

TEACHER AND STUDENT RESOURCE

Proofs About Parallelograms



NC Math 3 • NC.M3.G-CO.11

Congruence

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-CO.11 | NC Math 3 mathematics

This sample contains 23 complete selected pages from the 130-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-18, 35-36, 68, 93-94

Representative activities

M01: sample PDF page(s) 18; original page(s) 17

M02: sample PDF page(s) 19; original page(s) 18

O01: sample PDF page(s) 20; original page(s) 35

O02: sample PDF page(s) 21; original page(s) 36

Selected teacher prompts and keys: original page(s) 68, 93-94

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Prove theorems about parallelograms.

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

T1 Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

T2 Prove opposite angles of a parallelogram are congruent, distinguishing opposite equality from consecutive supplementation.

T3 Prove that each diagonal of a parallelogram bisects the other; distinguish equal halves within one diagonal from congruent diagonals.

T4 Prove that a parallelogram with congruent diagonals is a rectangle, and test every hypothesis without reversing a conditional or overclaiming a square.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M02, M03, M05, M12, M17, M19, M29, O01, O03, O05, O09, O23, O26

T2: M04, M06, M11, M21, M25, M27, O02, O06, O16, O22, O24, O28

T3: M07, M10, M16, M20, M23, M24, O04, O07, O10, O12, O13, O15, O18, O25, O27

T4: M08, M09, M13, M14, M15, M18, M22, M26, M28, M30, O08, O11, O14, O17, O19, O20, O21, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Build a proof from the definition

Learning target: Identify opposite sides, transversals and correct triangle correspondence.

Learning target: Prove opposite-side and opposite-angle congruence without circular reasoning.

Select M01–M06/O01–O06 across a diagnostic and model lesson. Retain the unmarked initial diagram. Have learners name the exact parallel pair before writing each angle equality; do not use a visual resemblance as a reason.

Session 2: Connect the diagonal theorems

Learning target: Establish diagonal bisection with a grounded congruence or coordinate proof.

Learning target: Derive a rectangle from congruent diagonals and preserve the parallelogram hypothesis.

Select M07–M23/O07–O23 across multiple lessons. Require an individual proof attempt before group comparison. Include at least one synthetic proof, one exact coordinate proof and one counterexample or failed-proof audit, rather than assigning the entire bank in one sitting.

Session 3: Repair a specific gap and certify transfer

Learning target: Repair correspondence, hypotheses, conditional direction or equality-versus-supplementation errors.

Learning target: Produce an independent theorem-based explanation or complete design certificate.

Choose M24–M27/O24–O27 from actual diagnostic evidence, then use fresh M28–M30/O28–O30 evidence without the model. A right numeric answer without a justified theorem chain does not demonstrate this proof standard.

PLANNING

Materials, access and pacing

Materials

Straightedge, pencil, eraser, equal-scale graph paper and highlighters for matching triangle vertices. A protractor is useful for constructing the 60-degree diagonal angle in O25, not for replacing a proof. Tracing paper can support the half-turn task. A calculator may check arithmetic after exact relationships are written; dynamic geometry software is optional.

Prerequisites

Identify a simple quadrilateral and name vertices in boundary order; use parallel-line angle theorems, linear pairs, ASA/AAS/SAS/SSS and corresponding parts of congruent triangles; compute exact distances and midpoints; solve linear equations; use translations and half-turns as isometries; distinguish necessary conditions from sufficient conditions. Area tasks use triangle area and common-altitude comparisons. The optional coordinate construction in O25 uses $\cos 60^\circ = 1/2$ and $\sin 60^\circ = \sqrt{3}/2$, which may be supplied.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

