

TEACHER AND STUDENT RESOURCE

Cross Sections and Solids of Rotation



NC Math 3 • NC.M3.G-GMD.4

Geometric Measurement & Dimension

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-GMD.4 | NC Math 3 mathematics

This sample contains 30 complete selected pages from the 144-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-18, 33-36, 71-72, 101-106

Representative activities

M01: sample PDF page(s) 19; original page(s) 18

M02: sample PDF page(s) 19; original page(s) 18

O01: sample PDF page(s) 20-21; original page(s) 33-34

O02: sample PDF page(s) 22-23; original page(s) 35-36

Selected teacher prompts and keys: original page(s) 71-72, 101-106

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

T1 Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

T2 Identify rotations of filled rectangles, triangles and half-disks about specified axes, distinguishing the generated solid from its boundary.

T3 Analyze interior and exterior rotation axes, cavities and composite generators using swept radial intervals and diagnostic cross-sections.

T4 Test sufficient hypotheses, finite-solid bounds and degenerate cases, construct counterexamples and distinguish true sections from projections.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M04, M05, M06, M07, M09, M26, M27, O01, O02, O04, O05, O06, O09, O10, O11

T2: M14, M15, M16, M17, O14, O15, O21

T3: M18, M19, M20, M21, M22, M23, M29, O16, O17, O18, O19, O20, O22, O23, O29

T4: M08, M10, M11, M12, M13, M24, M25, M28, M30, O03, O07, O08, O12, O13, O24, O25, O26, O27, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify planes and section shapes

Learning target: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Learning target: Test sufficient hypotheses, finite-solid bounds and degenerate cases, construct counterexamples and distinguish true sections from projections.

Use Warm-up and I Do to identify the entire section and justify its shape. Construct O01 O02 O04 and one cube section. Compare true lengths with projected appearance. Keep the finite-cylinder end-cap check explicit in O08.

Session 2 Generate and compare rotational sets

Learning target: Identify rotations of filled rectangles, triangles and half-disks about specified axes, distinguishing the generated solid from its boundary.

Learning target: Analyze interior and exterior rotation axes, cavities and composite generators using swept radial intervals and diagnostic cross-sections.

Use We Do to establish section reasoning, then Collaborative tasks to rotate filled generators about specified axes. Compare O14 O16 and O17 before exterior-axis cavities in O19 and the torus in O20. Each requested second construction receives its own blank workspace.

Session 3 Audit boundaries and recover models

Learning target: Analyze interior and exterior rotation axes, cavities and composite generators using swept radial intervals and diagnostic cross-sections.

Learning target: Test sufficient hypotheses, finite-solid bounds and degenerate cases, construct counterexamples and distinguish true sections from projections.

Use Independent practice to distinguish a filled solid from a swept surface, compare axes and diagnose incomplete models. Reteach the actual gap with O24-O27. Exit tasks require nonuniqueness evidence, a composite generator and a two-scan model decision.

PLANNING

Materials, access and pacing

Materials

Pencils, ruler and separate blank grids for the requested constructions. Optional clay solids, transparent models and paper shapes can support exploration after an initial prediction. Every schematic is dimension-labeled rather than a measured perspective drawing; a calculator is unnecessary for the exact results.

Prerequisites

Recognize plane figures and three-dimensional solids; use radius and diameter, the Pythagorean Theorem, basic coordinate distances and similar triangles; interpret coordinate equalities, inequalities and parallel or perpendicular directions. No calculus or conic-section equation fluency is required.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

Review convention

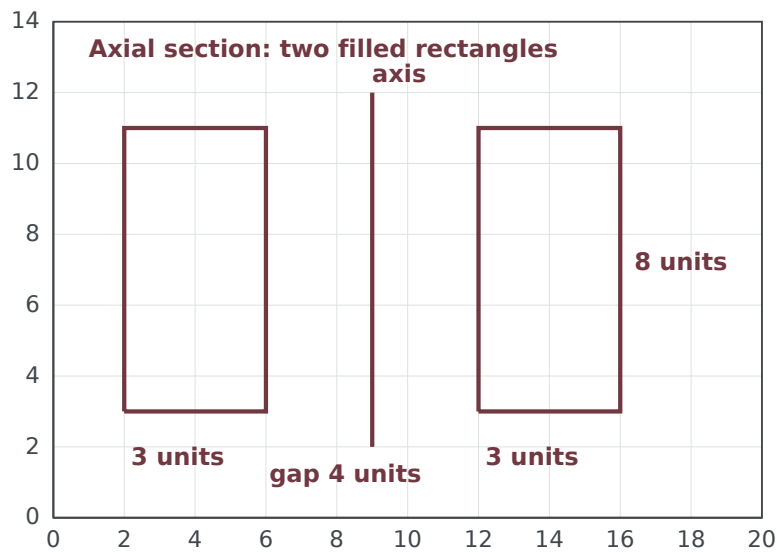
A cross-section is the entire intersection of the stated set and a plane. A closed solid includes its interior and boundary. A disk includes a circular interior; a circle is only its boundary. A ball is a solid; a spherical surface is only its boundary. An annulus includes material between two concentric circles. Boundary-only generators are stated explicitly; do not silently fill them. All rotations are full 360-degree turns about the specified line. Axis location and orientation are part of the data. Rotation sweeps a union of points, so overlapping paths do not add radius. Cylinders and cones are finite right circular solids unless stated otherwise. Interior, tangent, boundary-face and disjoint cuts are distinguished. Name an uncut elliptical section only after checking that finite end caps do not truncate it. Schematics use 2D projection coordinates for placement, not physical measurement; use labels and exact reasoning. Completed construction drawings are teacher-key evidence, never student templates. Accept other valid constructions satisfying every condition.

TEACHER MODEL

Track distance from the axis before naming the solid

The filled rectangle $2 \leq x \leq 5$, $0 \leq y \leq 8$ rotates about the y-axis. Identify the generated solid and its axial section. Explain why the width of the rectangle is neither the inner nor the outer radius.

