

TEACHER AND STUDENT RESOURCE

Arithmetic with Rational Expressions



NC Math 3 • NC.M3.A-APR.7

Arithmetic with Polynomial and Rational Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-APR.7 | NC Math 3 mathematics

This sample contains 20 complete selected pages from the 104-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 85

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 85

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.

- a. Add and subtract two rational expressions, $a(x)$ and $b(x)$, where the denominators of both $a(x)$ and $b(x)$ are linear expressions.
- b. Multiply and divide two rational expressions.

T1 Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

T2 Multiply and divide rational expressions by factoring, using reciprocals and preserving original and divisor-zero restrictions (objective b).

T3 Explain similarities with rational-number arithmetic, prove identities and diagnose limitations of cancellation and operation rules (objectives a and b).

T4 Use rational-expression arithmetic in models and representations while checking units, positivity and the original domain (objectives a and b).

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M02, M03, M08, M09, M10, M11, M15, M19, M21, M23, M25, O02, O05, O09, O17, O19, O25, O28

T2: M04, M05, M06, M07, M12, M13, M16, M18, M20, M22, M24, M26, M27, M29, O03, O04, O06, O16, O18, O20, O22, O24, O29

T3: M14, M28, M30, O01, O07, O10, O14, O15, O21, O26, O27, O30

T4: M17, O08, O11, O12, O13, O23

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Equivalent fractions and operation choice

Learning target: Connect number fraction rules to expressions with variable denominators.

Learning target: Use common denominators for sums and factors/reciprocals for products and quotients.

Use Warm-up and I Do tasks selectively. Ask learners to identify the operation before writing any numerator rule and to preserve a separate domain line.

Session 2: Domain-aware models and identities

Learning target: Explain extra restrictions caused by a zero divisor.

Learning target: Use arithmetic to model area, travel time and graphs or construct identities.

Use We Do and Collaborative tasks. Require each learner's first calculation or proposed restriction before group discussion. Compare a legal zero dividend with an illegal zero divisor.

Session 3: Diagnose and transfer

Learning target: Repair cancellation, subtraction and inversion errors with exact evidence.

Learning target: Complete fresh mixed-format tasks while retaining every original restriction.

Use Independent work, targeted Small-group tasks and Exit evidence. A selected option alone does not establish the arithmetic reasoning or domain understanding.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use fraction arithmetic, distributivity, factoring quadratics, polynomial identities and function domains. Distinguish a factor from a term. Know that division requires a defined nonzero divisor, while a zero dividend is allowed. Exact calculations and simple coordinate sketches support reasoning; no calculator is required.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Addition and subtraction tasks use linear original denominators, as required by objective a; their combined denominator may be quadratic. Multiplication and division may use higher-degree polynomials, as in objective b. A complete domain record excludes zeros of every original denominator and, in division, all allowed inputs where the divisor is zero. Restrictions survive cancellation. Equal formulas represent the same function only when their stated domains also agree. Physical contexts may impose additional positivity or integer conditions.

TEACHER MODEL

Read the operation before choosing the numerator rule

Compare $2/(x-1)+3/(x+2)$ and $[2/(x-1)]\div[3/(x+2)]$. Give a simplified form and full domain for each.

For addition, use common denominator $(x-1)(x+2)$: the numerator is $2(x+2)+3(x-1)=5x+1$. Thus the sum is $(5x+1)/[(x-1)(x+2)]$, $x\neq 1,-2$. For division, invert only the divisor: $[2/(x-1)]\cdot[(x+2)/3]=2(x+2)/[3(x-1)]$. Both original denominators still exclude 1 and -2; the divisor numerator 3 never vanishes, so there is no additional divisor-zero exclusion. A common denominator and a reciprocal solve different arithmetic problems, even though the original fractions look the same.

VISUAL ANCHOR

Same fraction laws, explicit variable restrictions

Add: $a/b + c/d = (ad + bc)/(bd)$, with b, d nonzero

Subtract: $a/b - c/d = (ad - bc)/(bd)$; subtract the entire second product

Multiply: $(a/b)(c/d) = ac/(bd)$, with b, d nonzero

Divide: $(a/b) \div (c/d) = ad/(bc)$, with b, d nonzero and c nonzero

Cancel common factors of a whole numerator and denominator, not terms across a sum

Record original restrictions before simplifying; keep canceled exclusions

A zero numerator may be allowed. A zero divisor is never allowed.