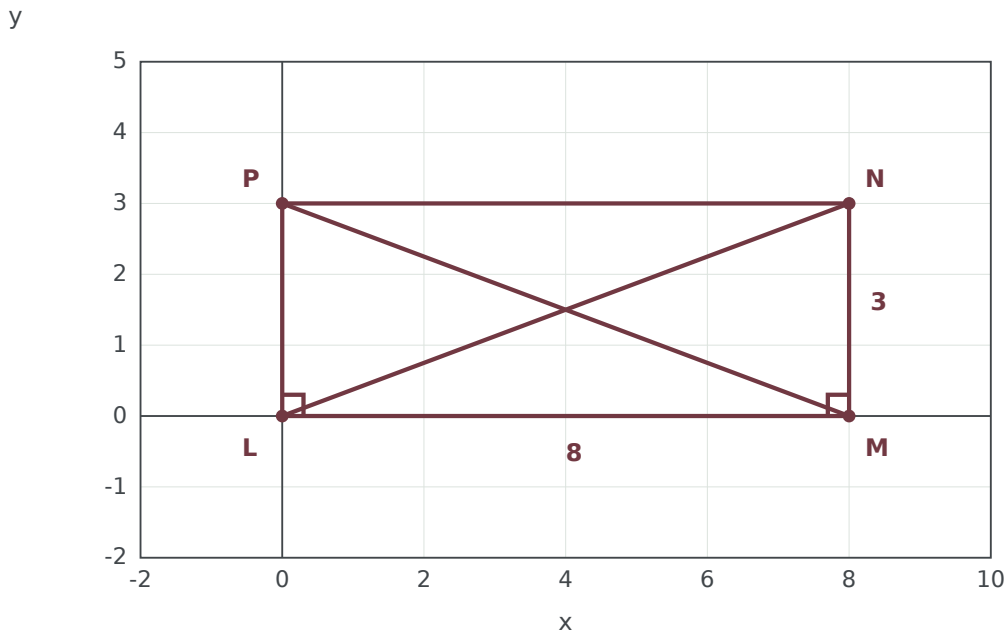
Review convention

A parallelogram is a simple, nondegenerate quadrilateral with both pairs of opposite sides parallel; vertices are named in boundary order unless an item explicitly asks learners to repair that order. A rectangle has four right angles; squares are included among rectangles. Diagrams are schematic and not physical-scale measurement templates. Only stated givens and proved facts may be used. A sketch, finite measurements or dragging examples may suggest a theorem but do not prove a universal claim. A proof may cite an earlier established theorem, but may not assume its own conclusion. CPCTC means corresponding parts of congruent triangles are congruent. Exact forms such as $\sqrt{20}=2\sqrt{5}$ are equivalent. All contexts and measurements are invented for instruction.

TEACHER MODEL

Equal diagonals supply SSS; supplementation supplies right angles

A parallelogram LMNP has congruent diagonals LN and MP. Show why that information proves a rectangle but does not prove a square. Record each assumption before drawing a conclusion.



Given: LM parallel to NP, MN parallel to LP, and $LN=MP$. Compare triangles LMN and MLP. $LM=ML$ by reflexivity; $MN=LP$ by the previously proved opposite-side theorem; $LN=MP$ is the extra diagonal hypothesis. SSS gives triangle LMN congruent to triangle MLP. Therefore angle LMN equals angle MLP by corresponding parts. These are consecutive angles in the parallelogram, so they sum to 180 degrees. Equal supplementary angles each measure 90 degrees. The other two angles equal their opposites, hence are right too: LMNP is a rectangle. Side lengths 8 and 3 provide a nonsquare instance, with both diagonals $\sqrt{73}$. Equality of diagonals has never asserted equality of adjacent sides.

VISUAL ANCHOR

A proof has premises, a bridge and an exact conclusion

Opposite sides: parallel pairs + alternate angles + shared diagonal $\rightarrow$ ASA $\rightarrow$ $AB=CD$ and $AD=BC$.

Opposite angles: $A+B=180$ and $C+B=180 \rightarrow A=C$; compare another common supplement for $B=D$.