Not to scale



Schematic coordinates; use stated labels

The nearest generating points are 2 from the axis and the farthest are 5 from it. At every height, the full turn sweeps the radial interval 2 through 5, giving a hollow cylinder with inner radius 2, outer radius 5 and height 8. Rectangle width 3 is wall thickness. An axial plane cuts two filled 3-by-8 rectangles separated by an empty width-4 gap. The entire solid remains connected around the axis even though this section has two components. First locate the axis, then track all occupied distances, and only then name the set.

VISUAL ANCHOR

Check the plane the generator and the axis

Right cylinder: perpendicular interior cut gives a disk; axial cut gives a diameter-by-height rectangle.

Axis-parallel offset cylinder cut: width is a chord, not automatically a diameter.

Pyramid or cone: parallel-base sections are similar; use distance from the apex for the linear scale.

Ball at center distance $d < R$: disk radius $\sqrt{R^2 - d^2}$; $d = R$ gives a point; $d > R$ gives no intersection.

A cube can have triangular, rectangular, pentagonal or hexagonal sections; the plane and its position matter.

Filled rectangle about a side: cylinder. Filled right triangle about a leg: cone. Filled half-disk about its diameter: ball.

Exterior axis: a rectangle can make a hollow cylinder; an offset filled disk can make a solid ring torus.

VISUAL ANCHOR CONTINUED

Check the plane the generator and the axis

Interior axis: use all absolute distances and take their union. Do not add overlapping swept regions.

A boundary curve, a filled region and a region with a cavity need not generate the same set.

A tilted infinite-cylinder section may be elliptical, but finite end caps can clip it. Check the bounds first.

A shape name without the plane or axis conditions can be misleading. Use the exact occupied region, distinguish true shape from perspective, and verify whether the result has positive area or three-dimensional interior.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask learners to name the set being cut before naming the shape. Use O01 to separate the prism height from the section dimensions, and have the learner finish both the perspective and true-shape drawings. In O02 ask “Where is the chord midpoint, and why is this angle right?” Require the full chord after finding a half. Use O03 to distinguish a smaller disk, a tangent point and no intersection; do not call all three circles. On O04 measure similarity from the apex, not from the base. Keep the base endpoint and apex endpoint in the radius rule.

Core task

A solid rectangular prism has a 9 cm by 5 cm base and height 7 cm. A plane parallel to the base is 2 cm above it. On one schematic, draw the prism and the cutting plane and label all given dimensions. On a separate schematic, draw the true shape of the cross-section and label its side lengths. Explain which dimensions change if the plane is raised to 6 cm above the base.

Teacher solution and discussion

Draw two parallel 9-by-5 rectangles connected by height-7 edges, then an interior plane parallel to them at height 2. Its intersection is a filled 9 cm by 5 cm rectangle; draw this true shape separately rather than measuring its perspective image. Raising the plane to height 6 changes only its location. Since $0 < 6 < 7$ and a prism has congruent parallel-base sections, both section side lengths stay 9 cm and 5 cm. The 2 cm or 6 cm cut height is not a side length of the section.

Evidence and response

Require two completed schematics, a properly located parallel cut, all three prism dimensions plus cut height, the true 9-by-5 section and an explanation that raising the interior plane does not change its dimensions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 by first enforcing all coordinate bounds, then locating actual face intersections. Do not extend the plane to imaginary intercepts outside the cube. In O06 verify the right angle with perpendicular directions rather than a perspective picture. For O07 assign one face to each consecutive edge and exclude the remaining face; simply counting listed points is not a proof of completeness. Think aloud through O08 in this order: finite height bounds, section family, true in-plane lengths, then projection comparison. A circle seen from above is not evidence of a circular section.

Core task

A solid cube is given by $0 \leq x, y, z \leq 3$. Construct a perspective schematic of its intersection with $x + y + z = 2$, labeling the three cut vertices A, B and C. State their coordinates, prove that the section is an equilateral triangle and find its side length. Explain why no additional face of the cube creates a fourth side.

Teacher solution and discussion

The cut vertices are $A=(2,0,0)$, $B=(0,2,0)$ and $C=(0,0,2)$. Join them inside the cube in the schematic. Every section point has nonnegative coordinates summing to 2, so each coordinate is at most 2, already below the upper cube limit 3. Therefore the faces $x=3$, $y=3$ and $z=3$ never meet this section; the three coordinate-zero faces supply all three sides. Distances AB, BC and CA each equal $\sqrt{2^2+2^2}=2\sqrt{2}$, establishing an equilateral triangle. Its filled interior is included because the cube is solid.

Evidence and response

Require a completed three-vertex cut in the cube, all exact vertex coordinates and equal-distance calculations, plus a bound excluding the three far faces and hence any fourth side. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

In O09 have one learner identify the complete triangle and another identify one of its right-triangle halves. Require both schematic drawings and all labels before discussing classification. Compare the two apex planes in O10 to show that naming a triangle does not determine its dimensions. On O11 contrast translation in a prism with similarity in a pyramid. In O12 retain both signed plane positions while recognizing that center distance itself is nonnegative. Use O13 to let students explain how a supporting plane can meet a polyhedron in a vertex, edge or face.

Core task

A right circular cone has base radius 5 cm and height 12 cm. Make one schematic showing an axial cutting plane and its triangular intersection within the cone, and a second showing the true section alone. Label the section base, height and both slant sides. Determine whether the section is equilateral, isosceles or right, and justify every classification you accept or reject.

Teacher solution and discussion

An axial plane contains the cone axis, so it meets the base in a diameter and the cone at its apex. The section is a filled triangle with base 10 cm and altitude 12 cm. Each half has legs 5 and 12, so the slant sides are 13 cm. Thus the complete section is isosceles with sides 13,13,10, not equilateral. For a right triangle the largest side squared would equal the sum of the other two squares; here $13^2 \neq 13^2 + 10^2$. It is not right, although its altitude divides it into two right triangles. Both schematics must show the entire section, not a single half.

Evidence and response

Require two complete labeled schematics, diameter 10, height 12 and slant sides 13; distinguish isosceles from equilateral and explicitly reject rightness for the whole triangle while recognizing the right-triangle halves. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Identify rotations of filled rectangles, triangles and half-disks about specified axes, distinguishing the generated solid from its boundary.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign generator, axis and swept-distance checks to different learners after private predictions. O14 needs two separate constructions: use the specified side as height and the perpendicular side as radius. On O15 keep the common hypotenuse visible while the axis roles change. O16 deliberately overlaps swept points from opposite sides; ask why visiting a point twice does not increase the radius. O17 should show the full filled frustum, not merely its outer surface. For O18 name the shared disk and the two disjoint interiors, then verify each cone's own halfway section.

