

TEACHER AND STUDENT RESOURCE

Composition, Domains and Inverse Functions



Precalculus • PC.F.5

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.F.5 | North Carolina Precalculus mathematics

This sample contains 21 complete selected pages from the 100-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 62, 77-78

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 62, 77-78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of function composition to build new functions from existing functions.

T1 PC.F.5.1 Compose algebraic functions in the specified order.

T2 PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

T3 PC.F.5.3 Find composite domains by preserving both stages of restrictions.

T4 PC.F.5.4 Build and interpret contextual composition models.

T5 PC.F.5.5 Decompose a composite rule into two nonidentity functions.

T6 PC.F.5.6 Find inverse functions algebraically and graphically with necessary restrictions.

T7 PC.F.5.7 Test inverse pairs using both compositions and their domains.

PC.F.5.1 Implement algebraic procedures to compose functions.

PC.F.5.2 Execute a procedure to determine the value of a composite function at a given value using algebraic, graphical, and tabular representations.

PC.F.5.3 Implement algebraic methods to find the domain of a composite function.

PC.F.5.4 Organize information to build models involving function composition.

PC.F.5.5 Deconstruct a composite function into two functions.

PC.F.5.6 Implement algebraic and graphical methods to find an inverse function of an existing function, restricting domains if necessary.

PC.F.5.7 Use composition to determine if one function is the inverse of another function.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M08, M15, M22, O01, O08, O15, O22

T2: M02, M09, M16, M23, O02, O09, O16, O23

T3: M03, M10, M17, M24, M29, O03, O10, O17, O24, O29

T4: M04, M11, M18, M25, O04, O11, O18, O25

T5: M05, M12, M19, M26, O05, O12, O19, O26

T6: M06, M13, M20, M27, M30, O06, O13, O20, O27, O30

T7: M07, M14, M21, M28, O07, O14, O21, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.F.5.1 Compose algebraic functions in the specified order.

Learning target: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

Learning target: PC.F.5.3 Find composite domains by preserving both stages of restrictions.

Learning target: PC.F.5.4 Build and interpret contextual composition models.

Learning target: PC.F.5.5 Decompose a composite rule into two nonidentity functions.

Learning target: PC.F.5.6 Find inverse functions algebraically and graphically with necessary restrictions.

Learning target: PC.F.5.7 Test inverse pairs using both compositions and their domains.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Function notation, solving equations, logarithm/radical domains, piecewise rules and coordinate graphs.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All variables and functions are real. $(f \circ g)(x) = f(g(x))$: the rightmost function acts first. A composite requires x in the domain of g AND $g(x)$ in the domain of f . A simplified rule does not restore excluded inputs. Tables define functions only at the listed inputs unless explicitly stated otherwise. Joined graph segments include their endpoints unless an open endpoint is shown. f^{-1} means inverse function, not reciprocal. An inverse swaps the original domain and range. Angles are radians.

TEACHER REFERENCE

A two-stage input audit

Composition is a route

Trace $x \rightarrow g(x) \rightarrow f(g(x))$. For a table, locate the inner output as a new input in the outer table. For a graph, read a y -value, return to the horizontal axis with that number, and read the second graph. Missing table entries do not authorize interpolation.

Domain before cancellation

Write $x \in \text{Dom}(g)$ and $g(x) \in \text{Dom}(f)$. Solve both constraints and intersect. An outer square root permits zero, but a square root in a denominator does not. Logarithms require strictly positive arguments.

Inverse is a two-way claim

A function has an inverse function only when it is one-to-one on the selected domain. Solve $y=f(x)$, choose the branch justified by the original domain, swap variables and state the inverse domain. Verify $f(g(x))=x$ and $g(f(x))=x$ where each composition is defined.