The symbols stand for values of polynomial expressions at an allowed input. Establish the domain first, apply the ordinary fraction law, then simplify the factors and verify.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T3: Explain similarities with rational-number arithmetic, prove identities and diagnose limitations of cancellation and operation rules (objectives a and b).

Optional bank: M01–M04; 001–004. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

001 asks how each fraction becomes an equivalent fraction, not just for a memorized cross-product rule. In 002, put brackets around both products before subtracting. 003 distinguishes the legal zero -2 from the forbidden canceled input 2 . For 004, list divisor zeros separately from denominator zeros before writing a reciprocal.

Core task

Calculate $\frac{2}{3} + \frac{1}{4}$ using equivalent fractions. Then combine $\frac{2}{x+1} + \frac{1}{x+1}$. Explain how the same fraction rule produces each numerator, and state every restriction in the variable expression.

Teacher solution and discussion

For the numbers, $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$, so the sum is $\frac{11}{12}$. For the expressions, $\frac{2}{x} = \frac{2(x+1)}{x(x+1)}$ and $\frac{1}{(x+1)} = \frac{x}{x(x+1)}$, giving $\frac{(3x+2)}{x(x+1)}$ for $x \neq 0, -1$. In both calculations, each numerator is multiplied by the factor used to make its denominator common. The denominators are not added. The variable factors must be nonzero so the original expressions and equivalence steps are valid.

Evidence and response

Link numerical and algebraic common-denominator reasoning explicitly, rather than present two unconnected calculations. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O05 to disprove a false formula with both a legal counterexample and a domain comparison. In O06, physically mark the divisor as the only expression inverted. O07 contrasts an additive inverse with a reciprocal. O08 makes the common whole visible but requires a separate explanation of why the algebraic domain exceeds the positive-length picture.

Core task

A learner claims $1/x + 1/(x+2) = 2/(2x+2)$. Disprove the identity at an input where both sides are defined, repair the algebra, and compare the domains of the original sum and the false proposed formula.

Teacher solution and discussion

At $x=1$, the original sum is $1+1/3=4/3$, while the proposed formula is $2/4=1/2$. Correct common-denominator addition gives $[(x+2)+x]/[x(x+2)]=2(x+1)/[x(x+2)]$, with $x \neq 0, -2$. The false formula is undefined at -1 but defined at 0 and -2 , so its domain differs as well as its values. At -1 the actual sum is $-1+1=0$, demonstrating that the false formula even excludes an allowed original input.

Evidence and response

Give a legal numerical counterexample, derive the repaired expression and compare the domains in both directions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

O09 and O10 ask for identities, so compare polynomial coefficients rather than rely on a single input. O11 sketches the sum itself with three branches; keep its zero at -1 distinct from its breaks at -2 and 0. O12 and O13 require units to decide whether to add times or divide area by width. Ask why a model's positivity requirement can be stronger than algebraic definedness.

Core task

Find constants A and B so that $(Ax+B)/(x-1)-(2x-3)/(x-1)=5/(x-1)$ for every $x \neq 1$. Prove the identity with your values and explain why one sample input alone would not determine both constants.

Teacher solution and discussion

On $x \neq 1$, combine numerators to get $[(A-2)x+(B+3)]/(x-1)$. Equality for all allowed inputs requires the polynomial numerator identity $(A-2)x+(B+3)=5$, so $A=2$ and $B=2$. Substituting gives $[(2x+2)-(2x-3)]/(x-1)=5/(x-1)$. A single input gives only one equation in A and B and generally leaves infinitely many pairs; the all-input identity fixes both the x coefficient and the constant term.

Evidence and response

Use coefficient comparison from an identity, verify both constants and distinguish all-input information from one evaluation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Explain similarities with rational-number arithmetic, prove identities and diagnose limitations of cancellation and operation rules (objectives a and b).

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

For O14, have one learner derive addition and another multiplication from number fractions, then reconcile the distinct numerators. O15 uses a parameter to expose a cancellation without removing the original hole. O16 audits three different failure reasons in one quotient. O17 compares two valid methods rather than treating the longer common-denominator route as wrong.

Core task

Let $a(x)$ and $c(x)$ be polynomials and let $b(x)$, $d(x)$ be nonzero linear polynomials. Derive the addition rule $a/b + c/d = (ad + bc)/(bd)$ on its appropriate domain. Contrast it with the multiplication rule, and give a numerical example showing why their numerators must not be confused.