Core task

A filled 3 cm by 5 cm rectangle can be rotated about the line containing either one of its sides. For each of the two different side-length choices, make a separate schematic showing the rectangle, the chosen axis and the completed generated solid. Label the solid's radius and height. Compare its axial section and a section perpendicular to the axis with those for the other rotation.

Teacher solution and discussion

About a 5 cm side, the perpendicular width is 3, so the result is a cylinder of radius 3 and height 5. Its axial section is a 6-by-5 rectangle, and any interior perpendicular section is a radius-3 disk. About a 3 cm side, the result is a cylinder of radius 5 and height 3, with a 10-by-3 axial rectangle and radius-5 perpendicular disks. Draw each generator and cylinder with its own axis clearly marked. The same rectangle generates noncongruent cylinders because switching the axis exchanges which dimension is height and which is radius; a rectangle side becomes a radius, not the cylinder diameter.

Evidence and response

Require two generator-and-solid schematics with distinct axes, all radii and heights, both diameter-based axial rectangles and both perpendicular disks, and an explicit axis-dependent noncongruence explanation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Analyze interior and exterior rotation axes, cavities and composite generators using swept radial intervals and diagnostic cross-sections.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Collect O19–O23 without giving the shape name or an answer-bearing drawing. In O19 look for the full cavity diameter, not just its radius, between the two axial rectangles. O20 asks for two genuinely different section views; check the disconnected axial disks and the out-of-plane explanation of connectivity. O21 separates equality from inequality and requires all tangent and disjoint cases. O22 must use the hypotenuse, not a familiar leg, as axis and label both unequal cone heights. O23 tests a perpendicular axis that leaves an annulus planar rather than making a solid.

Core task

The rectangle $0 \leq x \leq 3$, $0 \leq y \leq 8$ rotates fully about the vertical line $x = -2$ in its plane. Identify the solid and its dimensions. On separate schematics draw (1) its full section by a plane containing the rotation axis and (2) a section perpendicular to the axis halfway up. Label every width, gap or radius needed to distinguish material from the central cavity.

Teacher solution and discussion

Distances from $x = -2$ to the rectangle range from 2 to 5, so the result is a hollow circular cylinder with inner radius 2, outer radius 5 and height 8. In an axial plane, the material forms two filled rectangles, each width $5 - 2 = 3$ and height 8, separated by a gap of width $2 \cdot 2 = 4$ centered on the axis. The perpendicular mid-height section is an annulus with inner radius 2 and outer radius 5, not a disk. Draw and label both disconnected axial material regions and the annular material region; the gap is empty even though the cylinder as a whole surrounds it.

Evidence and response

Require the exterior-axis distance calculation, all three solid dimensions, two distinct completed section schematics, both axial rectangle widths and the full cavity gap, and both annular radii with material clearly distinguished. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Test sufficient hypotheses, finite-solid bounds and degenerate cases, construct counterexamples and distinguish true sections from projections.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O24 for confusion among interior, boundary and exterior axes. Draw an interval on a temporary teacher model, name its absolute distances and then remove that model before reassessment. O25 diagnoses the subtle boundary-only case: track all four sides, including the fixed axis edge, before taking their union. O26 distinguishes disks, surfaces and segments from positive-volume solids. For O27 write the finite-height interval first, identify both cap chords and then construct the clipped region. Do not repair a clipped-ellipse error by silently replacing the finite cylinder with an infinite one.

Core task

A student says, “A rectangle rotated about any parallel vertical line makes a hollow cylinder unless the line is exactly on a side.” Test the claim using the filled rectangle $-1 \leq x \leq 3$, $0 \leq y \leq 4$. Determine the result about the y -axis and about $x=5$. Give the radial interval at each height in both cases, and write a correct general rule for a vertical axis and a rectangle.

Teacher solution and discussion

About the y -axis, distances from the interval $[-1,3]$ include every value from 0 to 3, so the result is a filled cylinder of radius 3 and height 4, even though the axis passes through the interior rather than a side. About $x=5$, all rectangle points lie on one side and distances range from $5-3=2$ to $5-(-1)=6$, so the result is a hollow cylinder with inner radius 2, outer radius 6 and height 4. In general, if the vertical axis meets the rectangle’s horizontal interval, minimum radius is zero and the cylinder is filled. If it lies strictly outside, minimum radius is positive and the cylinder is hollow; maximum radius is the farthest endpoint distance.

Evidence and response

Give both full radial intervals and cylinder dimensions, use the interior-axis case as an explicit counterexample, and state a general rule that includes interior, boundary and exterior axes. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Test sufficient hypotheses, finite-solid bounds and degenerate cases, construct counterexamples and distinguish true sections from projections.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 requires three complete models and one useful new observation; a list of solid names is not enough. In O29 collect the exact generating polygon and both completed constructions, then check the two disk sections, the apex point and the outside plane. On O30 the first annulus is deliberately ambiguous: learners must use the second scan to distinguish the two stated candidates without claiming uniqueness among all solids. Record whether any remaining gap concerns plane location, axis distance, boundary versus interior, finite bounds or a missing justification, then choose the next task from that evidence.

Core task

A scanner reports one filled circular disk cross-section of radius 2 cm. Show that this observation alone does not identify the three-dimensional solid by giving three noncongruent examples: one ball, one right cylinder and one right cone. Specify all dimensions and the plane location for each, and verify the radius-2 section. State one additional observation that would distinguish your cylinder from your cone.

Teacher solution and discussion

One example is a ball of radius 2 cut through its center. Another is a right cylinder of radius 2 and height 6 cut perpendicular to its axis at height 3 above the base. A third is a right cone of base radius 4 and height 8 cut parallel to its base at height 4 above the base; its apex distance is 4, half the full height, so its section radius is $4 \cdot (4/8) = 2$. The solids are noncongruent despite sharing this section. For the chosen cylinder and cone, take a second parallel cut 1 cm higher: the cylinder radius stays 2, but the cone cut is then 3 below the apex and has radius $4 \cdot (3/8) = 3/2$. This additional observation separates the two models.

