

TEACHER AND STUDENT RESOURCE

Regression Models and Residual Evidence



NC Math 4 • NC.M4.AF.5

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M4.AF.5 | NC Math 4 mathematics

This sample contains 22 complete selected pages from the 103-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-17, 32-33, 62, 77-78

Representative activities

M01: sample PDF page(s) 18; original page(s) 17

M02: sample PDF page(s) 18; original page(s) 17

O01: sample PDF page(s) 19; original page(s) 32

O02: sample PDF page(s) 20; original page(s) 33

Selected teacher prompts and keys: original page(s) 62, 77-78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.
Its required companion lab is included with the complete paid download.
The sample does not include that complete companion archive.

ALIGNMENT

Standard and learning targets

Understand how to model functions with regression.

T1 NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

T2 NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

NC.M4.AF.5.1 Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.

NC.M4.AF.5.2 Compare residuals and residual plots of non-linear models to assess the goodness-of-fit of the model.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M02, M03, M04, M06, M07, M12, M16, M18, M19, M20, M26, M28, M29, M30, O01, O03, O05, O07, O09, O11, O13, O15, O17, O19, O21, O23, O25, O27, O29

T2: M01, M05, M08, M09, M10, M11, M13, M14, M15, M17, M21, M22, M23, M24, M25, M27, O02, O04, O06, O08, O10, O12, O14, O16, O18, O20, O22, O24, O26, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Learning target: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

A graphing calculator, spreadsheet with a solver, or Python with NumPy and SciPy is required for technology-designated tasks. Use the printed paired data or companion `regression_data.csv`. The reproducible `regression_lab.py` supplies the main fitted models and residual calculations. Pencils and graph paper support interpretation; technology does not replace a written check.

Prerequisites

Function families, scatterplots, evaluating formulas, square roots, logarithms, and interpreting units.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All data and contexts are invented for instruction. A residual is observed y minus predicted y . SSE means sum of squared residuals; here $RMSE = \sqrt{(SSE/n)}$, not a degrees-of-freedom adjusted statistic. Fit on the original y scale unless a task explicitly requests a log-response comparison. Round final coefficients to four decimals, retain full precision for calculations, and accept small technology-dependent rounding differences.

TECHNOLOGY LABORATORY

Run a regression and keep evidence

Data entry

Enter each x and its matching y in the same row. Make a scatterplot before fitting. The lettered data sets in the prompts also appear in `regression_data.csv`. All are invented classroom data, not measured scientific records.

Fit the requested family

Use $y = a + bx$, $y = a + bx + cx^2$, $y = a \cdot b^x$ with $a > 0$ and $b > 0$, or $y = a + b \ln x$ with $x > 0$. For a stated period 12, fit $y = d + p \sin(\pi x/6) + q \cos(\pi x/6)$ in radians. These sinusoidal coefficients determine amplitude $\sqrt{p^2 + q^2}$.

Original-response objective

Choose parameters to minimize $\sum(y - \text{predicted } y)^2$. Exponential regression based on a line through $\ln y$ instead minimizes squared log residuals and can give different coefficients. State the method. The packet keys use original- y least squares except where explicitly comparing methods.

Evidence to save

Record the data, model form and parameter estimates. Create predicted- y and residual columns; plot residual versus x with a horizontal zero reference. Compute SSE and RMSE. Write a contextual prediction with units and mark interpolation versus extrapolation.

Reproducible route

Run the companion `regression_lab.py` with Python 3, NumPy and SciPy. It fits all main data sets, prints coefficients and writes `regression_results.json`. Review the source to identify the model used; the output is a check, not a substitute for explaining the fit.

TECHNOLOGY LABORATORY

Compare models without overclaiming

A fair comparison

Compare the same observations, response units and residual definition. Inspect both numerical error and the residual plot. A smaller training SSE alone does not settle model choice when one model is more flexible or behaves implausibly outside the data.

What a pattern says

A U-shaped, wave-shaped or trending residual plot suggests a missing mean pattern. A widening fan suggests nonconstant error spread. A random-looking small sample supports, but does not prove, a useful model. A residual sum near zero can occur even for a poor model.

Sinusoidal fitting

For data H , fit $y = d + A \cos(wx + c)$, with $A \geq 0$ and $10 \leq \text{period} \leq 14$, so $2\pi/14 \leq w \leq 2\pi/10$. Try several starting periods and compare SSE. Equivalent phases differing by whole turns describe the same curve. Use radians. A local fit is not automatically the best available fit.