Diagonals: prove small triangles congruent $\rightarrow AE=CE$ AND $BE=DE$. These are two different equal pairs.

Rectangle test: parallelogram + $AC=BD \rightarrow$ SSS $\rightarrow$ equal consecutive angles $\rightarrow$ each 90° .

Keep the hypothesis: equal diagonals in an arbitrary quadrilateral do not prove a rectangle.

An exact counterexample defeats an “always” claim. Approximate measurement supports a conjecture, not a universal proof.

Do not replace “bisect” with “congruent,” “perpendicular” with “congruent,” or “rectangle” with “square.” Read the middle letter of every angle and the order of every triangle correspondence.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Begin with M01 and O01 before adding congruence marks. Ask: “Which two lines are parallel? Which segment is the transversal? Which angle has its vertex at the middle letter?” In O02 draw the extension past B, not toward A. Use O03 to separate opposite-side congruence from four equal sides. O04 distinguishes this proof standard from the preceding experimental-center standard: measurements motivate a claim but the present target requires a deductive argument.

Core task

ABCD is a parallelogram with AB parallel to CD and AD parallel to BC. On the supplied unmarked quadrilateral, draw diagonal AC. Name two alternate-interior-angle pairs that will compare triangles ABC and CDA. Name the shared side and write the vertex correspondence; do not assume any side congruences other than the shared side.

Teacher solution and discussion

Draw AC. AB parallel to CD gives angle BAC=angle DCA. BC parallel to AD gives angle BCA=angle DAC. AC=CA by the reflexive property. The correspondence is $A \leftrightarrow C$, $B \leftrightarrow D$, $C \leftrightarrow A$, so triangle ABC corresponds to triangle CDA. In each triangle the shared side lies between the two stated angles. These facts would support ASA without assuming the opposite-side theorem.

Evidence and response

Draw the requested diagonal; provide two correctly named transversal angle pairs, the reflexive side and a consistent ordered vertex map. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model a proof as premises $\rightarrow$ justified bridge $\rightarrow$ conclusion. In O05 highlight $EF \leftrightarrow GH$, $EH \leftrightarrow GF$ and the shared FH before using ASA; do not cite opposite sides as a premise. In O06 write both equations containing the same angle before subtracting. In O07 distinguish the theorem already established (opposite sides) from the theorem currently being proved (bisection). For O08 or the teacher model, pause after SSS: ask why equal consecutive angles must be 90 degrees, and why the same reasoning does not make the frame square.

Core task

Prove from the definition that opposite sides of every parallelogram are congruent. Use quadrilateral EFGH in boundary order and draw diagonal FH. Write a complete statement-and-reason argument. Include the ordered triangle congruence and both resulting opposite-side equalities; using the theorem being proved as a premise is not allowed.

Teacher solution and discussion

EF parallel to GH and EH parallel to FG are given by the definition. Draw FH. Angle EFH equals angle GHF because EF is parallel to GH with transversal FH. Angle EHF equals angle GFH because EH is parallel to FG with the same transversal. $FH = HF$ by reflexivity. Hence triangle EFH is congruent to triangle GHF by ASA, with $E \leftrightarrow G$, $F \leftrightarrow H$, $H \leftrightarrow F$. Corresponding parts give $EF = GH$ and $EH = GF$, the two opposite-side pairs. Neither of these conclusions was used to establish congruence.

Evidence and response

Show a completed labeled auxiliary-diagonal diagram; give two parallel-line angle reasons, reflexivity, ASA, correct correspondence and both CPCTC conclusions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: O09 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

In O09 allow negative b and ask why a right-leaning picture is not part of the hypotheses. In O10 calculate both midpoint formulas, retaining p and q . In O11 use two colors for the two diagonals, solve the within-diagonal equations independently, then compare the completed diagonals. In O12 verify the constructed fourth-point relationships, not only the midpoint arithmetic. In O13 ask a partner to read the vertex map aloud before using CPCTC; the exact unequal half-lengths make the correspondence error visible.