Evidence and response

Supply three fully dimensioned solids with valid cut positions and verified equal section radii, then give a specific additional measurement and predicted values that actually distinguish the chosen cylinder and cone. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A solid right circular cylinder has radius 4 cm and height 9 cm. A plane perpendicular to its axis passes 2 cm above the bottom base. What is the entire cross-section, including its interior?

- A. A rectangle 8 cm by 9 cm.
- B. Only a circle of radius 4 cm, without its interior.
- C. A disk of radius 2 cm.
- D. A disk of radius 4 cm.

Reasoning: _____

M02 | Warm-up | T1

A plane contains the axis of a solid right circular cylinder of radius 3 m and height 8 m. Which figure is the cross-section?

- A. A rectangle 6 m by 8 m.
- B. A disk of radius 3 m.
- C. A rectangle 3 m by 8 m.
- D. An isosceles triangle of height 8 m.

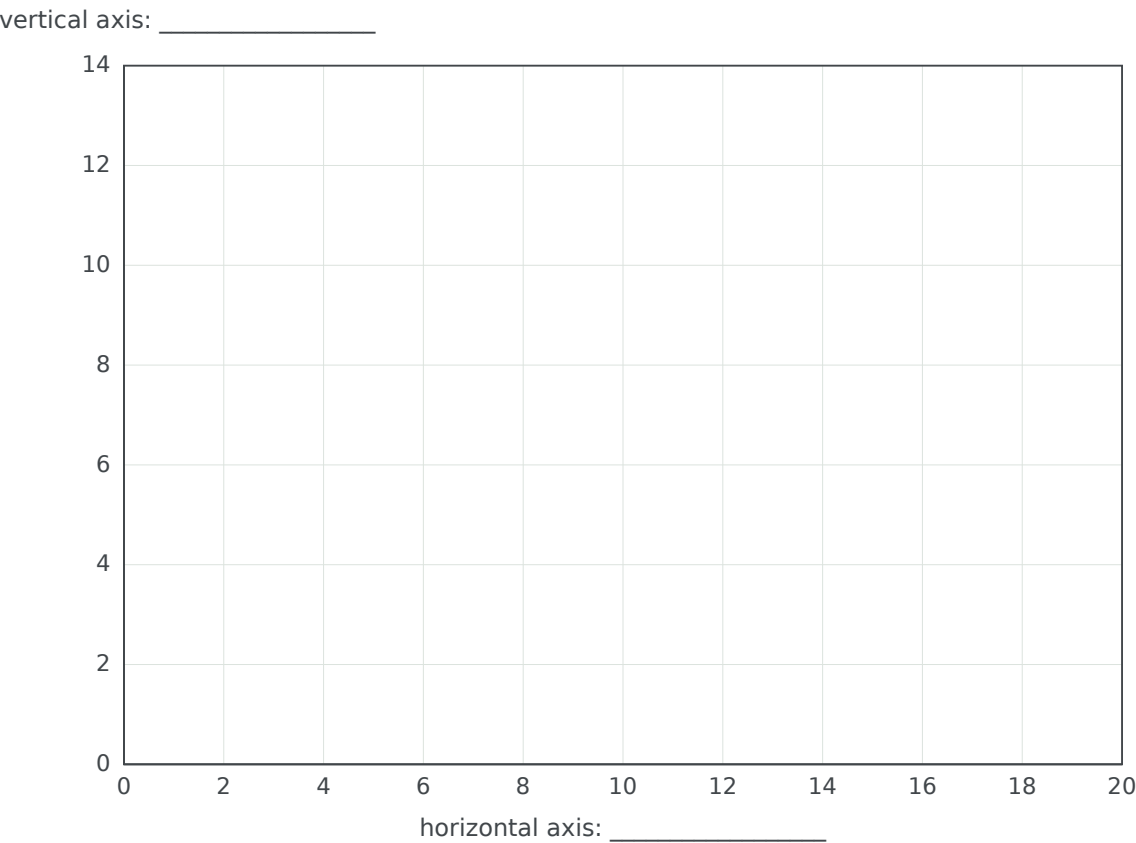
Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

A solid rectangular prism has a 9 cm by 5 cm base and height 7 cm. A plane parallel to the base is 2 cm above it. On one schematic, draw the prism and the cutting plane and label all given dimensions. On a separate schematic, draw the true shape of the cross-section and label its side lengths. Explain which dimensions change if the plane is raised to 6 cm above the base.