Teacher checking tolerance

Use about ± 0.01 for predictions from four-decimal coefficients unless a task requires tighter work. Different but equivalent sinusoidal forms are valid. For constructed data or recommendations, assess the saved workflow, calculations, residual evidence and scope of the claim, not a single preferred wording.

Tool documentation

NumPy least-squares reference:

<https://numpy.org/doc/stable/reference/generated/numpy.linalg.lstsq.html>

SciPy nonlinear curve fitting reference:

https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html

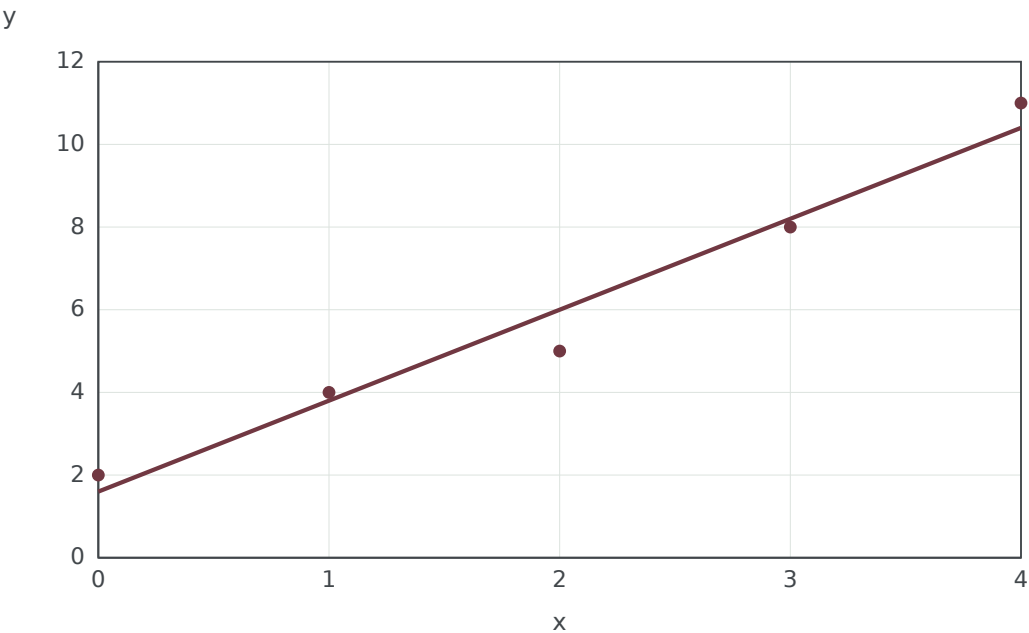
TEACHER MODEL

Fit first then inspect the errors

Data A.

Input x	0	1	2	3	4
Output y	2	4	5	8	11

Fit a line with technology, predict at $x = 2$ and compare the prediction with the recorded response.



Enter paired rows without rearranging columns separately. Fitting $y = a + bx$ gives $a = 1.6$ and $b = 2.2$. At $x = 2$, predicted $y = 6$; observed $y = 5$, so residual = -1 . The full residual list is $0.4, 0.2, -1, -0.2, 0.6$. $SSE = 1.6$ and $RMSE = \sqrt{(1.6/5)} \approx 0.5657$. Plot all residuals rather than judging the model from a single prediction.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Data A.

Input x	0	1	2	3	4
Output y	2	4	5	8	11

Use technology to fit a least-squares line. Predict y at $x = 2.5$, interpret its slope as response units per input unit, and compute the residual at $x = 2$.

Teacher solution and discussion

The fitted line is $y = 1.6 + 2.2x$. Prediction at 2.5 is 7.1. Each one-unit increase in x corresponds to an estimated increase of 2.2 response units. At $x = 2$, predicted y is 6 but observed y is 5, so the residual is -1. Save the data and fit output.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Data C.

Input x

Output y

0	1	2	3	4
10	12.5	16.5	20	25

Treat x as elapsed days and y as an invented culture index. Fit $y = a \cdot b^x$ using technology and original-y least squares. Interpret the growth factor and solve for the real time when the model reaches index 30.

Teacher solution and discussion

The fit is $y \approx 10.164776(1.253753)^x$, an estimated 25.3753% increase per day. Solve $x = \ln(30/10.164776)/\ln(1.253753) \approx 4.7858$ days. This is beyond the observed interval $[0, 4]$, so it is an extrapolated model prediction, not a confirmed observation.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Data E.

