

TEACHER AND STUDENT RESOURCE

Exponential Rates Across Time Intervals



NC Math 3 • NC.M3.A-SSE.3

Seeing Structure in Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-SSE.3 | NC Math 3 mathematics

This sample contains 20 complete selected pages from the 99-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 63, 81

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 63, 81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.

T1 Interpret a factor or relative rate with its time interval, units and modeled quantity.

T2 Use exponent properties to rewrite one model for longer, shorter, fractional or reverse intervals.

T3 Derive interval factors from ratios, tables, graphs and symbolic relationships.

T4 Evaluate equivalence claims, rounding, changing rates and assumptions behind interval conversions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M03, M07, M11, M23, M25, M28, O01, O24, O27

T2: M02, M04, M05, M09, M10, M16, M20, M29, O02, O03, O05, O07, O17, O18, O21, O30

T3: M06, M12, M13, M26, M27, O06, O08, O10, O13, O20, O22, O29

T4: M08, M14, M15, M17, M18, M19, M21, M22, M24, M30, O04, O09, O11, O12, O14, O15, O16, O19, O23, O25, O26, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Name the interval and quantity

Learning target: Connect factors and percent changes to units.

Learning target: Use powers and roots to express the same model on a new interval.

Select Warm-up and I Do tasks. Before using a calculator, label the physical duration of one base factor.

Session 2: Derive and critique

Learning target: Use ratios and time gaps to infer factors.

Learning target: Distinguish exact rewrites from approximations and endpoint replacements.

Use We Do and Collaborative tasks. Keep exact roots until the final reporting step and require an algebraic check.

Session 3: Transfer to unfamiliar clocks

Learning target: Account for time shifts, signed quantities and real-domain assumptions.

Learning target: Produce independently justified interval-rate forms.

Use Independent tasks, targeted reteach and Exit tasks. Record the quantity, factor, interval and evidence separately.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use exponent product, quotient and power rules for positive bases; interpret percent change as a multiplier; divide quantities with matching units. The $e/\ln$ items extend the same interval structure for learners ready for logarithmic notation.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Unless a task states otherwise, use positive nonzero initial amounts and positive exponential bases. Time may be real within the stated model. Distinguish an exact algebraic rewrite from a rounded approximation and from a new model matching only selected endpoints. A growth factor b corresponds to relative change $b-1$; for decay the positive lost fraction is $1-b$. State any calendar conversion explicitly. Signed deviations and zero initial values occur only in tasks that name those exceptions.

TEACHER MODEL

Change the factor and the period count together

A process is $Q(h)=300(1.44)^{(h/2)}$, with h in hours. Rewrite it to show hourly and half-hour factors, and interpret the rates.

Since $1.44=(1.2)^2$, the hourly form is $300(1.2)^h$: 20% growth each hour. The half-hour factor is $\sqrt{1.2}$, so $Q(h)=300[\sqrt{1.2}]^{(2h)}$. The half-hour relative growth is $\sqrt{1.2}-1$, about 9.54%, not 10%. For either rewrite, combining exponents recovers the original; at $h=2$ all forms give 432.

VISUAL ANCHOR

One physical process, three clocks

Original interval T: $Q(t) = A b^{(t/T)}$

New interval δ : factor $b^{(\delta/T)}$; number of new periods t/δ

Equivalent form: $Q(t) = A [b^{(\delta/T)}]^{(t/\delta)}$

Check: $(\delta/T)(t/\delta) = t/T$. Relative change is factor minus 1.

A correct conversion changes the factor and period count together while preserving the initial value.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Interpret a factor or relative rate with its time interval, units and modeled quantity.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask what one increase of the exponent represents, not merely what number appears as the base. Use O01 to separate a percentage increase from a retained fraction. With O02, substitute one full base interval before attempting a shorter interval; this reveals a reversed clock conversion without supplying the corrected formula.

Core task

Three descriptions refer to one process: “25% growth per 2 hours,” “factor 1.25 per 2 hours,” and “factor 0.25 per 2 hours.” Identify which agree, repair the incorrect description, and write a model with initial amount 80 using time h in hours.

Teacher solution and discussion

The first two agree: 25% growth means retaining the original 100% and adding 25%, so the multiplier is 1.25. A factor 0.25 instead means retaining 25%, or losing 75%. The growth model is $Q(h)=80(1.25)^{(h/2)}$. The exponent counts 2-hour intervals; $Q(0)=80$ and $Q(2)=100$.

Evidence and response

Distinguish relative change from multiplier and connect the model exponent to the named interval. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Use exponent properties to rewrite one model for longer, shorter, fractional or reverse intervals.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 by replacing t with δu before applying a power rule. State which symbols measure durations and which count intervals. Use O07 to show that a fixed calendar convention, not an unspoken real-world month length, makes two conversion routes agree.

Core task

Let $Q(t) = A b^{(t/T)}$, with $A > 0$, $b > 0$ and $T > 0$. Introduce a new clock $u = t/\delta$, where $\delta > 0$ is the duration of one new clock unit. Derive Q as a function of u and identify its new per-unit factor. Explain what stays unchanged.