Core task

For $a > 0$ and $c > 0$, let $A(0,0)$, $B(a,0)$, $C(a+b,c)$, $D(b,c)$, where b may be positive, zero or negative. Prove that $ABCD$ is a parallelogram and prove both pairs of opposite sides congruent using the distance formula. Explain why the argument still works when b is negative, rather than assuming the picture leans right.

Teacher solution and discussion

AB and DC have the same displacement $(a,0)$. AD and BC have the same displacement (b,c) . Thus the two opposite-side pairs are parallel. Because a and c are positive, the quadrilateral is nondegenerate. $AB=CD=a$. The other distances satisfy $AD^2=b^2+c^2$ and $BC^2=b^2+c^2$, so $AD=BC=\sqrt{b^2+c^2}$. These identities hold for any real b because the signed horizontal displacement is squared. Negative b makes the upper edge shift left; it does not alter the opposite-pair equalities or the parallel displacements.

Evidence and response

Establish nondegeneracy and parallelism, calculate both opposite pairs symbolically, and explicitly account for negative shear. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Prove that a parallelogram with congruent diagonals is a rectangle, and test every hypothesis without reversing a conditional or overclaiming a square.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign premise-checker, diagram-maker, proof-writer and skeptic roles after each learner writes an initial claim. O14 requires both conditional directions with different congruence criteria; do not let the forward SAS proof be mistaken for the reverse. O15 requires equal-area reasoning without claiming four congruent triangles. O16 makes the half-turn an actual construction. O17 checks an entire counterexample family and names the missing parallelogram hypothesis. O18 requires replacing a circular premise, not merely relabeling it as “given.” Collect individual explanations after group comparison.

Core task

Two partners are responsible for opposite directions of a claim. Partner 1 proves that a rectangle has congruent diagonals; Partner 2 proves that a parallelogram with congruent diagonals is a rectangle. Write both proofs for ABCD and state the precise biconditional that the group may conclude. Specify where each direction uses its distinct hypothesis.

Teacher solution and discussion

Forward: In rectangle ABCD, $AB=BA$ and $BC=AD$ by parallelogram side congruence. Angles ABC and BAD are both 90 degrees by the rectangle hypothesis. SAS gives triangle ABC congruent to triangle BAD; corresponding third sides give $AC=BD$. Reverse: In a parallelogram with $AC=BD$, the same triangles have $AB=BA$, $BC=AD$ and $AC=BD$, so they are congruent by SSS. Hence consecutive angles ABC and BAD are congruent; they are also supplementary, so each is 90 degrees. Their opposite angles are right too, establishing a rectangle. Therefore, among nondegenerate parallelograms, being a rectangle is equivalent to having congruent diagonals. The unrestricted claim about every quadrilateral is not established.

Evidence and response

Keep forward SAS and reverse SSS arguments distinct and state a biconditional with the necessary parallelogram domain. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Prove that a parallelogram with congruent diagonals is a rectangle, and test every hypothesis without reversing a conditional or overclaiming a square.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O19–O23 without hinting at the decisive theorem. O19 assesses exact diagonal evidence in a tilted nonsquare rectangle. O20 needs both sufficiency and necessity for its unique parameter, with two completed drawings. O21 tests whether common perimeter and center really determine shape. O22 deliberately has no valid blueprint: accept the inconsistency conclusion only after all forced angle checks. O23 separates invariant opposite-pair relationships from unchanged individual lengths and from whole-figure translation. Record neutral access supports separately from theorem-identifying cues.

Core task

A tilted display frame has ordered vertices $A(1,1)$, $B(7,3)$, $C(6,6)$, $D(0,4)$. Prove it is a parallelogram, then prove it is a rectangle by comparing its diagonals. Determine whether it is a square. Draw both diagonals and label their common midpoint. Use exact distances rather than judging the tilted picture.