Input x

Output y

0	1	2	3	4	5	6	7	8	9	10	11
10.2	11.8	13.7	13.9	13.3	12.2	9.8	8.1	6.7	5.8	6.6	8.1

Use technology to fit a sinusoid whose period is 12. Report its midline, amplitude and period, then predict the response at $x = 12$. Use radians.

Teacher solution and discussion

The fit is $y \approx 10.016667 + 3.977425 \sin(\pi x/6) + 0.033932 \cos(\pi x/6)$. Midline = 10.016667; amplitude = $\sqrt{(3.977425^2 + 0.033932^2)} \approx 3.9776$; period = 12. At $x = 12$ the prediction is about 10.0506. This is just beyond the recorded range ending at 11.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Two candidate models are tested on the same held-out responses 4, 8, 12. P predicts 3, 9, 13 and Q predicts 4, 7, 15. Their training SSE values were 9 for P and 1 for Q. Compute held-out SSE and recommend the stronger held-out performer.

Teacher solution and discussion

Held-out residuals for P are 1, -1, -1, giving $SSE = 3$. For Q they are 0, 1, -3, giving $SSE = 10$. P performs better on these held-out rows despite worse training SSE. Three held-out cases provide limited evidence, so avoid an absolute claim about all future predictions.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Data B.

Input x

-2	-1	0	1	2
9.2	2.7	1.2	3.1	8.8

Output y

Use the fitted quadratic to predict at $x = 10$. Contrast this with prediction at $x = 1$. Explain how you would label both results in a report.

Teacher solution and discussion

Using $y \approx 1.028571 - 0.04x + 1.985714x^2$ gives about 199.2 at $x = 10$ and 2.9743 at $x = 1$. The latter is within the measured interval $[-2, 2]$; the former is distant extrapolation. Report 199.2 only as a model-based extrapolation with strong uncertainty about applicability, not as a verified outcome.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Data B.

Input x	-2	-1	0	1	2
Output y	9.2	2.7	1.2	3.1	8.8

A learner sorts y separately into 1.2, 2.7, 3.1, 8.8, 9.2 while leaving x in increasing order. Fit a line to this corrupted pairing and contrast it with the original line. Explain why a stronger-looking trend is not an improvement.

Teacher solution and discussion

The original line is $y = 5 - 0.04x$. The separately sorted responses give $y = 5 + 2.21x$. The apparent increasing trend was created by breaking the observed pairs. It is not a better model of the original data. Restore the original row pairings and document the data-entry error before further analysis.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

For $x = 0, 1, 2, 3$, observed y values are 5, 8, 14, 26. A proposed exponential is $y = 3 \cdot 2^x$. Compute and plot its residuals. Fit a vertical shift $y = 3 \cdot 2^x + c$ by minimizing SSE and explain the revised errors.

Teacher solution and discussion

Original predictions are 3, 6, 12, 24, so residuals are 2, 2, 2, 2 and $SSE = 16$. With shift c , every residual is $2 - c$, so $SSE = 4(2 - c)^2$ is minimized at $c = 2$. The revised model $y = 3 \cdot 2^x + 2$ has zero errors on these rows. The uniformly positive residuals indicated a missing upward offset.

Evidence and response

Ask learners to save the entered paired data, model form, fitted parameters, predicted values and residual plot. A family name or supplied equation alone is not evidence of constructing a regression. Ask what the residual pattern and context reveal beyond a small SSE. Do not treat fit as proof of causation or reliable distant prediction.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T2**

A model predicts 18 units, but the observed response is 21 units. What is the residual?

- A. 3 units
- B. -3 units
- C. 39 units
- D. 21 units

Reasoning: _____

M02 | Warm-up | T1

For equally spaced inputs, the response rises by nearly the same amount each step. Which family is the most reasonable first model to fit?

- A. Exponential
- B. Sinusoidal
- C. Linear
- D. Quadratic solely because there are three points

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Data A.

Input x	0	1	2	3	4
Output y	2	4	5	8	11

Use technology to fit a least-squares line. Predict y at x = 2.5, interpret its slope as response units per input unit, and compute the residual at x = 2.

