical factor first makes the remaining structure easier to see, and verify the product.

Teacher solution and discussion

The greatest common numerical factor is 6: $6x^2-24=6(x^2-4)$. The bracket is a difference of squares, so the full factorization is $6(x-2)(x+2)$. Multiplying the conjugate binomials gives x^2-4 and multiplying by 6 returns $6x^2-24$. Factoring out 6 reveals simple squares rather than requiring square roots of the original coefficients.

Evidence and response

Apply common-factor and difference-of-squares structures in sequence and recover the original expression. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Which factorization uses the difference-of-squares structure of x^2-49 ?**

- A. $(x-49)(x+1)$
- B. $(x-7)(x+7)$
- C. $(x-7)^2$
- D. $x(x-49)$

Reasoning: _____

M02 | Warm-up | T1**Which structure explains $x^2+10x+25$?**

- A. It is $(x+10)^2$ because the middle coefficient is 10.
- B. It is $(x+25)^2$ because the constant is 25.
- C. It is $(x+5)^2$ because $10x=2 \cdot x \cdot 5$.
- D. It is $(x+5)(x-5)$ because $25=5^2$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Identify common factors, square/cube identities and multivariable patterns.

Sort x^2-16 , x^2+16 and $x^2-8x+16$ into difference of squares, sum of squares and perfect-square trinomial. Factor each as far as possible using real linear factors, and explain why their similar terms do not justify the same rule.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Identify common factors, square/cube identities and multivariable patterns.

Use a four-region area model to expand $(x+3)(x+4)$. Label every partial product, combine like terms, and explain why the two middle products combine but x^2 and the constant do not. Assume $x > 0$ for the area interpretation.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Identify common factors, square/cube identities and multivariable patterns.****Answer: B. $(x-7)(x+7)$**

Recognize $49=7^2$. The identity $u^2-v^2=(u-v)(u+v)$ gives $x^2-49=(x-7)(x+7)$. The opposite middle terms cancel when expanded.

Check each alternative

A. This expands to $x^2-48x-49$.

B. Correct. Recognize $49=7^2$. The identity $u^2-v^2=(u-v)(u+v)$ gives $x^2-49=(x-7)(x+7)$. The opposite middle terms cancel when expanded.

C. This introduces a $-14x$ term and a positive constant 49.

D. This produces x^2-49x , not x^2-49 .

Evidence to assess

Identify both squares and explain cancellation of the cross terms.

M02 • T1: Identify common factors, square/cube identities and multivariable patterns.**Answer: C. It is $(x+5)^2$ because $10x=2 \cdot x \cdot 5$.**

The first and last terms are x^2 and 5^2 , and the middle term is twice their base product. Hence the expression is the perfect square $(x+5)^2$.

Check each alternative

A. That square has middle term $20x$ and constant 100.

B. The constant of that square is 625.

C. Correct. The first and last terms are x^2 and 5^2 , and the middle term is twice their base product. Hence the expression is the perfect square $(x+5)^2$.

D. Opposite factors eliminate the middle term and give a negative constant.

Evidence to assess

Check all three terms, especially the double-product term.

WORKED TEACHER KEYS

001**001 • T1: Identify common factors, square/cube identities and multivariable patterns.**

x^2-16 is a difference of squares: $(x-4)(x+4)$. x^2+16 is a sum of squares and has no real linear factors because it is positive for every real x . $x^2-8x+16$ is $(x-4)^2$ because its middle term is $-2 \cdot x \cdot 4$. The signs and the presence of the middle term distinguish the patterns; matching only x^2 and 16 is insufficient.

Evidence to assess

Classify all three structures, justify the real-factor limitation and identify the double-product term.

WORKED TEACHER KEYS

002

002 • T1: Identify common factors, square/cube identities and multivariable patterns.

Partition one side into x and 3 , the other into x and 4 . The regions have areas $x^2, 4x, 3x, 12$. Their sum is $x^2 + 7x + 12$. The terms $4x$ and $3x$ have the same variable part and combine to $7x$; x^2 and 12 have different variable parts. The polynomial identity is valid for every real x , although the literal positive-length area model uses $x > 0$.

	x	4
x	x^2	$4x$
3	$3x$	12

Schematic teacher model; lengths are represented by labels.

Evidence to assess

Connect each product to a region and distinguish the geometric-model domain from the algebraic identity.