Teacher solution and discussion

Since $t = \delta u$, $Q = A b^{(\delta u/T)} = A [b^{(\delta/T)}]^u$. The new clock factor is $b^{(\delta/T)}$. The initial amount A and the physical value at every elapsed duration are unchanged; only the numerical time coordinate and the base used for one coordinate unit change. Setting $u = T/\delta$ recovers $Q = Ab$ after one original base period.

Evidence and response

Substitute the new clock correctly and prove the physical model is preserved. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Evaluate equivalence claims, rounding, changing rates and assumptions behind interval conversions.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

For O09, retain all measured readings and their stated intervals. Do not round the data until ratios look constant. For O10, divide the two supplied values to eliminate A, then reconstruct A and verify both points. A table gap of 3 units needs a cube-root conversion.

Core task

Measurements at days 0,1,2 are 100.0,109.9,121.2. Each reading has an agreed absolute tolerance of ± 0.3 . Test whether the model $Q(t)=100(1.10)^t$ is consistent with every reading. Explain why rounding the data into exact ratios or declaring a proved daily rate would overstate the evidence.

Teacher solution and discussion

The model predicts 100,110,121. The reading intervals are [99.7,100.3], [109.6,110.2], and [120.9,121.5], so all three predictions lie inside the corresponding intervals. Thus the data are consistent with 10% daily exponential growth at this tolerance. The observed ratios are not exactly equal, and adjusting readings to force equality would discard measurement evidence. Agreement within tolerance supports the model for these observations but does not prove the exact law for all times.

Evidence and response

Compare each prediction with its stated tolerance and distinguish consistency from exact identification or proof. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Evaluate equivalence claims, rounding, changing rates and assumptions behind interval conversions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign separate roles for exact algebra, numerical verification and model-scope critique. In O15 compare both the annual endpoint and an intermediate value; the annual matching factor is useful but cannot replace the full changing-rate path without qualification. Require each learner to explain the distinction.

Core task

Construct an exponential model with initial amount 100 whose monthly growth is strictly greater than 2% but whose annual growth is less than 30%. Assume 12 equal months per year. Give one model, verify both inequalities and describe the full allowed interval of monthly factors.

Teacher solution and discussion

Let the monthly factor be q . The conditions are $q > 1.02$ and $q^{12} < 1.30$, so $1.02 < q < (1.30)^{1/12}$, approximately 1.022104. Choose $q = 1.021$: $Q(m) = 100(1.021)^m$ has 2.1% monthly growth, and its annual factor $(1.021)^{12}$ is approximately 1.2832, below 1.30. Any factor in the stated open interval works; equality at either endpoint is excluded.

Evidence and response

Translate both strict rate conditions into factor inequalities, construct a valid example and state the entire allowable interval. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Evaluate equivalence claims, rounding, changing rates and assumptions behind interval conversions.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O19 to assess whether the learner propagates rounding through the requested duration rather than looking only at the base error. Use O20 to test coordinated graph and formula conversion. In O21, record whether the learner distinguishes logarithmic slope k from the effective factor e^k .

Core task

A model doubles every 10 time units. Its exact one-unit factor is $q=2^{(1/10)}$. Compare the factors obtained by rounding q to two and three decimal places. Over 20 units, which rounded model keeps the factor within 0.5% relative error of the exact factor? Show the error calculation and retain the exact model as the equivalence statement.

Teacher solution and discussion

The exact 20-unit factor is $q^{20}=4$. The rounded bases are 1.07 and 1.072. Their 20-unit factors are 1.07^{20} and 1.072^{20} . Compute relative errors $|1.07^{20}-4|/4$ and $|1.072^{20}-4|/4$: approximately 3.26% and 0.424%, respectively. Thus the three-decimal base meets the 0.5% criterion and the two-decimal base does not. Both are approximations; the exact identity remains $2^{(t/10)}=(2^{(1/10)})^t$.

Evidence and response

Propagate the rounded factor over the full interval and compare relative, not absolute, error with the stated tolerance. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Interpret a factor or relative rate with its time interval, units and modeled quantity.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for reversed units, O25 for invalid zero ratios, O26 for endpoint-only fitting, or O27 for confusion between a signed value and its magnitude. Restore the precise missing condition, then use a fresh task rather than copying a completed rewrite.

Core task

A rate label says “factor b per T hours.” Explain why $A b^{(t/T)}$ is dimensionally meaningful when t is measured in hours, while $A b^{(Tt)}$ generally is not. Check what each exponent would be at one complete base interval $t=T$.

Teacher solution and discussion

The ratio t/T divides hours by hours and gives a dimensionless count of base periods. At $t=T$, its exponent is 1, so the output is Ab as required. The product Tt has units of hours squared, not a period count; numerically at $t=T$ it becomes T^2 rather than 1 (except accidental special numerical choices). The exponent must encode how many base intervals elapsed, not multiply two durations.

