

TEACHER AND STUDENT RESOURCE

When Graphs Meet



NC Math 1 • NC.M1.A-REI.11

Intersections, equations and numerical solutions

Seven instructional sections with 30 multiple-choice and 30 open/performance tasks, detailed teaching scripts, original graphs, student grids, refinement tables and worked keys.

Free sample | Contents

NC.M1.A-REI.11 | NC Math 1 mathematics

This sample contains 20 complete selected pages from the 108-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 31-32, 61, 76-77

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 31

O02: sample PDF page(s) 18; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76-77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, and/or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values.

T1 Explain why shared graph points correspond to equation solutions.

T2 Determine and interpret intersections and their input values.

T3 Approximate solutions by refining a table of function values.

T4 Use graphing technology and verify numerical intersections.

T5 Interpret solutions within a stated domain and context.

Linear, quadratic and exponential pairs appear throughout. Graphing-tool performances and successive table refinements provide direct evidence; choice questions alone do not establish these skills.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M05, M24, O01, O02, O24

T2: M03, M06, M07, M10, M11, M19, M20, M21, M28, O03, O07, O08, O12, O27

T3: M04, M08, M09, M13, M16, M18, M22, M25, M27, O04, O06, O09, O11, O13, O17, O20, O22, O25, O29

T4: M12, M17, M23, M26, M29, O05, O10, O16, O18, O19, O23, O26, O28

T5: M14, M15, M30, O14, O15, O21, O30

PLANNING

Materials, access and pacing

Materials

Pencils, straightedge, these student grids, and a school-approved graphing calculator or graphing application with two-function plots, a table, and an intersection or trace/zoom feature. A device is required for tasks labeled TECHNOLOGY. A scientific calculator supports exponential table work. No paid product is specified.

Prerequisites

Evaluate linear, quadratic and exponential functions; plot ordered pairs; distinguish an input from an output; compare decimal numbers. Teach needed arithmetic before collecting independent evidence.

Across several sessions

Use a warm-up, a model and one shared task first. Continue with collaborative and independent tasks in later sessions; use reteaching and exit evidence to choose the next step. Select two to four tasks at a time. This is a reusable bank, not a 60-item one-sitting test.

Neutral access

Read text aloud, enlarge grids, allow dictation or an accessible equivalent graphing tool, and provide keyboard or pointing supports. Preserve the input/output reasoning and tool operation demand. Record inaccessible technology evidence as N/O until an accessible performance is possible.

Protect first evidence

Keep first independent work before sharing keys. Record neutral access supports separately from answer-supplying cues. A teacher reference graph does not replace the learner's own graph or tool record.