Teacher solution and discussion

$AB=(6,2)=DC$ and $AD=(-1,3)=BC$, so both pairs of opposite sides are parallel. The two directions are nonparallel. $AC=(5,5)$, giving $AC^2=50$; $BD=(-7,1)$, giving $BD^2=50$. The congruent-diagonals theorem therefore establishes a rectangle. $AB^2=40$ while $BC^2=10$, so adjacent sides have lengths $2\sqrt{10}$ and $\sqrt{10}$, and the rectangle is not a square. The midpoint of AC is $((1+6)/2, (1+6)/2)=(7/2, 7/2)$; BD gives the same midpoint. The completed diagram shows the tilted rectangle and both congruent, bisected diagonals.

Evidence and response

Verify parallel sides before applying the conditional theorem, compute both diagonal distances and distinguish a rectangle from a square with side evidence. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Prove opposite angles of a parallelogram are congruent, distinguishing opposite equality from consecutive supplementation.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose the actual error. O24 targets “equal” versus “right”; preserve the correct portion of the learner’s reasoning. O25 makes the two diagonal pairs physically distinct and uses an oblique crossing so that a rectangle is not suggested. O26 repairs boundary order before naming opposite sides; O27 checks nondegeneracy before applying a coordinate theorem. After the repair, collect a different unsupported task rather than treating a copied marked diagram as mastery.

Core task

For parallelogram WXYZ, a student writes this chain: “ $w+x=180$; $x+y=180$; therefore $w=y$; therefore all four angles equal 90 .” Here w,x,y,z are the consecutive interior-angle measures. Identify which inference is invalid. Retain the valid work, finish a proof that $x=z$, and give a specific non-right-angle quadruple satisfying all correct equations. Explain what extra information would be needed to force right angles.

Teacher solution and discussion

The first two equations are valid consecutive-supplement relationships, and subtracting x correctly gives $w=y$. The invalid inference is that opposite-angle equality forces 90 degrees. Also $w+z=180$ by the other parallel-side pair; comparing with $w+x=180$ gives $x=z$. A valid non-right quadruple is $(w,x,y,z)=(70,110,70,110)$, which has equal opposite angles and supplementary consecutive angles. To force right angles one could additionally know $w=x$: equality plus $w+x=180$ gives $w=x=90$, and the opposite-angle results give $y=z=90$. Equal opposite angles alone are already true in every parallelogram.

Evidence and response

Preserve valid deductions, pinpoint the overreach, prove the missing opposite pair and give a verified non-right counterexample with a sufficient extra condition. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Prove opposite angles of a parallelogram are congruent, distinguishing opposite equality from consecutive supplementation.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 asks for all four named angles and a justified classification. O29 links cross-diagonal half equality to full diagonal congruence and includes a constructed nonsquare example. O30 is a full design certificate with coordinates, hypotheses, midpoint, diagonal computations and conclusion. Collect the first response before exposing the key. Score a correct theorem choice separately from a valid complete proof; a diagram or final answer alone is insufficient.

Core task

Prove that a parallelogram cannot have exactly one right angle. Name a hypothetical parallelogram LMNP with angle $L=90$ degrees. Use both opposite-angle congruence and consecutive-angle supplementation, naming the affected vertices, to determine every angle. State the strongest shape classification justified without side-length information.

Teacher solution and discussion

Angle N is opposite L, so $N=L=90$ degrees. Angle M is consecutive to L, so $M=180-90=90$ degrees. Angle P is also consecutive to L (or opposite M), so $P=90$ degrees. Thus one right angle forces all four right angles, and exactly one right angle is impossible. The figure is a rectangle. It cannot be classified as a square without further information such as equal adjacent side lengths.