Evidence and response

Use units and the one-period check together; do not rely on an accidental numerical value of T . Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Evaluate equivalence claims, rounding, changing rates and assumptions behind interval conversions.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 checks why positive roots are selected; O29 distinguishes amount changes from relative changes; O30 integrates factor, period count and exact equivalence. Require a stated quantity and interval before accepting an unlabelled percentage. Keep neutral calculation supports separate from hints about which exponent to use.

Core task

A two-period retention factor is 0.81. Solving $q^2=0.81$ gives $q=0.9$ or $q=-0.9$. Explain why the positive root is used for a positive real-time exponential decay model. What would the negative choice do at integer steps and at a half-step?

Teacher solution and discussion

The positive factor $q=0.9$ preserves positivity and yields real values q^t for every real t . A negative factor -0.9 would alternate the sign of a positive initial amount at integer steps; its square still matches 0.81 after two steps. At a half-step, $(-0.9)^{(1/2)}$ is not real. Thus matching a longer-interval factor is not enough: positivity and the real-time domain select the positive root.

Evidence and response

Use both the positivity and real-domain requirements to choose the root, rather than discard a solution without explanation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A culture is modeled by $N(t)=150(1.08)^t$, where t counts days. What does the base reveal?

- A. Its initial size is 1.08 organisms.
- B. The culture increases by 8% of its current size each day.
- C. It increases by 108% each day.
- D. It adds exactly 8 organisms each day.

Reasoning: _____

M02 | Warm-up | T2

For $V(t)=900(0.64)^{(t/2)}$, t is in years. Which equivalent form displays the one-year factor?

- A. $V(t)=900(0.8)^t$.
- B. $V(t)=450(0.64)^t$.
- C. $V(t)=900(0.32)^t$.
- D. $V(t)=900(0.64)^{(2t)}$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Interpret a factor or relative rate with its time interval, units and modeled quantity.

Three descriptions refer to one process: “25% growth per 2 hours,” “factor 1.25 per 2 hours,” and “factor 0.25 per 2 hours.” Identify which agree, repair the incorrect description, and write a model with initial amount 80 using time h in hours.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Use exponent properties to rewrite one model for longer, shorter, fractional or reverse intervals.

A student models 18% growth per 4 months by $Q(m)=200(1.18)^{(4m)}$, where m is elapsed months. Diagnose the clock error, repair the model and rewrite it to display a one-month factor. Verify the value after 4 months.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Interpret a factor or relative rate with its time interval, units and modeled quantity.

Answer: B. The culture increases by 8% of its current size each day.

For any t , $N(t+1)/N(t)=1.08$. Thus each one-day interval multiplies the current amount by 1.08, giving relative change $1.08-1=0.08=8\%$.

Check each alternative

- A. The coefficient 150 gives the initial value.
- B. Correct. For any t , $N(t+1)/N(t)=1.08$. Thus each one-day interval multiplies the current amount by 1.08, giving relative change $1.08-1=0.08=8\%$.
- C. A factor of 1.08 retains the original 100% and adds 8%.
- D. Eight percent is a relative change, not a fixed number of organisms.

Evidence to assess

State both the percent and the one-day interval; distinguish relative from additive change.

M02 • T2: Use exponent properties to rewrite one model for longer, shorter, fractional or reverse intervals.

Answer: A. $V(t)=900(0.8)^t$.

Since $0.64=(0.8)^2$, $(0.64)^{(t/2)}=[(0.8)^2]^{(t/2)}=(0.8)^t$. The yearly retention is 80%, so the yearly decrease is 20%, not half of 36%.

Check each alternative

- A. Correct. Since $0.64=(0.8)^2$, $(0.64)^{(t/2)}=[(0.8)^2]^{(t/2)}=(0.8)^t$. The yearly retention is 80%, so the yearly decrease is 20%, not half of 36%.
- B. Changing the coefficient changes the initial value and does not convert the time interval.
- C. Halving the interval requires a square root of the factor, not division by 2.
- D. This doubles the number of two-year factors rather than halving their duration.

Evidence to assess

Take the positive square root and preserve the coefficient and time variable.

WORKED TEACHER KEYS

001 / 002**001 • T1: Interpret a factor or relative rate with its time interval, units and modeled quantity.**

The first two agree: 25% growth means retaining the original 100% and adding 25%, so the multiplier is 1.25. A factor 0.25 instead means retaining 25%, or losing 75%. The growth model is $Q(h)=80(1.25)^{(h/2)}$. The exponent counts 2-hour intervals; $Q(0)=80$ and $Q(2)=100$.

Evidence to assess

Distinguish relative change from multiplier and connect the model exponent to the named interval.

002 • T2: Use exponent properties to rewrite one model for longer, shorter, fractional or reverse intervals.

The exponent $4m$ counts four periods per month, but the stated base period lasts 4 months. Use $Q(m)=200(1.18)^{(m/4)}=200[(1.18)^{(1/4)}]^m$. The monthly factor is the positive fourth root of 1.18. At $m=4$, the repaired expression gives $200(1.18)=236$; the incorrect expression uses 16 repetitions and does not match the stated rate.

Evidence to assess

Correct the period count, identify the root factor and test at one full base interval.