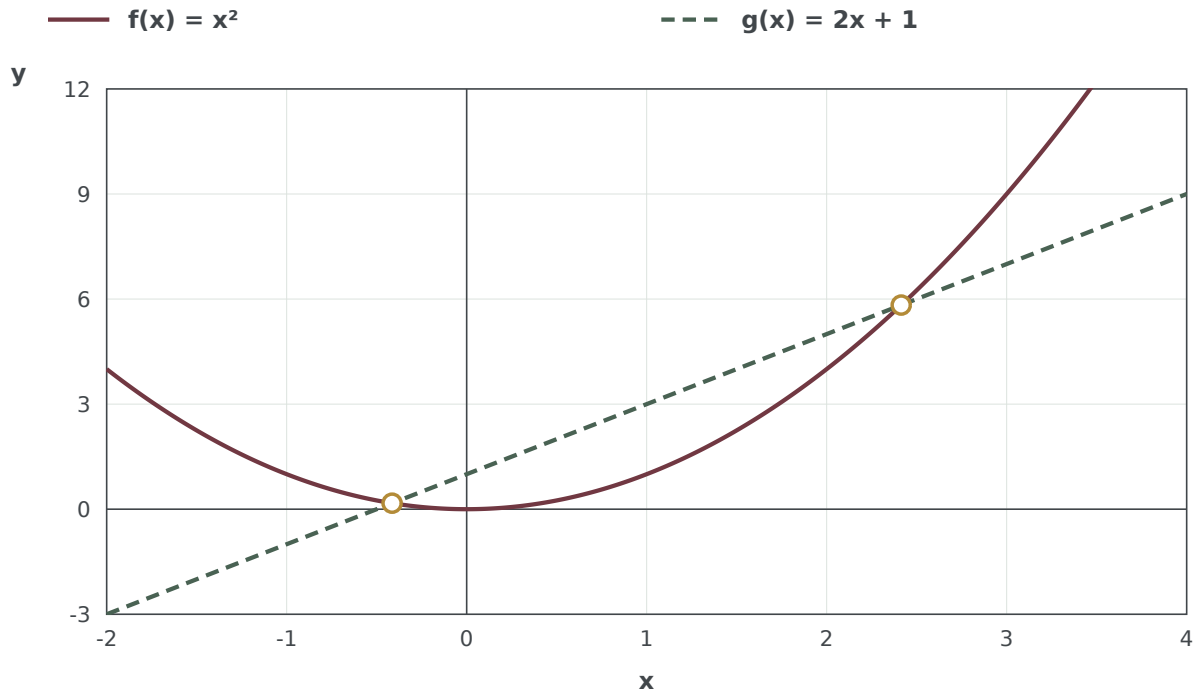
Rounding and domains

Use real inputs unless a task states a restricted domain. Report approximate inputs with $\approx$. Round to the requested place using the usual school rule (a following digit of 5 or more rounds the preceding digit up in magnitude). Retain unrounded values in calculations. Table tasks require a narrowing interval; a rounded output equality alone is not proof.

TEACHER MODEL

One input, equal outputs

Compare $f(x) = x^2$ and $g(x) = 2x + 1$. Find and interpret their intersections. Show how a refined table supports a graphing estimate.



At a shared point (a, b) , $b = f(a)$ and $b = g(a)$, hence $f(a) = g(a)$. Here $x^2 - 2x - 1 = 0$ gives $x = 1 \pm \sqrt{2}$, approximately -0.414214 and 2.414214 . The shared outputs are approximately 0.171573 and 5.828427 . For the positive solution, $d(2.414) = -0.000604$ and $d(2.415) = 0.002225$; continuity places the root between those inputs, so $x \approx 2.41$ to hundredths. A school-approved tool can locate the same intersections. Record its window and method, then check both original outputs. The teacher reference graph below is generated from the formulas; it is not a software test.

METHOD ANCHOR

Graph, refine, check

Why an input is a solution

A point (a, b) on both graphs gives $f(a) = b$ and $g(a) = b$. Therefore $f(a) = g(a)$. Conversely, if $f(a) = g(a) = b$, the point (a, b) belongs to both graphs. Touching counts, and coincident graphs may give infinitely many inputs.

Graphing technology routine

Enter each complete formula in a separate y-slot. Set the task's x- and y-window. Plot both functions and use intersection, trace/zoom, or a tool table to refine every visible shared point. Search separate neighborhoods for separate roots and inspect domain endpoints. Record tool, window, method and unrounded coordinates. Copy a labeled sketch, then check both original outputs.

Successive table routine

Define $d(x) = f(x) - g(x)$. Find endpoint values with opposite signs; these continuous function types have a zero between them. Narrow the input interval by tenths, hundredths and then thousandths as requested. Keep enough output digits to preserve the signs. Stop only when the bracket settles the requested rounding; report an approximate input and common output.

Important limitations

Same-sign endpoint values do not prove no root: a graph can touch zero, or two roots may lie between samples. A finite window does not prove the full real solution count. Rounded outputs that look equal need finer checking. Changing a noninteger root into an integer may destroy equality.