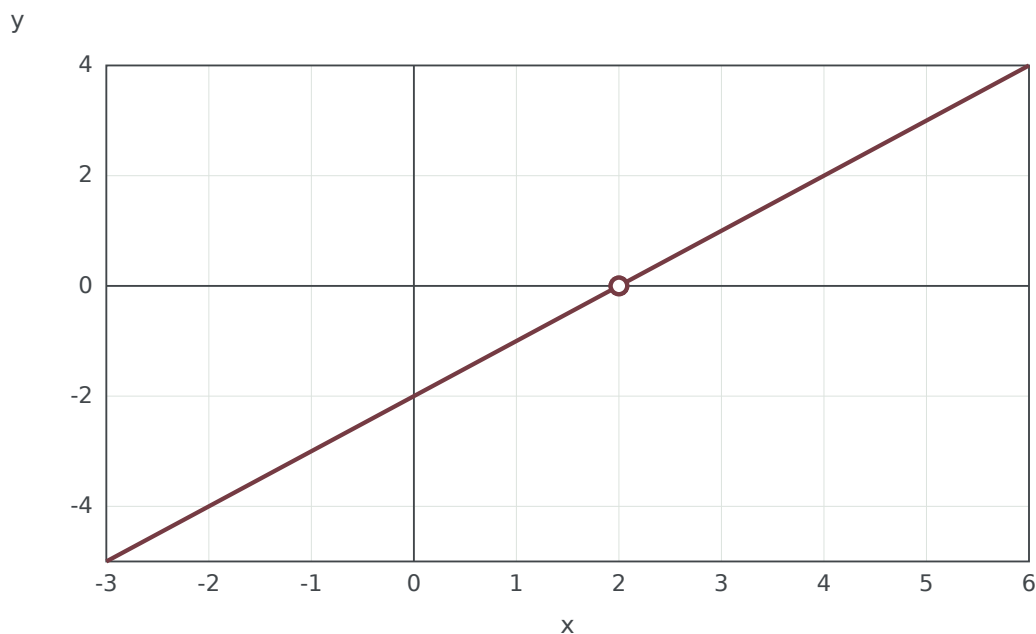
Original bank and transfer

These tasks are newly written for all seven Precalculus objectives. They extend familiar composition ideas through nested restrictions, multiple decompositions, nonprincipal inverse branches, finite-domain tables, contextual order and symbolic inverse criteria.

TEACHER MODEL

A composition domain survives simplification

Let $f(u)=1/u$ and $g(x)=1/(x-2)$. Is $f(g(x))$ the entire line $x-2$?



No. First require $x \neq 2$ so g is defined. Its output is never zero, so f adds no new restriction. The reciprocal simplifies to $x-2$ only for $x \neq 2$. The graph has a hole at $(2, 0)$, and its range excludes 0. A formula and its domain together define a function.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: PC.F.5.1 Compose algebraic functions in the specified order.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Let $f(x) = (x-1)/(x+2)$ and $g(x) = x^2$. Find $f \circ g$ and $g \circ f$, state each domain and use one allowed input to show the functions differ.

Teacher solution and discussion

$f(g(x)) = (x^2 - 1)/(x^2 + 2)$, defined for every real x because $x^2 + 2 > 0$. $g(f(x)) = [(x-1)/(x+2)]^2$ has domain $x \neq -2$. At $x=0$, the first composition is $-1/2$ and the second is $1/4$. Both are defined there, so they are different functions.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T5: PC.F.5.5 Decompose a composite rule into two nonidentity functions.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Decompose $H(x) = e^{-(x-1)^2}$ into $f(g(x))$ in two different ways, without using an identity function. State the range of each inner function and explain why both composites have domain all real numbers.

Teacher solution and discussion

One choice is $g(x) = -(x-1)^2$ and $f(u) = e^u$; the inner range is $(-\infty, 0]$. A second is $g(x) = x-1$ and $f(u) = e^{-u^2}$; the inner range is all real numbers. Both inner functions and both outer functions are defined for every real input, so both composites have domain all real numbers and produce the same H .

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

The supplied joined graph defines f on $[-2, 4]$. Let $g(x)=x-1$. Find $f(g(1))$, $f(g(3))$, $f(g(5))$ and $g(f(0))$; show the intermediate value for each. Explain why $f(g(6))$ is undefined.

Teacher solution and discussion

$g(1)=0$ then $f(0)=1$; $g(3)=2$ then $f(2)=3$; $g(5)=4$ then $f(4)=-1$. For the reverse route, $f(0)=1$ then $g(1)=0$. Finally $g(6)=5$, outside the graph domain $[-2, 4]$, so $f(g(6))$ is undefined; do not extend the last segment beyond its endpoint.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T7: PC.F.5.7 Test inverse pairs using both compositions and their domains.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Let $f: \mathbb{R} \rightarrow [0, \infty)$ be $f(x) = x^2$, and let $g: [0, \infty) \rightarrow \mathbb{R}$ be $g(x) = \sqrt{x}$. Compute both compositions with their domains. Explain why one identity alone does not prove a two-sided inverse, and repair the pair by restricting f .

Teacher solution and discussion

$f(g(x)) = x$ for $x \geq 0$. But $g(f(x)) = |x|$ for all real x , which differs from x when $x < 0$. Thus g is a right inverse in the first composition but not a two-sided inverse of f on $\mathbb{R}$; f maps x and $-x$ to the same output. Restrict f to $[0, \infty)$, with range $[0, \infty)$, and take g with the same domain/range. Then both compositions equal the identity on $[0, \infty)$.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T5: PC.F.5.5 Decompose a composite rule into two nonidentity functions.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Give two nonidentity decompositions of $H(x)=(2x+1)^6$, one with a cubic outer function and one with a sixth-power outer function. Compare their inner ranges and explain why this does not change H .

Teacher solution and discussion

For a cubic outer function, take $f(u)=u^3$ and $g(x)=(2x+1)^2$; the inner range is $[0, \infty)$. For a sixth-power outer function, take $f(u)=u^6$ and $g(x)=2x+1$; its inner range is $\mathbb{R}$. The power identity $[(2x+1)^2]^3=(2x+1)^6$ holds for every real x . The intermediate functions differ, but their final outputs agree on the same domain $\mathbb{R}$.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: PC.F.5.3 Find composite domains by preserving both stages of restrictions.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Let $f(u) = \sqrt{4 - u^2}$ and $g(x) = \ln x$. Find the full domain of $f \circ g$. Check both boundary values and explain why $x = 0$ is never admissible.

