

TEACHER AND STUDENT RESOURCE

Rearranging Formulas with Meaning



NC Math 1 • NC.M1.A-CED.4

Creating Equations • Science and mathematics formulas

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.A-CED.4 | NC Math 1 mathematics

This sample contains 19 complete selected pages from the 92-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

T1 Rearrange product and quotient formulas for a named quantity.

T2 Isolate a quantity in formulas with sums, differences, or grouped expressions.

T3 Collect and factor repeated occurrences of the quantity of interest.

T4 Solve squared-quantity formulas and apply the stated domain.

T5 Check solutions and determine when a rearrangement is valid.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M02, M05, M06, M07, M10, M11, M25, M26, O01, O02, O03, O05, O06, O10, O11, O19, O24, O26, O29

T2: M03, M04, M08, M09, M12, M14, M15, M16, M17, M23, M24, M27, M28, O04, O08, O09, O12, O14, O15, O17, O23, O25, O28

T3: M13, M18, M30, O13, O18

T4: M19, M20, M21, M22, O07, O20, O21, O22

T5: M29, O16, O27, O30

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use equal operations on both sides of an equation; interpret products, quotients, grouping symbols, signed numbers, and square roots; factor a common factor; distinguish length, area, and volume units. Every science formula and its assumptions are supplied; prior recall of a science formula is not required.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Work over the real numbers and use the compatible units supplied in each task. Keep π , fractions, and square roots exact unless a decimal is requested. The symbol $\sqrt{}$ denotes the nonnegative square root; $\pm$ lists both signs when the domain allows them. Division requires a nonzero divisor. Preserve original restrictions and check that calculated values satisfy the stated physical or mathematical domain. Accept algebraically equivalent forms with the same restrictions.

TEACHER MODEL

One quantity appears in two terms

A closed rectangular prism has surface area $S = 2lw + 2lh + 2wh$. Length l and width w are positive. Solve for height h , explain which values of S yield a positive height, and check $S = 94 \text{ cm}^2$, $l = 5 \text{ cm}$, and $w = 4 \text{ cm}$.

Name the target: h . The term $2lw$ does not contain h , so subtract it from both sides: $S - 2lw = 2lh + 2wh$. Factor the common h : $S - 2lw = h(2l + 2w)$. Since $l, w > 0$, the divisor $2l + 2w$ is positive. Divide to get $h = (S - 2lw)/(2l + 2w)$. A positive height requires $S > 2lw$; equality would give zero height. With the data, $h = (94 - 40)/(10 + 8) = 54/18 = 3 \text{ cm}$. Check in the original formula: $2(5)(4) + 2(5)(3) + 2(4)(3) = 40 + 30 + 24 = 94 \text{ cm}^2$. Surface area divided by a length gives a length, consistent with h .

VISUAL ANCHOR

Keep the two sides equal

A rearrangement flow for the prism formula $S = 2lw + 2lh + 2wh$. Target: h . Length and width are positive. Read each row as an equality-preserving move.

Move	New equation	Why it is valid
Subtract $2lw$	$S - 2lw = 2lh + 2wh$	Same subtraction on both sides
Factor h	$S - 2lw = h(2l + 2w)$	Distributive property
Divide	$h = (S - 2lw)/(2l + 2w)$	$2l + 2w > 0$
Check units	Area $\div$ length = length	Correct units for h

Substitute $l=5$ cm, $w=4$ cm and $S=94$ cm²: $h=(94-40)/18=3$ cm. Positive height also requires $S>2lw$; algebra alone does not supply the contextual restriction.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Rearrange product and quotient formulas for a named quantity.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

A rectangle has area $A = lw$, with positive length l and width w . Solve the formula for l . Then find l when $A = 48 \text{ cm}^2$ and $w = 6 \text{ cm}$. Show a substitution check and explain why $w = 0$ is excluded.

Teacher solution and discussion

Divide both sides by $w > 0$: $l = A/w$. With the given values, $l = 48/6 = 8 \text{ cm}$. Check: $(8 \text{ cm})(6 \text{ cm}) = 48 \text{ cm}^2$. Zero width is excluded by the rectangle domain and would make A/w undefined.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Rearrange product and quotient formulas for a named quantity.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

A triangle has area $A = bh/2$, with $A > 0$ and $h > 0$. Solve for the base b . Use $A = 35 \text{ m}^2$ and $h = 7 \text{ m}$ to find b . Explain the effect of the factor $1/2$ and check the result.

Teacher solution and discussion

Multiply by 2 to undo the factor $1/2$: $2A = bh$. Divide by $h > 0$: $b = 2A/h$. Then $b = 70/7 = 10 \text{ m}$. Check: $(10 \text{ m})(7 \text{ m})/2 = 35 \text{ m}^2$. The positive conditions ensure a positive base.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Isolate a quantity in formulas with sums, differences, or grouped expressions.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

For constant acceleration, $v = u + at$, where velocities and acceleration are signed and $t > 0$. Solve for a . Find a when $v = 18$ m/s, $u = 6$ m/s, and $t = 4$ s. Check your answer and state the division restriction.

Teacher solution and discussion

Subtract u : $v - u = at$. Divide by t : $a = (v - u)/t$. The division requires $t \neq 0$, satisfied by $t > 0$. Thus $a = (18 - 6)/4 = 3$ m/s². Check: $6 + (3)(4) = 18$ m/s.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Isolate a quantity in formulas with sums, differences, or grouped expressions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

A heating model is $Q = mc(T_f - T_i)$. Assume $m > 0$ and $T_f > T_i$. Solve for c . A 10 g sample absorbs 900 J while warming from 20°C to 50°C. Find c in $J/(g \cdot ^\circ C)$, verify it, and explain why equal initial and final temperatures would prevent this division.