What to collect

For technology: the learner's tool name, window, method, coordinates, sketch and substitution check. For a table: successive input brackets, both outputs or signed differences, a justified precision statement and an intersection explanation. Teacher figures in this packet are formula-generated references; no graphing software is claimed to have been tested.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, functions and allowed input domain**Representation / tool and window / table bracket****Original-output check and justification****Solution input, common output, units and precision**

WARM-UP

Teaching script

Core: O01 • T1: Explain why shared graph points correspond to equation solutions.

Optional bank: M01–M04; O01–O04. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Ask for a private sketch first. Say, “Point to the input coordinate, then to the common output. How do both formulas verify that point?” Collect the initial answer before discussing the lines.

Core task

Graph $f(x) = x + 1$ and $g(x) = 5 - x$ on the supplied grid. Label both lines and their intersection. State the solution of $f(x) = g(x)$, then explain why the input, rather than the output, solves the equation.

Worked teacher reasoning

For f , plot $(0, 1)$, $(2, 3)$, $(4, 5)$. For g , plot $(0, 5)$, $(2, 3)$, $(4, 1)$. The shared point is $(2, 3)$. Thus $x = 2$ because $f(2) = 3 = g(2)$. The number 3 is the common output. Equating gives $x + 1 = 5 - x$, so $2x = 4$.

Evidence and response

If a learner reports 3 as the solution, ask them to evaluate each function at input 3. The unequal results reveal why the shared output is not the requested input. Follow with O02.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

I DO

Teaching script

Core: 005 • T4: Use graphing technology and verify numerical intersections.

Optional bank: M05–M08; O05–O08. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Use a school-approved graphing tool in front of learners. Say, “I will enter each entire expression, set the given window and inspect both sides of the y-axis.” Locate one crossing, then deliberately search for the second. Record the unrounded coordinates before rounding the input. Substitute into both original formulas.

Core task

TECHNOLOGY: In a school-approved graphing tool enter $y_1 = x^2$ and $y_2 = 4 - x$. Use $-4 \leq x \leq 4$ and $-1 \leq y \leq 12$. Find every visible intersection with an intersection feature or trace/zoom. Record the tool, window and method, sketch both curves, and report the solution inputs to the nearest hundredth. Check both outputs at each unrounded tool input.

Worked teacher reasoning

The intersections are approximately $(-2.561553, 6.561553)$ and $(1.561553, 2.438447)$. Therefore $x \approx -2.56$ and 1.56 . Solving $x^2 + x - 4 = 0$ gives $(-1 \pm \sqrt{17})/2$, confirming both. At the negative unrounded root, both outputs are about 6.561553; at the positive unrounded root, both are about 2.438447. The teacher graph is a mathematical reference, not a report of tested software. Accept another recorded school-approved tool with the stated window and reliable trace/refinement evidence.

Evidence and response

If learners find only the positive root, restore the stated negative- x window and trace the left crossing. If the technology is inaccessible, provide access instruction first; use O06 separately for table evidence rather than labeling it a tool performance.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

WE DO

Teaching script

Core: 009 • T3: Approximate solutions by refining a table of function values.

Optional bank: M09–M13; O09–O13. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Ask learners to choose the first whole-input bracket. Build columns x , f , g and $f - g$ together. Say, “Which endpoint should we keep, and which finer input should replace the other?” Refine one place at a time. Ask partners to justify the last rounding interval.

Core task

Use successive tables to approximate the positive solution of $x^2 = 7$ to the nearest hundredth. Begin with whole-number inputs, then refine to an interval of width 0.001. Record $f(x)$, $g(x)$ and their difference at each endpoint.

Worked teacher reasoning

The whole-number bracket is $(2, 3)$, with differences -3 and 2 . Refine to $(2.6, 2.7)$, $(2.64, 2.65)$, and $(2.645, 2.646)$. Last differences are -0.003975 and 0.001316 . Thus the positive input is $x \approx 2.65$. Both graphs have output 7 at the actual shared input.