Teacher solution and discussion

The inner logarithm requires $x > 0$. The outer root requires $4 - (\ln x)^2 \geq 0$, so $-2 \leq \ln x \leq 2$.

Exponentiation, which is increasing, gives $e^{-2} \leq x \leq e^2$. Both endpoints are positive and make the radicand zero, so they are included. $x = 0$ is excluded before the outer stage because $\ln 0$ is undefined.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T7: PC.F.5.7 Test inverse pairs using both compositions and their domains.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Let $f(x)=ax+b$ and $g(x)=cx+d$ on all real numbers. Derive necessary and sufficient conditions on a , b , c , d for f and g to be inverses. Explain what fails if $a=0$.

Teacher solution and discussion

$f(g(x))=acx+ad+b$ and $g(f(x))=cax+cb+d$. Both equal x identically exactly when $ac=1$, $ad+b=0$ and $cb+d=0$. Equivalently $a \neq 0$, $c=1/a$ and $d=-b/a$; the last constant equation then follows automatically. If $a=0$, f is constant and cannot be one-to-one on $\mathbb{R}$; also $ac=1$ is impossible.

Evidence and response

Ask learners to name the inner output before applying the outer rule, record restrictions before simplifying, label intermediate units in contextual models, and verify both inverse compositions on the appropriate domains. A matching formula without its valid domain is incomplete evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Let $f(x)=2x-1$ and $g(x)=x^2+3$. Which is $(f \circ g)(x)$?

- A. $2x^2+5$
- B. $4x^2-4x+4$
- C. $2x^3-x^2+6x-3$
- D. $2x^2+2$

Reasoning: _____

M02 | Warm-up | T2

A table has inputs $x=-1, 0, 1, 2$, with $f(x)=2, -1, 0, 1$ and $g(x)=1, 2, -1, 0$ in that order. What is $f(g(0))$?

- A. -1
- B. 1
- C. 2
- D. 0

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.F.5.1 Compose algebraic functions in the specified order.

Let $f(x) = (x-1)/(x+2)$ and $g(x) = x^2$. Find $f \circ g$ and $g \circ f$, state each domain and use one allowed input to show the functions differ.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

Functions f and g are defined only on $\{0, 1, 2, 3\}$. In input order 0, 1, 2, 3, their values are f : 2, 0, 3, 1 and g : 1, 2, 0, 3. Complete a table of $f(g(x))$ and $g(f(x))$ for all four inputs. Explain one row as a two-stage lookup.

x	g(x)	f(g(x))	f(x)	g(f(x))

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: PC.F.5.1 Compose algebraic functions in the specified order.****Answer: A. $2x^2+5$** Substitute the whole inner output: $f(g(x))=2(x^2+3)-1=2x^2+5$.**Check each alternative**

- A. Correct. Substitute the whole inner output: $f(g(x))=2(x^2+3)-1=2x^2+5$.
- B. This is $g(f(x))=(2x-1)^2+3$, the reversed order.
- C. This is the product $f(x)g(x)$, not composition.
- D. This fails to multiply the inner constant 3 by 2 before subtracting 1.

M02 • T2: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.**Answer: B. 1**Read $g(0)=2$, then $f(2)=1$.**Check each alternative**

- A. This is $f(0)$, omitting the inner stage.
- B. Correct. Read $g(0)=2$, then $f(2)=1$.
- C. This is $g(0)$, stopping after the inner stage.
- D. This is $f(1)$, using the wrong inner output.

WORKED TEACHER KEYS

001

001 • T1: PC.F.5.1 Compose algebraic functions in the specified order.

$f(g(x)) = (x^2 - 1)/(x^2 + 2)$, defined for every real x because $x^2 + 2 > 0$. $g(f(x)) = [(x - 1)/(x + 2)]^2$ has domain $x \neq -2$. At $x = 0$, the first composition is $-1/2$ and the second is $1/4$. Both are defined there, so they are different functions.

Evidence to assess

Full evidence: Two correctly ordered formulas, distinct original domains and a valid shared-input counterexample. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002

002 • T2: PC.F.5.2 Evaluate compositions from formulas, tables and graphs.

For $x=0, 1, 2, 3$ respectively, $f(g(x))$ has values 0, 3, 2, 1 and $g(f(x))$ has values 0, 1, 3, 2. For example, at $x=1$, $g(1)=2$ then $f(2)=3$, whereas $f(1)=0$ then $g(0)=1$. Every inner output belongs to the listed domain, so all entries exist.

Completed two-stage lookup table

x	g(x)	f(g(x))	f(x)	g(f(x))
0	1	0	2	0
1	2	3	0	1
2	0	2	3	3
3	3	1	1	2

Evidence to assess

Full evidence: All eight composite values, one explicit lookup route and the finite-domain check.
Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.