Teacher solution and discussion

Require $b(x) \neq 0$ and $d(x) \neq 0$. Then $a/b = ad/(bd)$ and $c/d = bc/(bd)$, because each fraction is multiplied by a form of 1 with a nonzero denominator. Adding gives $(ad + bc)/(bd)$. Multiplication instead gives $ac/(bd)$, with no cross-product addition. For example $2/3 + 1/4 = 11/12$ whereas $(2/3)(1/4) = 2/12 = 1/6$. Both operations may use denominator 12, but they represent different arithmetic and therefore have different numerators. Any later cancellation keeps the original common-domain restrictions.

Evidence and response

Derive the addition rule from equivalence, contrast multiplication explicitly and retain the nonzero hypotheses. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

On O19, preserve the original $x=1$ exclusion through both requested output forms. O20 checks numerical factors as well as polynomial factors. O21 recovers terms from sum and difference and then verifies both identities. O22 distinguishes an allowed zero numerator from a forbidden zero divisor. O23 must verify the original dimensions after solving the simplified area equation.

Core task

Combine $(x^2-1)/(x-1)+2/(x+1)$. Give both a single-fraction form and a polynomial-plus-fraction form, preserving the original domain. Explain why the first simplification step does not restore $x=1$.

Teacher solution and discussion

The original domain excludes $x=1$ and -1 . Factoring $x^2-1=(x-1)(x+1)$ simplifies the first term to $x+1$ only on $x \neq 1$. Thus a polynomial-plus-fraction form is $x+1+2/(x+1)$, with both exclusions retained. Combining gives $[(x+1)^2+2]/(x+1)=(x^2+2x+3)/(x+1)$, on the same original domain. At $x=1$ this last formula has value 3, but the original first term is $0/0$ and the original sum is undefined. A simpler denominator does not describe the entire inherited domain by itself.

Evidence and response

Produce two equivalent forms for the same restricted function and explain the canceled input explicitly. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Multiply and divide rational expressions by factoring, using reciprocals and preserving original and divisor-zero restrictions (objective b).

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for an identically zero divisor, O25 for subtraction across a numerator, O26 for division order, or O27 for a candidate outside the common domain. Ask for a specific legal example or complete domain argument rather than a slogan. After the discussion, use another task without a cue that names the needed operation.

Core task

Let $F(x) = 2/(x+1)$ and $G(x) = 1/(x-1) - 1/(x-1)$. Is $F(x)/G(x)$ defined at any real input? Give a complete domain argument, including $x=1$ and $x=-1$, and explain why replacing G by the symbol 0 does not produce a usable reciprocal.

Teacher solution and discussion

At $x=1$, G is undefined because its original terms are undefined. At $x=-1$, F is undefined, although G is defined and equals zero. At every other real input, both inner expressions are defined but $G=0$. Thus no real input makes division by G valid. The simplification $G=0$ is only on $x \neq 1$, and even there its reciprocal $1/G$ would require division by zero. Neither cancellation nor a limiting idea gives a value to this original quotient.

Evidence and response

Check the exceptional inputs and the entire remaining domain, recognizing an identically zero divisor rather than an isolated excluded point. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 checks linear-denominator addition and reduction. O29 combines factor division with two output forms and hidden restrictions. O30 asks for a valid construction and a proof of impossibility under tighter numerator restrictions. Credit equivalent exact forms only when their original-domain record is complete, and record neutral access supports separately from supplied algebraic steps.

Core task

Combine $(3x+1)/(x-2) + (x-4)/(x+2)$ into a single reduced fraction. Show both cross-products, explain why no denominator factor cancels, and state the domain.

Teacher solution and discussion

The numerator over $(x-2)(x+2)$ is $(3x+1)(x+2) + (x-4)(x-2)$. These products are $3x^2+7x+2$ and x^2-6x+8 , so the numerator is $4x^2+x+10$. At $x=2$ it is 28, and at $x=-2$ it is 24, neither zero; therefore neither denominator factor cancels. The result is $(4x^2+x+10)/(x^2-4)$, with $x \neq 2, -2$. The factored denominator records the two restrictions clearly even if the final displayed denominator is expanded.

Evidence and response

Combine two linear-denominator expressions, test possible common factors and state both restrictions independently of the chosen display form. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Simplify $2/(x+1)+5/(x+1)$ on its original domain.**

- A. 7, for all real x .
- B. $7/(x+1)$, $x \neq -1$.
- C. $10/(x+1)$, $x \neq -1$.
- D. $7/(2x+2)$, $x \neq -1$.