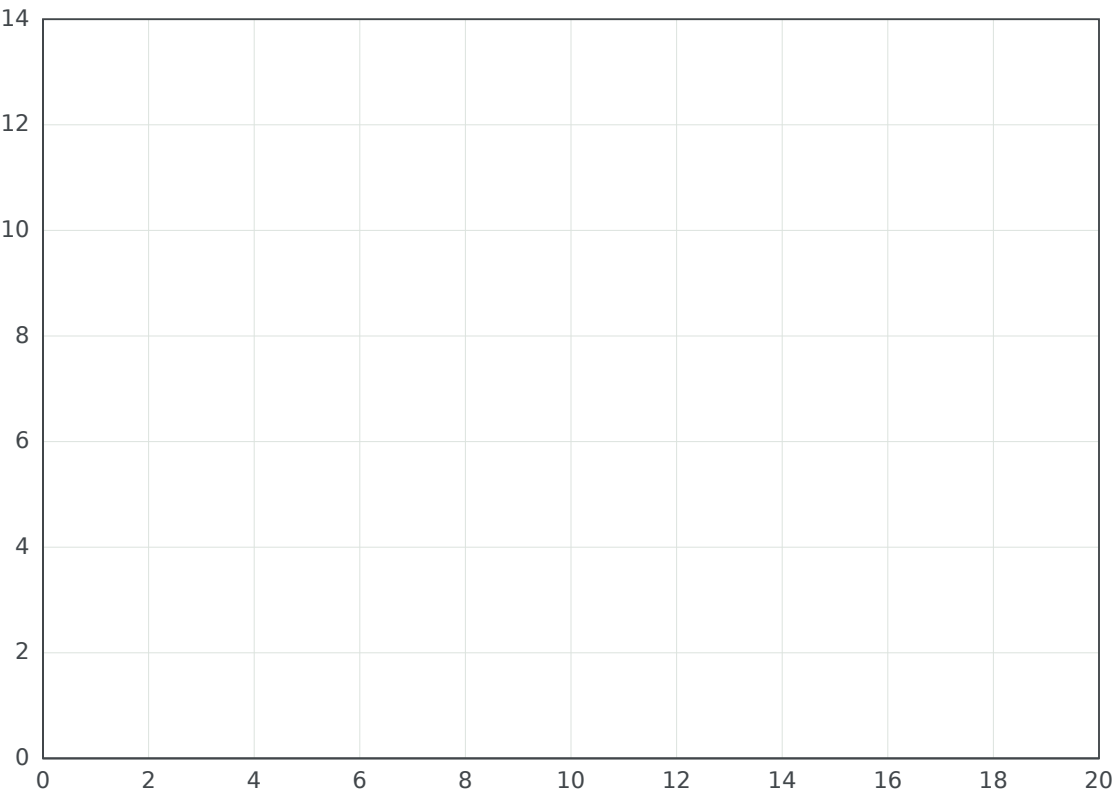
Explanation and check

WARM-UP WORKSPACE

001 • Construction 2

Use this separate grid for the other construction requested in the task. Label your figure and record the evidence that distinguishes it.

vertical axis: _____



horizontal axis: _____

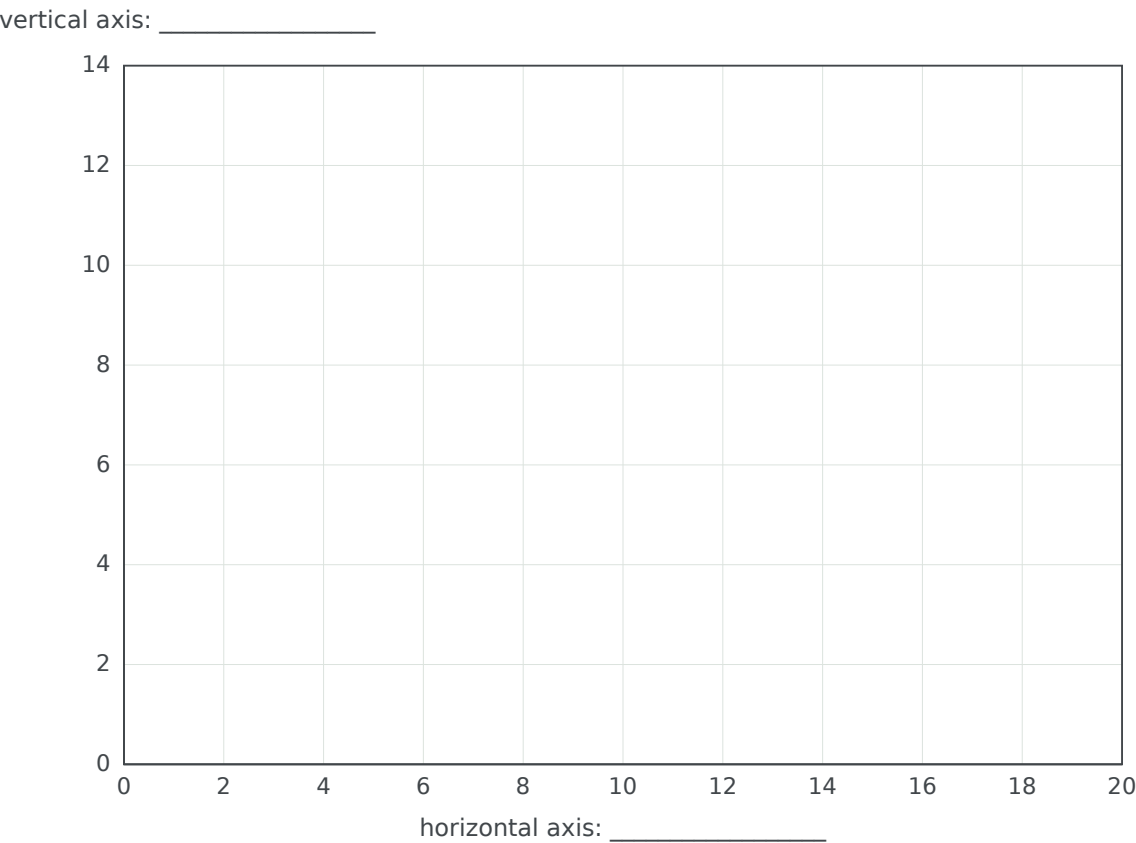
Explanation and check

WARM-UP

002 • Show your reasoning

T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

A right circular cylinder has radius 13 cm and height 9 cm. A cutting plane is parallel to its axis and 5 cm from the axis. Draw a base-view diagram showing the chord, its midpoint and the right triangle needed to find the chord length. On a separate schematic, draw and label the actual cross-section. Explain why the width is neither 13 cm nor 26 cm.