Teacher solution and discussion

Divide by $m(T_f - T_i)$, which is positive: $c = Q/[m(T_f - T_i)]$. The temperature change is $50 - 20 = 30^\circ C$, so $c = 900/(10 \cdot 30) = 3 J/(g \cdot ^\circ C)$. Check: $10(3)(50 - 20) = 900 J$. If $T_f = T_i$, the divisor is zero; with $Q = 900$ the original formula would then be impossible.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Rearrange product and quotient formulas for a named quantity.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Kinetic energy is $E = mv^2/2$. With $E > 0$ and speed $v > 0$, solve for mass m . Find m for $E = 45$ J and $v = 3$ m/s. A learner proposes $m = 2E/v$. Use the original formula to explain why the square cannot be dropped.

Teacher solution and discussion

Multiply by 2 and divide by $v^2 > 0$: $m = 2E/v^2$. Thus $m = 90/9 = 10$ kg. Check: $10(3^2)/2 = 45$ J. The proposed expression gives $90/3 = 30$; substituting that mass gives $30(9)/2 = 135$ J, not 45 J. The factor multiplying m is $v^2/2$, so both velocity factors matter.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Rearrange product and quotient formulas for a named quantity.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

The wave formula $v = f\lambda$ relates positive speed v , frequency f , and wavelength λ . Solve for f . Find f for $v = 340$ m/s and $\lambda = 2$ m. State the frequency in Hz (1/s), explain the units, and check the product.

Teacher solution and discussion

Divide by $\lambda > 0$: $f = v/\lambda$. Thus $f = (340 \text{ m/s})/(2 \text{ m}) = 170/\text{s} = 170 \text{ Hz}$. The meter units cancel, leaving inverse seconds. Check: $(170/\text{s})(2 \text{ m}) = 340 \text{ m/s}$.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Isolate a quantity in formulas with sums, differences, or grouped expressions.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Within its stated range of use, a linear expansion model is $L = L_0(1 + \alpha\Delta T)$, with $L_0 > 0$ and $\alpha > 0$. Solve for temperature change ΔT . Use $L = 10.02$ m, $L_0 = 10$ m, and $\alpha = 0.00002$ per $^{\circ}\text{C}$. Find ΔT exactly, verify it, and state what restriction keeps the modeled final length positive.

Teacher solution and discussion

Subtract L_0 after expanding: $L - L_0 = L_0\alpha\Delta T$. Divide by $L_0\alpha > 0$: $\Delta T = (L - L_0)/(L_0\alpha)$. Thus $\Delta T = 0.02/[10(0.00002)] = 0.02/0.0002 = 100^{\circ}\text{C}$. Check: $10[1 + 0.00002(100)] = 10(1.002) = 10.02$ m. A positive final length requires $1 + \alpha\Delta T > 0$, equivalently $\Delta T > -1/\alpha$; the given increase satisfies it.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

For motion at constant speed, distance $d = vt$, where $t > 0$ is elapsed time.

Which formula gives speed v ?

- A. $v = t/d$
- B. $v = dt$
- C. $v = d/t$
- D. $v = d - t$

Reasoning: _____

M02 | Warm-up | T1

Density is $\rho = m/V$, with mass $m > 0$ and volume $V > 0$. Which formula gives V in terms of m and ρ ?

- A. $V = m\rho$
- B. $V = m - \rho$
- C. $V = \rho/m$
- D. $V = m/\rho$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Rearrange product and quotient formulas for a named quantity.

A rectangle has area $A = lw$, with positive length l and width w . Solve the formula for l . Then find l when $A = 48 \text{ cm}^2$ and $w = 6 \text{ cm}$. Show a substitution check and explain why $w = 0$ is excluded.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Rearrange product and quotient formulas for a named quantity.

Average speed is $v = d/t$, where $d > 0$ and $t > 0$. Solve for elapsed time t . A trip covers 84 km at an average speed of 28 km/h. Find t , include units, and verify the original formula.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Rearrange product and quotient formulas for a named quantity.****Answer: C. $v = d/t$**

Divide both sides of $d = vt$ by the nonzero time t : $d/t = v$. Distance divided by time has speed units.

Check each alternative

- A. The ratio t/d reverses distance and time; its units are reciprocal speed.
- B. Multiplying d by t repeats multiplication rather than undoing it.
- C. Correct: dividing both sides by t gives $v = d/t$.
- D. Subtracting t does not undo multiplication by t and mixes distance with time.

M02 • T1: Rearrange product and quotient formulas for a named quantity.**Answer: D. $V = m/p$**

Multiply by V to obtain $pV = m$. Because positive mass and volume imply $p > 0$, divide by p : $V = m/p$.

Check each alternative

- A. This multiplies by p instead of dividing by it.
- B. This subtracts the coefficient p instead of undoing its multiplication by V .
- C. This reverses the quotient obtained by solving $pV = m$.
- D. Correct: $pV = m$ becomes $V = m/p$.

WORKED TEACHER KEYS

001 / 002**001 • T1: Rearrange product and quotient formulas for a named quantity.**

Divide both sides by $w > 0$: $l = A/w$. With the given values, $l = 48/6 = 8$ cm. Check: $(8 \text{ cm})(6 \text{ cm}) = 48 \text{ cm}^2$. Zero width is excluded by the rectangle domain and would make A/w undefined.

002 • T1: Rearrange product and quotient formulas for a named quantity.

Multiply by t : $vt = d$. Because d and t are positive, $v > 0$. Divide by v : $t = d/v = 84/28 = 3$ h. Check: $84 \text{ km} / 3 \text{ h} = 28 \text{ km/h}$. Distance divided by speed gives time.