

TEACHER AND STUDENT RESOURCE

Rational and Irrational Number Arguments



NC Math 2 • NC.M2.N-RN.3

The Real Number System

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.N-RN.3 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use the properties of rational and irrational numbers to explain why:

- the sum or product of two rational numbers is rational;
- the sum of a rational number and an irrational number is irrational;
- the product of a nonzero rational number and an irrational number is irrational.

T1 Prove rational numbers are closed under addition and multiplication.

T2 Explain rational-plus-irrational and nonzero-rational-times-irrational results.

T3 Use counterexamples to distinguish valid claims from overgeneralizations.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M02, O02, M11, O11, M13, O13, M21, O21, M24, O24

T2: M03, O03, M04, O04, M05, O05, M08, O08, M09, O09, M10, O10, M12, O12, M15, O15, M17, O17, M18, O18, M23, O23, M26, O26, M28, O28, M30, O30

T3: M06, O06, M07, O07, M14, O14, M16, O16, M19, O19, M20, O20, M22, O22, M25, O25, M27, O27, M29, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Prove rational numbers are closed under addition and multiplication.

Learning target: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Learning target: Use counterexamples to distinguish valid claims from overgeneralizations.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Prove rational numbers are closed under addition and multiplication.

Learning target: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Learning target: Use counterexamples to distinguish valid claims from overgeneralizations.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Rational numbers as integer ratios with nonzero denominators; known irrationality of square roots of nonsquare positive integers; elementary proof by contradiction.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Known irrational radicals such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{6}$ may be used. A finite displayed decimal is rational. A nonterminating decimal is not automatically irrational: repeating decimals are rational.

TEACHER MODEL

Why the nonzero condition matters

Prove $3\sqrt{2}$ is irrational, then explain why the same proof cannot establish that $0\sqrt{2}$ is irrational.

Assume $3\sqrt{2}=r$ is rational. Dividing by 3 gives $\sqrt{2}=r/3$, which would be rational, a contradiction. For multiplier 0, division by the multiplier is undefined. In fact $0\sqrt{2}=0$, which is rational. A hypothesis is necessary when the proof uses it.

VISUAL ANCHOR

Claims and their necessary conditions

The hypotheses determine what the conclusion can guarantee.

Inputs	Operation	Guaranteed result
rational and rational	sum or product	rational
rational and irrational	sum	irrational
nonzero rational and irrational	product	irrational
irrational and irrational	sum or product	not determined by type alone

Use a proof for an always statement; use one valid counterexample to refute an always statement. The zero exception is mathematical, not a formatting detail.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Prove rational numbers are closed under addition and multiplication.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Have learners identify the exact number type and the operation separately. Ask for integer-ratio evidence when a number is rational, and avoid treating the presence of a radical or decimal as sufficient classification.

Core task

Add $\frac{5}{6}$ and $\frac{7}{8}$ exactly. Then generalize: show why $\frac{a}{b} + \frac{c}{d}$ is rational when a, b, c, d are integers and $b, d \neq 0$.

Teacher solution and discussion

The numerical sum is $\frac{41}{24}$. In general $\frac{a}{b} + \frac{c}{d} = \frac{(ad+bc)}{(bd)}$. Its numerator and denominator are integers, and $bd \neq 0$, so the sum is rational.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Think aloud through a contradiction proof. Clearly state the assumed rational result, use rational closure to isolate the known irrational number, and name the contradiction. Point out every division by a nonzero quantity.

Core task

Using the irrational number $\sqrt{2}$, explain why the word nonzero is necessary in the rational-times-irrational theorem. Give the counterexample and the corrected statement.

Teacher solution and discussion

Take rational $r=0$ and irrational $s=\sqrt{2}$. Their product is 0, which is rational. The corrected claim is: if r is a nonzero rational number and s is irrational, then rs is irrational. Its proof divides by r , so zero must be excluded.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Invite learners to supply the missing closure step. Compare a worked example with a general statement, asking whether an example proves all cases or only refutes an always claim.

Core task

Let q be rational and u irrational. Classify $q+5u$ and $q+u-u$, proving each claim. Do not use decimal approximations.

Teacher solution and discussion

$5u$ is irrational by the nonzero rational multiplier result, so $q+5u$ is irrational. The second expression simplifies to q , which is rational. Cancellation must be considered before classification.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Use counterexamples to distinguish valid claims from overgeneralizations.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign partners to construct examples and challenge their input conditions. Require exact simplification before classification. Each group should produce a counterexample when a proposed generalization fails.

Core task

Let u be irrational and v real. Prove that if uv is a nonzero rational number, then v is irrational. Explain why the nonzero-product condition cannot be dropped.

Teacher solution and discussion

A nonzero product forces $v \neq 0$. If v were rational, uv would be irrational by the nonzero rational multiplier theorem, contradicting the given rational product. If the product is allowed to be zero, $v=0$ is a rational counterexample for any irrational u .

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Use counterexamples to distinguish valid claims from overgeneralizations.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect unprompted classifications and proofs. Do not tell learners which quantity to isolate. A correct label without a reason is incomplete evidence of the explanatory standard.