Evidence and response

If outputs are rounded so early that the sign disappears, retain more digits and rebuild the endpoint comparison. If a learner reports 7 as x , check 7^2 and return to the input column. Use O11 as a fresh exponential transfer task.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T5: Interpret solutions within a stated domain and context.

Optional bank: M14–M18; O14–O18. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Use reader, tool operator, checker and explainer roles; rotate after the core task. Each learner predicts the likely break-even interval before tool use. The checker substitutes the unrounded input into both dollar models; the explainer states time and dollars separately.

Core task

TECHNOLOGY: A continuous production model has cost $C(t) = 12 + 4t$ and revenue $R(t) = 3t^2$ dollars, with $0 \leq t \leq 6$ hours. Graph both in a window $0 \leq t \leq 6$, $0 \leq y \leq 110$. Find the break-even time to the nearest hundredth hour. Record your tool, window and method; sketch the curves, give the common dollar value and verify it.

Worked teacher reasoning

Break-even requires $12 + 4t = 3t^2$. The roots are $(4 \pm \sqrt{160})/6$; only $t \approx 2.774852$ is in the domain. Thus break-even is about 2.77 hours at approximately \$23.10. At the unrounded input, cost and revenue are both about 23.099407. The negative algebraic root is not a time in this model. The equality input is time, whereas the shared output is dollars.

Evidence and response

If the negative root is included, ask which given domain condition it violates. If the rounded time gives slightly unequal displayed dollars, explain approximation and check at the unrounded input. Use O15 to transfer the interpretation to successive tables.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Use graphing technology and verify numerical intersections.

Optional bank: M19–M23; O19–O23. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Give the printed task and neutral supports only. Require a personal tool record, labeled sketch and two original-output checks. Keep the first response. Ask no leading question that points to either root before collecting evidence.

Core task

TECHNOLOGY: Graph $y_1 = x^2$ and $y_2 = 2x + 5$ on $-4 \leq x \leq 5$, $-4 \leq y \leq 25$. Record your tool, window and intersection method. Sketch both curves, find every visible solution to the nearest hundredth, and check both outputs at each unrounded input. Explain why you report x-coordinates.

Worked teacher reasoning

The equation $x^2 - 2x - 5 = 0$ has roots $1 \pm \sqrt{6}$, about -1.449490 and 3.449490 . Intersections are near $(-1.449490, 2.101021)$ and $(3.449490, 11.898979)$. Report $x \approx -1.45$ and 3.45 . Both original outputs agree at each root. Inputs solve the equation; the second coordinates are the corresponding common outputs.

Evidence and response

After collection, distinguish a missing second root from a rounding error or an input/output reversal. Choose O22 for table refinement or O24 for coordinate meaning; record any new mathematical cue separately.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Explain why shared graph points correspond to equation solutions.

Optional bank: M24–M27; O24–O27. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Begin with the learner’s original claim. Build a two-column input/output check for $x = 4$ before drawing. Say, “Does this input make both outputs equal?” Then construct the two lines and connect the successful input to the shared point.

Core task

A learner graphs $f(x) = x + 2$ and $g(x) = 3x - 2$, then claims that $x = 4$ solves $f(x) = g(x)$. Draw both graphs, identify their shared point and test $x = 4$ in both formulas. Write a correction explaining the input/output mix-up.

Worked teacher reasoning

The lines meet at $(2, 4)$: $f(2) = 4$ and $g(2) = 4$. Thus the solution is $x = 2$. Testing the claim gives $f(4) = 6$ and $g(4) = 10$, so $x = 4$ fails. The learner used the intersection’s output as its input. A shared point (a, b) tells us $f(a) = g(a) = b$, not necessarily $f(b) = g(b)$.