Representation / calculation

Explanation / check

WARM-UP

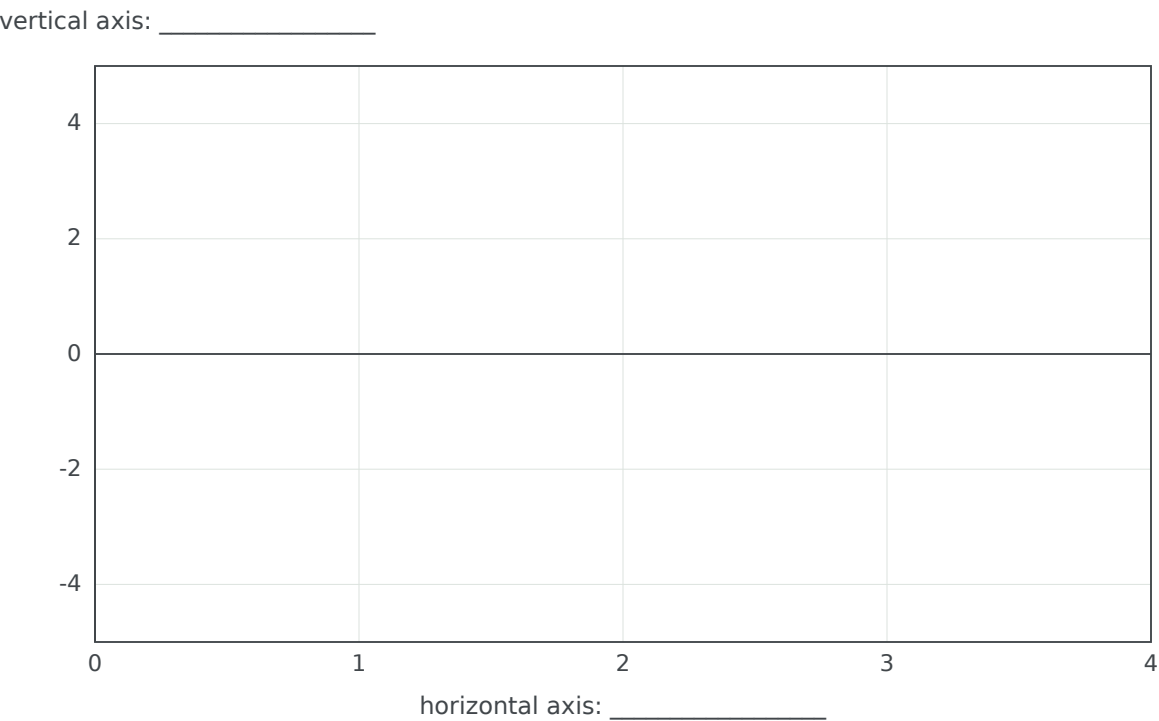
002 • Show your reasoning

T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Data F.

Input x	0	1	2	3	4
Output y	1	3	9	19	33

Compare the supplied linear model $y = 8x - 3$ and quadratic model $y = 2x^2 + 1$. Compute each residual list and SSE. Plot the line-model residuals and explain what their pattern shows.



Explanation and check

WORKED TEACHER KEYS

M01 / M02

M01 • T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Answer: A. 3 units

Residual = $21 - 18 = 3$ units. The model underpredicts.

Check each alternative

- A. Correct. Residual = $21 - 18 = 3$ units. The model underpredicts.
- B. That reverses observed minus predicted.
- C. Adding the two values does not measure prediction error.
- D. The observed response alone is not a residual.

M02 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

Answer: C. Linear

Approximately constant first differences support beginning with a linear model, then checking residuals.

Check each alternative

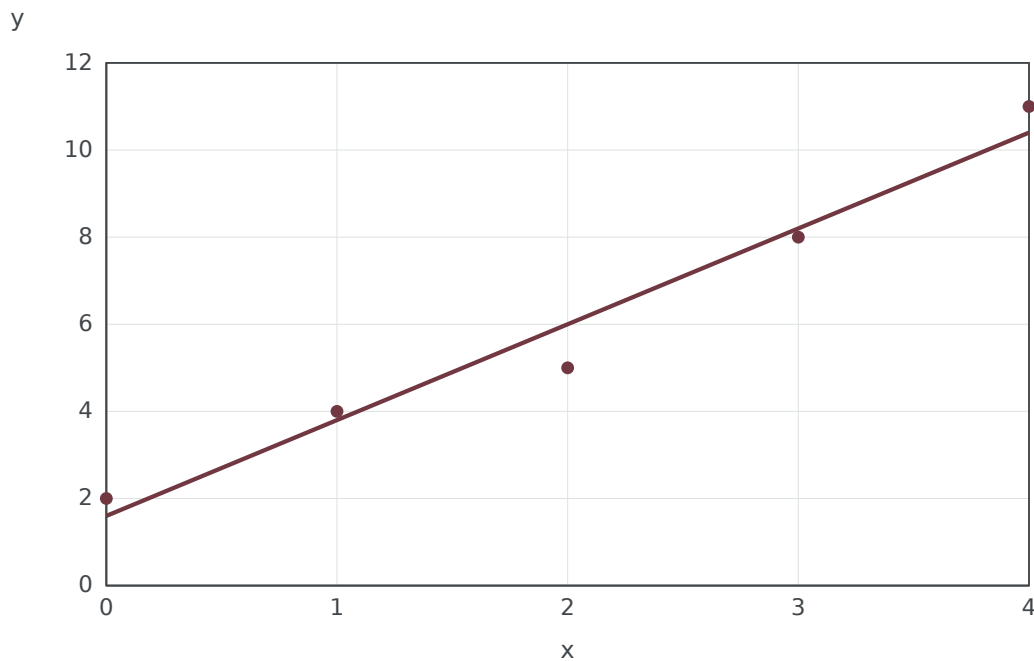
- A. Exponential growth has approximately constant ratios rather than constant differences.
- B. No repeated oscillation is described.
- C. Correct. Approximately constant first differences support beginning with a linear model, then checking residuals.
- D. The number of observations does not by itself select the family.