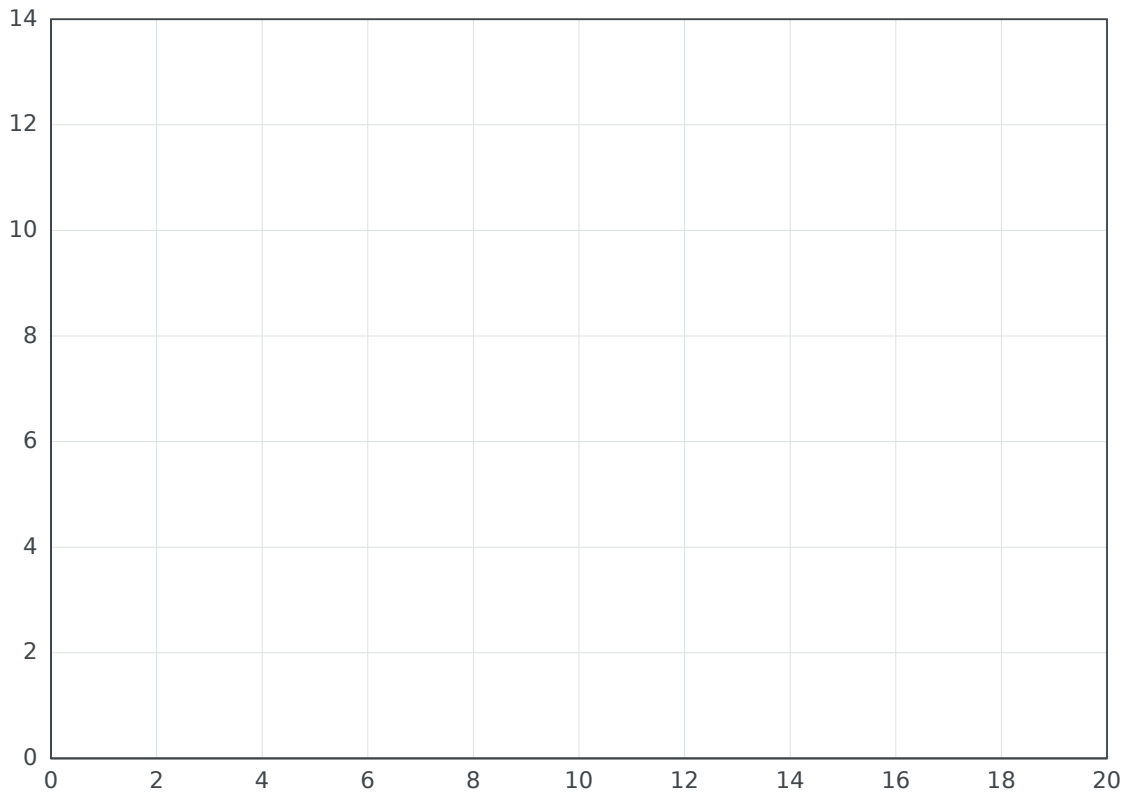
Explanation and check

WARM-UP WORKSPACE

002 • Construction 2

Use this separate grid for the other construction requested in the task. Label your figure and record the evidence that distinguishes it.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01

M01 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Answer: D. A disk of radius 4 cm.

The cut lies strictly between the two bases because $0 < 2 < 9$. A plane perpendicular to the axis is parallel to the circular bases of a right cylinder. Every such interior section is a congruent filled disk, so its radius is 4 cm. The disk boundary is a circle, but the entire cross-section includes interior points.

Check each alternative

A. That rectangle would come from a plane containing the cylinder axis, not a plane perpendicular to it.

B. A solid includes its interior, so its intersection with this plane includes the whole disk, not just the circular boundary.

C. The 2 cm measures the location of the cut, not a changing radius; a cylinder has the same radius throughout its height.

D. Correct. The cut lies strictly between the two bases because $0 < 2 < 9$. A plane perpendicular to the axis is parallel to the circular bases of a right cylinder. Every such interior section is a congruent filled disk, so its radius is 4 cm. The disk boundary is a circle, but the entire cross-section includes interior points.

Evidence to assess

Use both the plane orientation and its interior location, preserve the cylinder radius, and distinguish a disk from its circular boundary.

WORKED TEACHER KEYS

M02

M02 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Answer: A. A rectangle 6 m by 8 m.

In each base plane, the axial plane passes through the center and cuts a diameter $2r=6$ m. The bases are 8 m apart, and the two side segments are perpendicular to them because the cylinder is right. Thus the filled cross-section is a 6 m by 8 m rectangle.

Check each alternative

- A. Correct. In each base plane, the axial plane passes through the center and cuts a diameter $2r=6$ m. The bases are 8 m apart, and the two side segments are perpendicular to them because the cylinder is right. Thus the filled cross-section is a 6 m by 8 m rectangle.
- B. A disk comes from a plane perpendicular to the axis, not one containing it.
- C. An axial plane crosses the full base diameter, not only one radius.
- D. A cylinder does not taper to an apex; its two parallel side generators make a rectangle.

Evidence to assess

Identify the full diameter and the perpendicular height as the two rectangle dimensions, and connect the right-angle conclusion to the right cylinder.

WORKED TEACHER KEYS

001

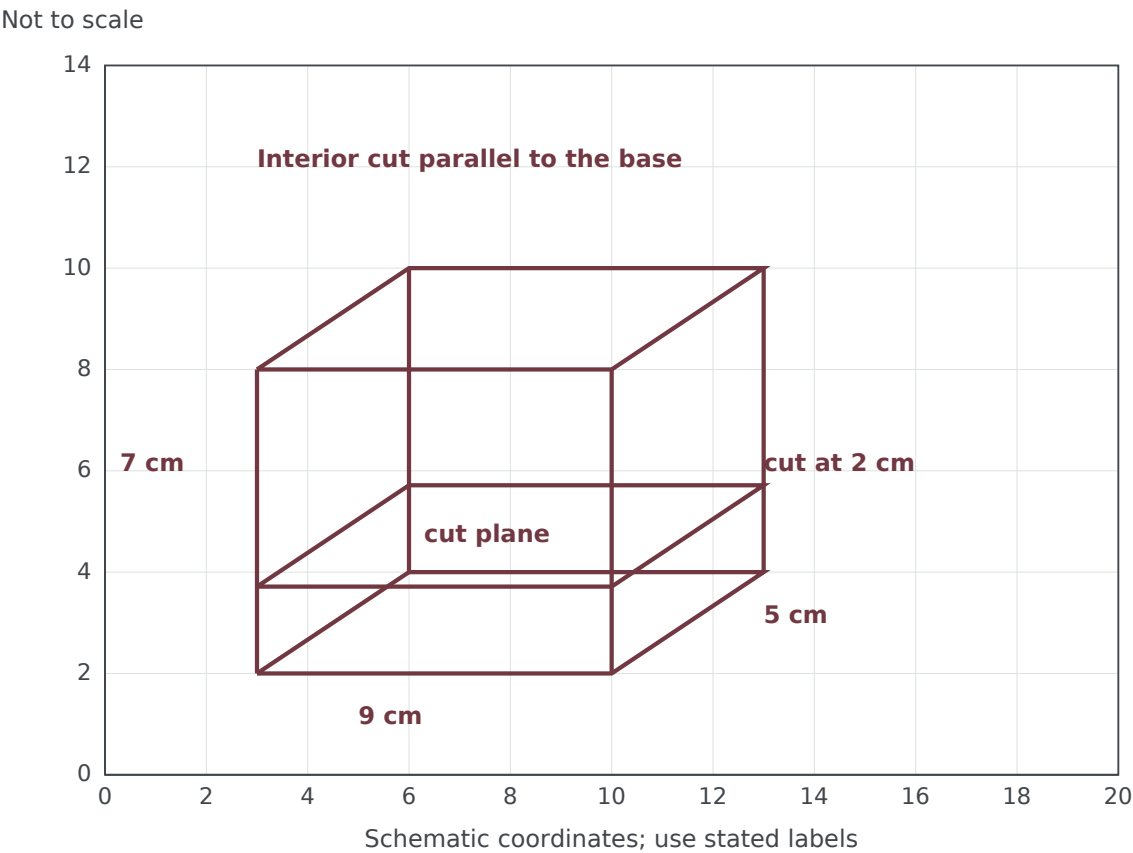
001 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