Evidence and response

If the learner uses an output as input again, let them label a point as (input, output) and evaluate both formulas. Follow with O27 to check whether they understand that touching also counts. Preserve supported work separately from the first attempt.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

EXIT TICKET

Teaching script

Core: 028 • T4: Use graphing technology and verify numerical intersections.

Optional bank: M28–M30; O28–O30. Use across meetings as needed. Suggested core discussion/work: 8–20 minutes; adjust for access, not a speed criterion.

Launch and teacher language

Collect an individual technology record and concise intersection explanation. Allow adequate tool access time. Select O29 or O30 in another short meeting when table or domain evidence is still needed; do not treat one exit task as evidence for every target.

Core task

TECHNOLOGY EXIT: Graph $y_1 = x^2$ and $y_2 = 3x + 1$ on $-2 \leq x \leq 5$, $-5 \leq y \leq 25$. Record tool, window and method. Sketch the graphs, report both solution inputs to the nearest hundredth and check each using the original formulas. Explain the connection between the intersection and the equation.

Worked teacher reasoning

$x^2 - 3x - 1 = 0$ gives $x = (3 \pm \sqrt{13})/2$, approximately -0.302776 and 3.302776 . The shared points are near $(-0.302776, 0.091673)$ and $(3.302776, 10.908327)$. Therefore $x \approx -0.30$ or 3.30 . At either unrounded input $f(x)$ and $g(x)$ have equal output, which is why the first coordinate is a solution.

Evidence and response

If either root is missing, schedule a window/second-neighborhood search. If both roots are right but the explanation is absent, ask for the shared-output link on a fresh pair of functions before assigning a reasoning rating.

Closure

Ask the learner to state the solution input(s), common output(s), and one check that links the representation to the equation. A selected choice alone does not identify a unique misconception.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1: Explain why shared graph points correspond to equation solutions.****Two graphs intersect at $(-2, 7)$. Which value is a solution of $f(x) = g(x)$?**

- A. 2
- B. 7
- C. $(-2, 7)$
- D. -2

Reasoning: _____

M02 | Warm-up | T1: Explain why shared graph points correspond to equation solutions.**For $f(x) = 2x + 1$ and $g(x) = 7 - x$, which point lies on both graphs?**

- A. $(2, 5)$
- B. $(0, 1)$
- C. $(3, 7)$
- D. $(5, 2)$

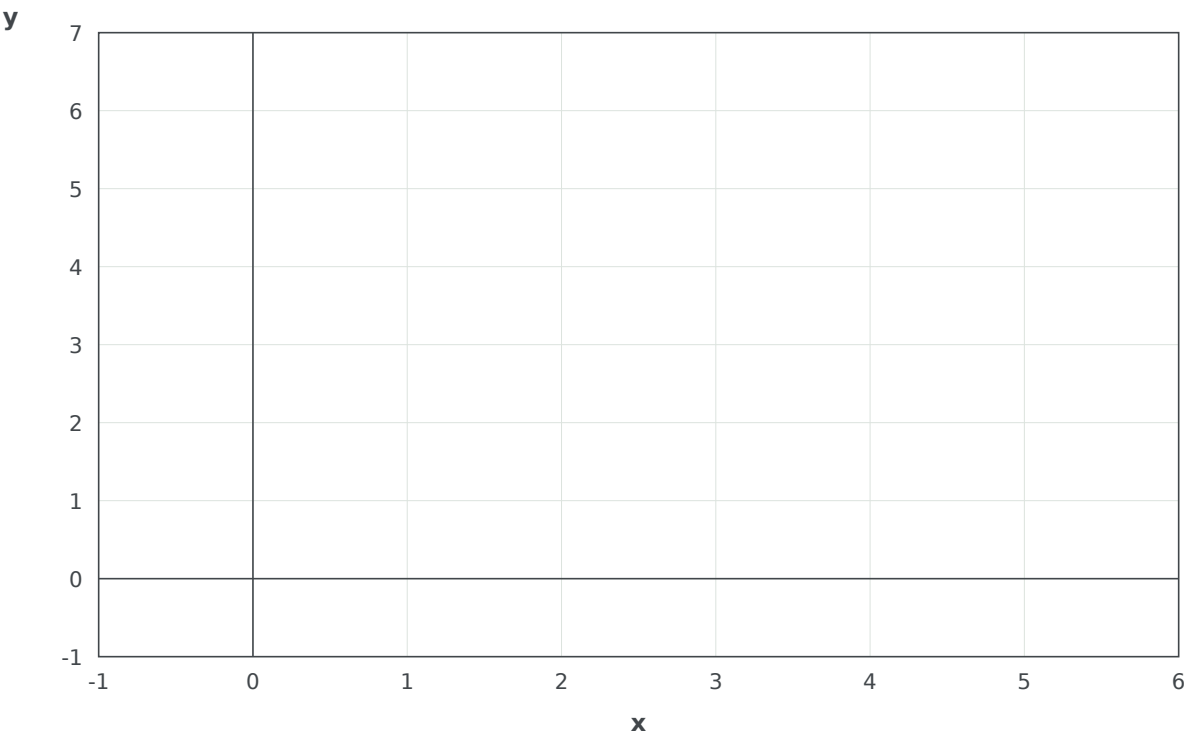
Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Explain why shared graph points correspond to equation solutions.