Core task

Two real numbers sum to 10. Give one example where both are rational and one where both are irrational. Prove that a pair with exactly one irrational number is impossible.

Teacher solution and discussion

Rational example: 3 and 7. Irrational example: $\sqrt{2}$ and $10-\sqrt{2}$. Both irrationals are justified by rational-plus-irrational reasoning. If exactly one were irrational and the other rational, their sum would be irrational, contradicting the rational value 10.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Prove rational numbers are closed under addition and multiplication.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Diagnose whether the gap is rational closure, the nonzero hypothesis, or overgeneralization about two irrationals. Use one contrasting example pair and then ask for a new proof or counterexample.

Core task

Repair this argument: Every a/b with integer a, b is rational; therefore rational numbers are closed under division. State the needed restrictions and give a division that violates the unrestricted claim.

Teacher solution and discussion

The definition needs $b \neq 0$. For rational $r = a/b$ and $s = c/d$, with $b, d \neq 0$, the quotient $r/s = ad/(bc)$ is rational only when $s \neq 0$, equivalently $c \neq 0$. Division by rational zero, such as $(1/2)/0$, is undefined, not rational. Closure under division is valid only for nonzero divisors.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Explain rational-plus-irrational and nonzero-rational-times-irrational results.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Require a short proof that uses the appropriate hypotheses, not a decimal guess. Check whether the learner distinguishes always statements, possible outcomes, and claims that need a counterexample.

Core task

Classify $(\frac{1}{3})(6+9\sqrt{2})-2$. Simplify it exactly, then justify the classification without decimal calculation.

Teacher solution and discussion

Distribute: $(\frac{1}{3})(6+9\sqrt{2})-2=2+3\sqrt{2}-2=3\sqrt{2}$. This is irrational because 3 is a nonzero rational multiplier and $\sqrt{2}$ is irrational. The rational terms cancel, but the nonzero irrational term remains.

Evidence and response

Assess the reason, not only the classification. Ask which assumed rational quantity would force a known irrational number to be rational. If a student says all irrational sums or products are irrational, request one cancellation or square-root counterexample.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Why is $\frac{2}{3} + \frac{4}{5}$ rational?**

- A. Every sum of real numbers is rational.
- B. A nonterminating decimal is always irrational.
- C. It equals the integer ratio $\frac{22}{15}$.
- D. Both denominators must be the same before the sum is rational.

Reasoning: _____

M02 | Warm-up | T1**Which argument proves that $\frac{2}{3} \times \frac{4}{5}$ is rational?**

- A. Its decimal must terminate.
- B. Any product of fractions is an integer.
- C. The product is $\frac{8}{15}$, a ratio of integers.
- D. All products of real numbers are rational.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Prove rational numbers are closed under addition and multiplication.

Add $\frac{5}{6}$ and $\frac{7}{8}$ exactly. Then generalize: show why $\frac{a}{b} + \frac{c}{d}$ is rational when a, b, c, d are integers and $b, d \neq 0$.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Prove rational numbers are closed under addition and multiplication.

Calculate $\frac{5}{6} \times \frac{7}{8}$ exactly and prove the product of any two rational numbers is rational.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01**M01 • T1: Prove rational numbers are closed under addition and multiplication.****Answer: C. It equals the integer ratio 22/15.**

Use a common denominator: $2/3 + 4/5 = 22/15$. A ratio of integers with nonzero denominator is rational; the original denominators need not match.

Check each alternative

- A. For example $0 + \sqrt{2}$ is irrational.
- B. Repeating nonterminating decimals are rational.
- C. Use a common denominator: $2/3 + 4/5 = 22/15$. A ratio of integers with nonzero denominator is rational; the original denominators need not match.
- D. A common denominator is a calculation method, not a condition on the original numbers.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02**M02 • T1: Prove rational numbers are closed under addition and multiplication.****Answer: C. The product is $8/15$, a ratio of integers.**

Multiply numerators and denominators: $2/3 \times 4/5 = 8/15$. Both are integers and the denominator is nonzero, establishing rationality without needing decimal expansion.

Check each alternative

- A. A repeating decimal is also rational.
- B. A rational product need not be an integer.
- C. Multiply numerators and denominators: $2/3 \times 4/5 = 8/15$. Both are integers and the denominator is nonzero, establishing rationality without needing decimal expansion.
- D. For example $1 \times \sqrt{2}$ is irrational.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001**001 • T1: Prove rational numbers are closed under addition and multiplication.**

The numerical sum is $4\frac{1}{24}$. In general $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$. Its numerator and denominator are integers, and $bd \neq 0$, so the sum is rational.

Evidence to assess

Require correct classification plus a closure-based proof or valid counterexample. Check nonzero denominators and multipliers. Decimal appearance alone is not sufficient evidence of irrationality.

WORKED TEACHER KEYS

002**002 • T1: Prove rational numbers are closed under addition and multiplication.**

The product is $35/48$. Generally $(a/b)(c/d)=ac/(bd)$; ac and bd are integers, and $bd \neq 0$. Thus the product is rational.

Evidence to assess

Require correct classification plus a closure-based proof or valid counterexample. Check nonzero denominators and multipliers. Decimal appearance alone is not sufficient evidence of irrationality.