Draw two parallel 9-by-5 rectangles connected by height-7 edges, then an interior plane parallel to them at height 2. Its intersection is a filled 9 cm by 5 cm rectangle; draw this true shape separately rather than measuring its perspective image. Raising the plane to height 6 changes only its location. Since $0 < 6 < 7$ and a prism has congruent parallel-base sections, both section side lengths stay 9 cm and 5 cm. The 2 cm or 6 cm cut height is not a side length of the section.

WORKED CONSTRUCTION CONTINUATION

001 • Construction 1

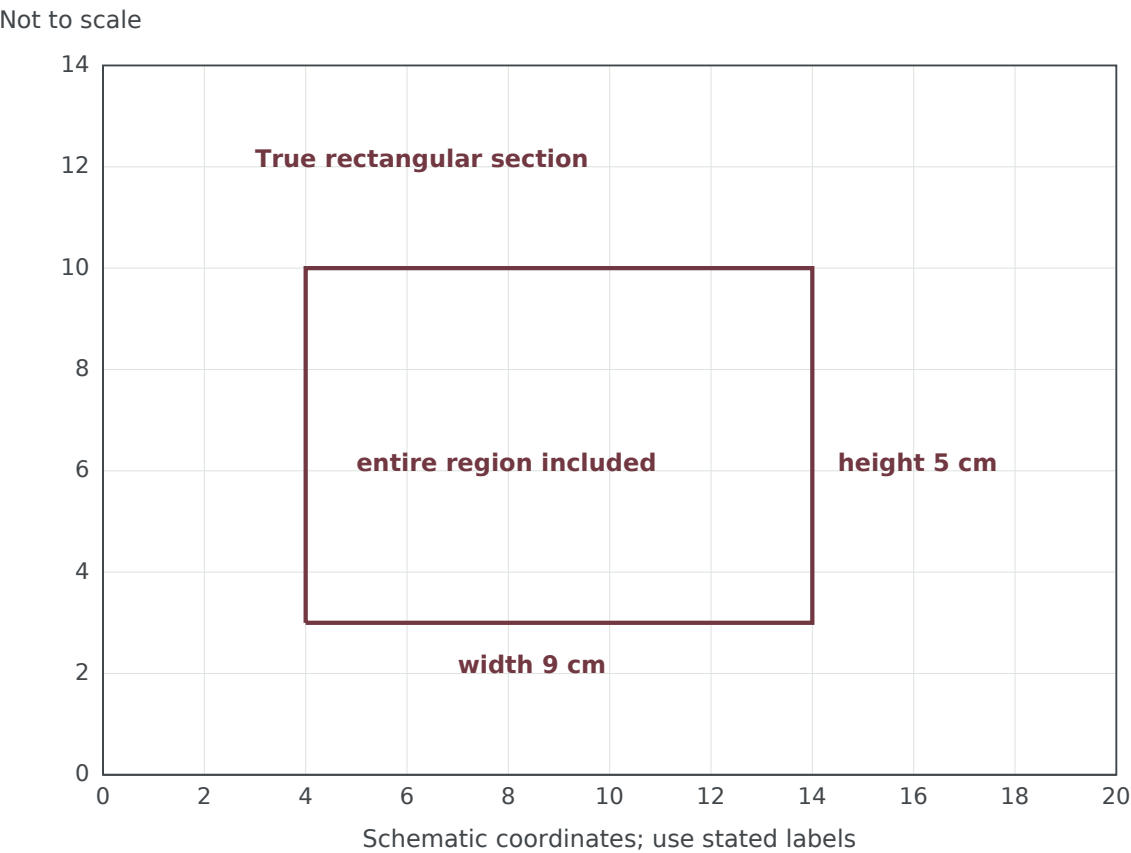
Prism with the parallel plane at height 2 cm



WORKED CONSTRUCTION CONTINUATION

001 • Construction 2

The actual cross-section is a filled 9 cm by 5 cm rectangle



Evidence to assess

Require two completed schematics, a properly located parallel cut, all three prism dimensions plus cut height, the true 9-by-5 section and an explanation that raising the interior plane does not change its dimensions.