Graph $f(x) = x + 1$ and $g(x) = 5 - x$ on the supplied grid. Label both lines and their intersection. State the solution of $f(x) = g(x)$, then explain why the input, rather than the output, solves the equation.



Intersection coordinates / original-output checks / explanation

WARM-UP

002 • Show your reasoning

T1: Explain why shared graph points correspond to equation solutions.

For $f(x) = 3x - 2$ and $g(x) = x + 4$, complete a table at $x = 0, 1, 2, 3, 4$. Find the shared point and explain how its coordinates establish a solution of $f(x) = g(x)$.

Record the requested inputs and both outputs. Use extra rows for checks.

Input x	f(x)	g(x)	Difference f – g

Shared point / solution input / justification

WORKED MULTIPLE-CHOICE KEYS

M01 / M02**M01 • T1: Explain why shared graph points correspond to equation solutions.****Answer: D. -2**

At $(-2, 7)$, $f(-2) = 7$ and $g(-2) = 7$. Therefore $x = -2$ solves $f(x) = g(x)$.

- A. Changing the sign changes the input; the intersection has $x = -2$.
- B. 7 is the common output, not the input.
- C. The ordered pair describes the intersection; the equation asks for an input value.
- D. Correct: At $(-2, 7)$, $f(-2) = 7$ and $g(-2) = 7$. Therefore $x = -2$ solves $f(x) = g(x)$.

M02 • T1: Explain why shared graph points correspond to equation solutions.**Answer: A. $(2, 5)$**