WORKED TEACHER KEYS

001

001 • T1: NC.M4.AF.5.1 Use technology to construct linear, quadratic, exponential, logarithmic and sinusoidal regression models and solve problems.

The fitted line is $y = 1.6 + 2.2x$. Prediction at 2.5 is 7.1. Each one-unit increase in x corresponds to an estimated increase of 2.2 response units. At $x = 2$, predicted y is 6 but observed y is 5, so the residual is -1. Save the data and fit output.



Evidence to assess

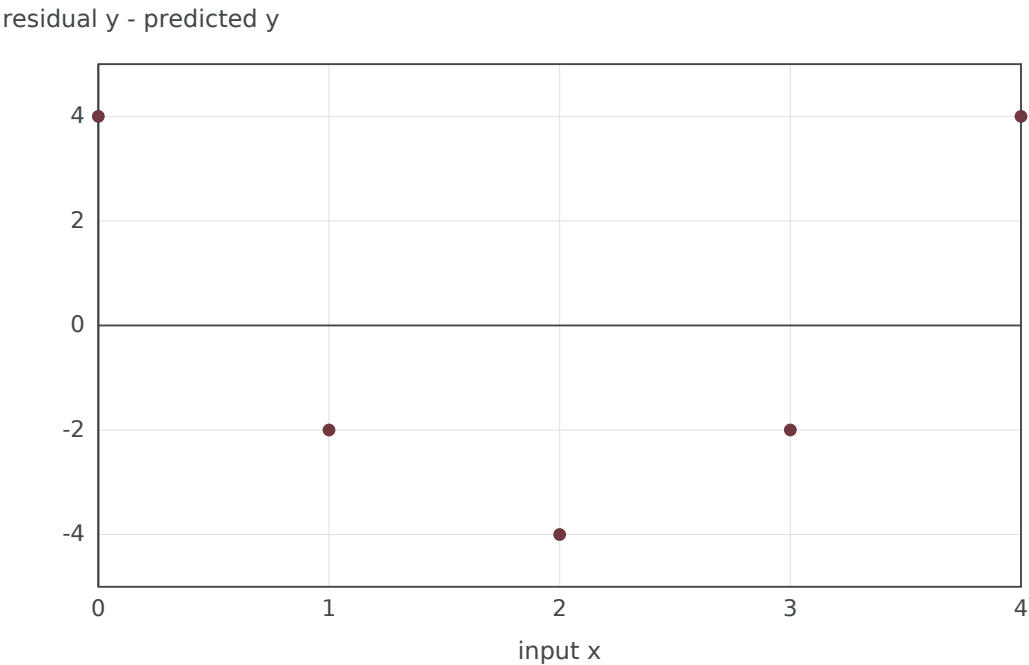
Full evidence: Technology evidence, full model, prediction, slope meaning and signed residual.
 Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002

002 • T2: NC.M4.AF.5.2 Compare residual values and plots for nonlinear models to assess fit and justify model choices.

Line predictions are -3, 5, 13, 21, 29; residuals are 4, -2, -4, -2, 4, with $SSE = 56$. The quadratic predicts all five responses exactly, so all residuals and SSE are zero. The line residuals form a U-shaped pattern, showing missed curvature. Exact fit describes these invented data, not a guarantee for all future data.



Evidence to assess

Full evidence: Both residual lists, SSE values, plotted pattern and appropriately limited conclusion. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.