WORKED TEACHER KEYS

002

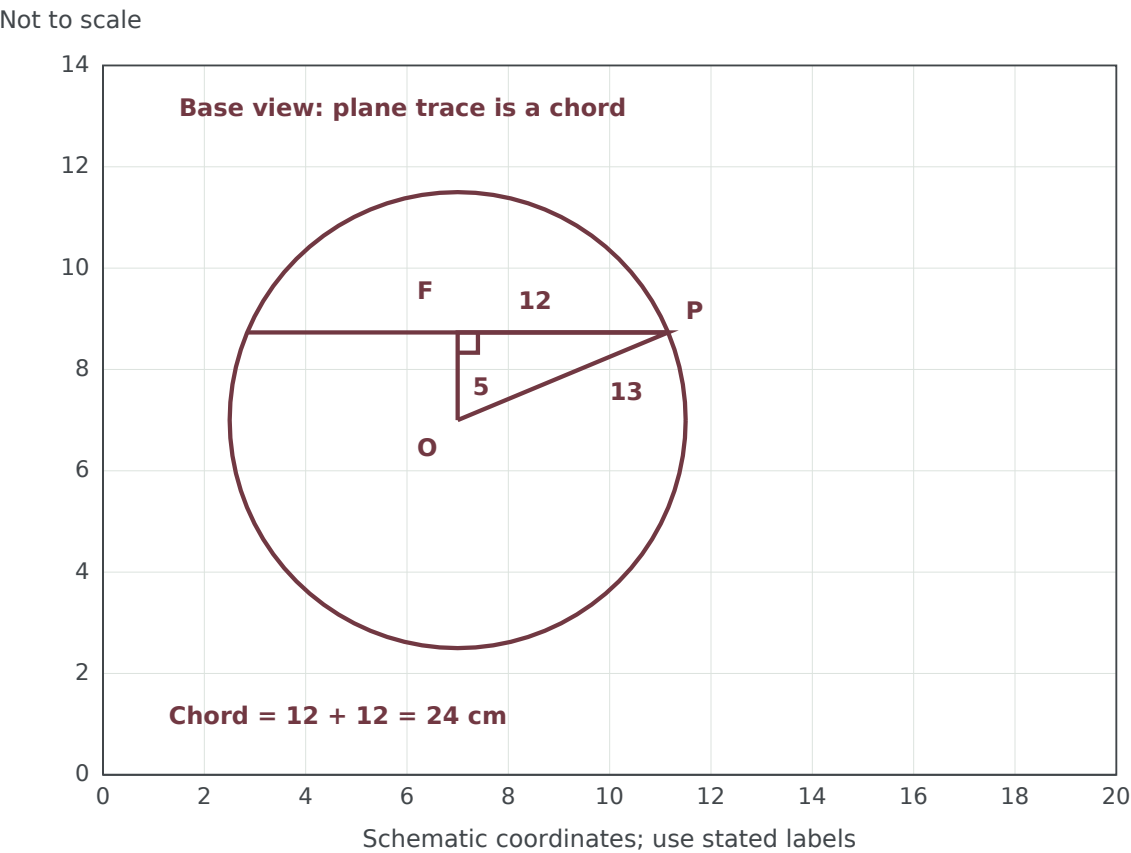
002 • T1: Identify and justify cross-section shapes and dimensions using plane orientation, position, solid structure and similarity.

In the base circle, join center O to the chord midpoint F with $OF=5$, and join O to a chord endpoint P with $OP=13$. OF is perpendicular to the chord, so $FP^2=13^2-5^2=144$ and $FP=12$. The complete chord is 24 cm. The plane extends along the full 9 cm height, so the true section is a filled 24-by-9 rectangle. Width 13 confuses a radius with a chord, while width 26 assumes the plane passes through the axis. The positive offset makes this chord shorter than the diameter.

WORKED CONSTRUCTION CONTINUATION

002 • Construction 1

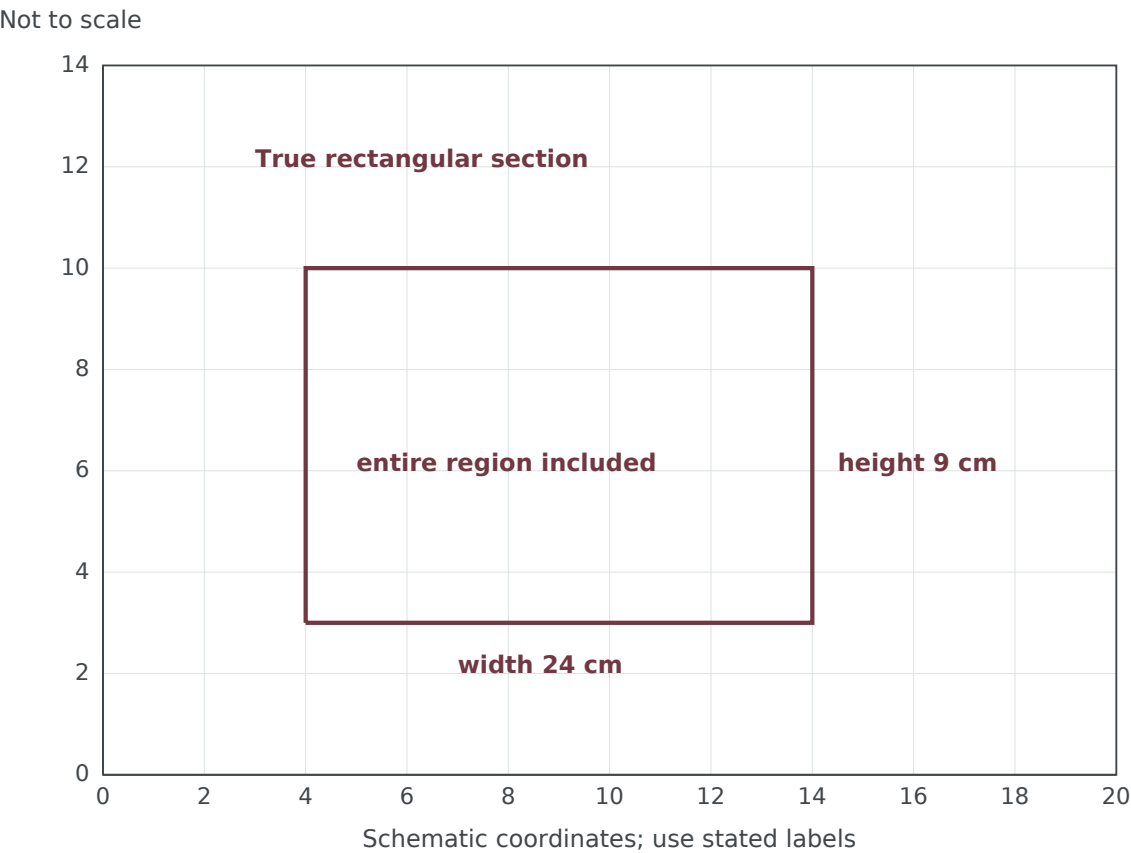
Base view with radius 13 offset 5 and half-chord 12



WORKED CONSTRUCTION CONTINUATION

002 • Construction 2

True axis-parallel section with width 24 and height 9



Evidence to assess

Require the chord midpoint, perpendicular center-to-chord segment and correct radius in the base view; show the 12 cm half-chord calculation, a separate completed 24-by-9 rectangle and both rejected-width explanations.