Reasoning: _____

M02 | Warm-up | T1**What is $(x+4)/(x-2)-(x-1)/(x-2)$?**

- A. 5, for all real x .
- B. $5/(x-2)$, $x \neq 2$.
- C. $(2x+3)/(x-2)$, $x \neq 2$.
- D. $3/(x-2)$, $x \neq 2$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T3: Explain similarities with rational-number arithmetic, prove identities and diagnose limitations of cancellation and operation rules (objectives a and b).

Calculate $\frac{2}{3} + \frac{1}{4}$ using equivalent fractions. Then combine $\frac{2}{x+1} + \frac{1}{x+1}$. Explain how the same fraction rule produces each numerator, and state every restriction in the variable expression.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Combine $(2x+3)/(x-1)-(x-4)/(x+2)$ into one reduced fraction. Show each cross-product, preserve the subtraction sign, state the domain, and check the result at $x=0$.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Answer: B. $7/(x+1)$, $x \neq -1$.

Both fractions count units of size $1/(x+1)$, so add $2+5$ while keeping the denominator: $7/(x+1)$. The denominator is nonzero exactly when $x \neq -1$. This is the same common-denominator rule used for rational numbers.

Check each alternative

- A. The numerator has no common factor $x+1$ to cancel, and $x=-1$ is excluded.
- B. Correct. Both fractions count units of size $1/(x+1)$, so add $2+5$ while keeping the denominator: $7/(x+1)$. The denominator is nonzero exactly when $x \neq -1$. This is the same common-denominator rule used for rational numbers.
- C. This multiplies the numerators rather than adds them.
- D. A common denominator stays fixed when adding the numerators.

Evidence to assess

Keep a shared denominator unchanged and carry its original restriction into the result.

M02 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

Answer: B. $5/(x-2)$, $x \neq 2$.

Place parentheses around the whole subtracted numerator:
 $[(x+4)-(x-1)]/(x-2) = (x+4-x+1)/(x-2) = 5/(x-2)$. The restriction $x \neq 2$ remains. The subtraction sign applies to both terms in $x-1$.

Check each alternative

- A. The common denominator does not disappear when subtracting numerators.
- B. Correct. Place parentheses around the whole subtracted numerator:
 $[(x+4)-(x-1)]/(x-2) = (x+4-x+1)/(x-2) = 5/(x-2)$. The restriction $x \neq 2$ remains. The subtraction sign applies to both terms in $x-1$.
- C. This adds the numerator expressions rather than subtracts them.
- D. Subtracting $x-1$ gives $-x+1$, not $-x-1$.

Evidence to assess

Distribute the negative sign across the entire numerator and preserve the denominator.

WORKED TEACHER KEYS

001 / 002**001 • T3: Explain similarities with rational-number arithmetic, prove identities and diagnose limitations of cancellation and operation rules (objectives a and b).**

For the numbers, $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$, so the sum is $\frac{11}{12}$. For the expressions, $\frac{2}{x} = \frac{2(x+1)}{[x(x+1)]}$ and $\frac{1}{(x+1)} = \frac{x}{[x(x+1)]}$, giving $\frac{(3x+2)}{[x(x+1)]}$ for $x \neq 0, -1$. In both calculations, each numerator is multiplied by the factor used to make its denominator common. The denominators are not added. The variable factors must be nonzero so the original expressions and equivalence steps are valid.

Evidence to assess

Link numerical and algebraic common-denominator reasoning explicitly, rather than present two unconnected calculations.

002 • T1: Add and subtract rational expressions with linear denominators using equivalent fractions and complete domain records (objective a).

The common denominator is $(x-1)(x+2)$, so the numerator is $(2x+3)(x+2) - (x-4)(x-1)$. Expanding gives $(2x^2 + 7x + 6) - (x^2 - 5x + 4) = x^2 + 12x + 2$. Neither denominator factor cancels: this numerator is 15 at $x=1$ and -18 at $x=-2$. The domain is $x \neq 1, -2$. At 0, the original difference is $-3 - (-2) = -1$; the combined fraction gives $2/(-2) = -1$. The identity, not merely this sample check, verifies the full simplification.

Evidence to assess

Expand and subtract both products, justify that no common factor remains and verify one legal input.