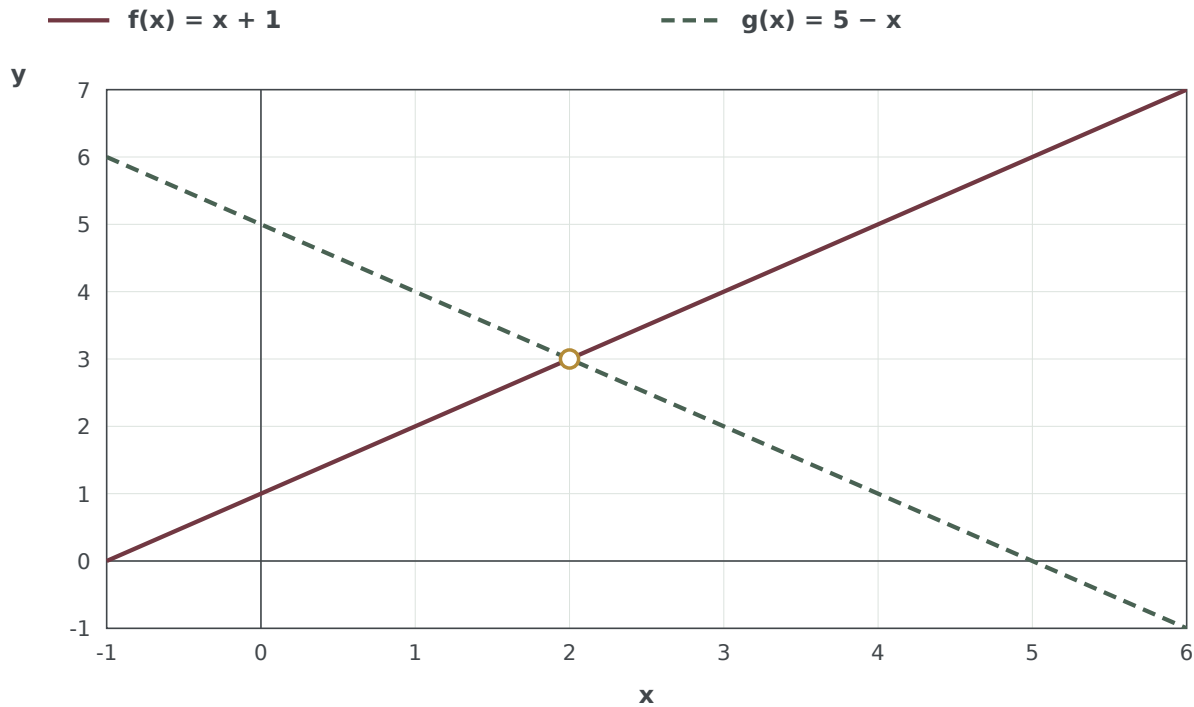
$2(2) + 1 = 5$ and $7 - 2 = 5$, so both graphs contain $(2, 5)$.

- A. Correct: $2(2) + 1 = 5$ and $7 - 2 = 5$, so both graphs contain $(2, 5)$.
- B. $f(0) = 1$ but $g(0) = 7$.
- C. $f(3) = 7$ but $g(3) = 4$.
- D. At $x = 5$ the outputs are 11 and 2; only g gives 2.

WORKED OPEN/PERFORMANCE KEYS

001 • Teacher reference**T1: Explain why shared graph points correspond to equation solutions.**

For f , plot $(0, 1)$, $(2, 3)$, $(4, 5)$. For g , plot $(0, 5)$, $(2, 3)$, $(4, 1)$. The shared point is $(2, 3)$. Thus $x = 2$ because $f(2) = 3 = g(2)$. The number 3 is the common output. Equating gives $x + 1 = 5 - x$, so $2x = 4$.

**Evidence to assess**

Assess the requested representation, the input/output connection, numerical accuracy and a justified conclusion. A tool result alone is insufficient: require the stated check and the learner's record of the method.

WORKED OPEN/PERFORMANCE KEYS

002 • Teacher reference

T1: Explain why shared graph points correspond to equation solutions.

The rows (x, f, g) are (0, −2, 4), (1, 1, 5), (2, 4, 6), (3, 7, 7), (4, 10, 8). At x = 3 both outputs are 7, so (3, 7) is on both graphs and x = 3 is a solution. Solving $3x - 2 = x + 4$ gives $2x = 6$, confirming the table.

Outputs and differences are rounded to six decimal places; input endpoints are shown as used.

x	f(x)	g(x)	f(x) – g(x)
0	-2.000000	4.000000	-6.000000
1	1.000000	5.000000	-4.000000
2	4.000000	6.000000	-2.000000
3	7.000000	7.000000	+0.000000
4	10.000000	8.000000	+2.000000

Evidence to assess

Assess the requested representation, the input/output connection, numerical accuracy and a justified conclusion. A tool result alone is insufficient: require the stated check and the learner’s record of the method.