Evidence and response

Determine all three remaining named angles with valid theorem reasons and make a rectangle, not unsupported square, conclusion. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 • Select and justify

M01 | Warm-up | T1

A simple quadrilateral ABCD is named in order around its boundary. Which information is exactly the definition needed to begin a proof that ABCD is a parallelogram?

- A. AB and BC have the same length.
- B. AB is parallel to CD and BC is parallel to AD.
- C. AC and BD have the same length.
- D. Angle A and angle B have equal measures.

Reasoning: _____

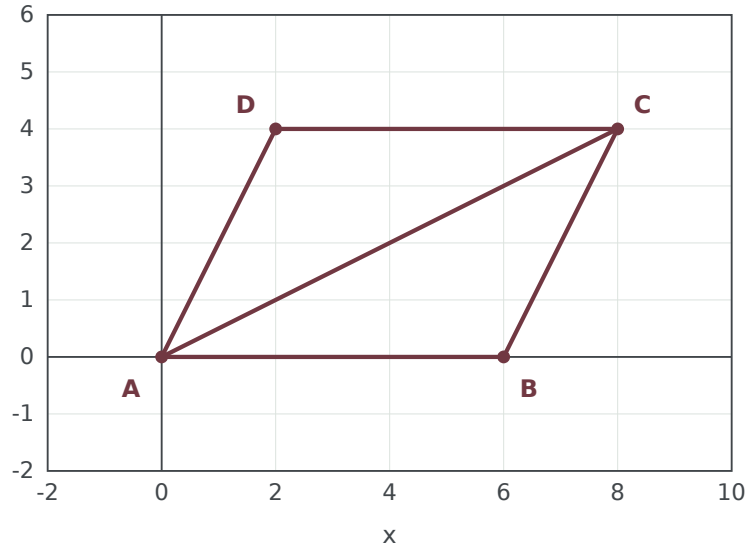
MULTIPLE-CHOICE BANK

M02 • Select and justify

M02 | Warm-up | T1

In parallelogram $ABCD$, diagonal AC is drawn. Which angle equality follows directly from AB parallel to CD with AC as the transversal?

y



- A. Angle BAC equals angle DCA.
- B. Angle BAC equals angle BCA.
- C. Angle BAC equals angle CAD.
- D. Angle ABC equals angle ACD.

Reasoning: _____

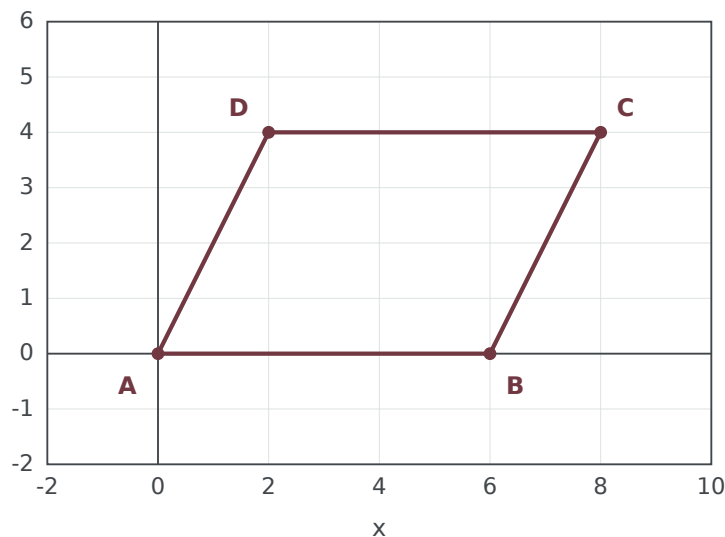
WARM-UP

001 • Show your reasoning

T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

ABCD is a parallelogram with AB parallel to CD and AD parallel to BC. On the supplied unmarked quadrilateral, draw diagonal AC. Name two alternate-interior-angle pairs that will compare triangles ABC and CDA. Name the shared side and write the vertex correspondence; do not assume any side congruences other than the shared side.

y



Representation / calculation

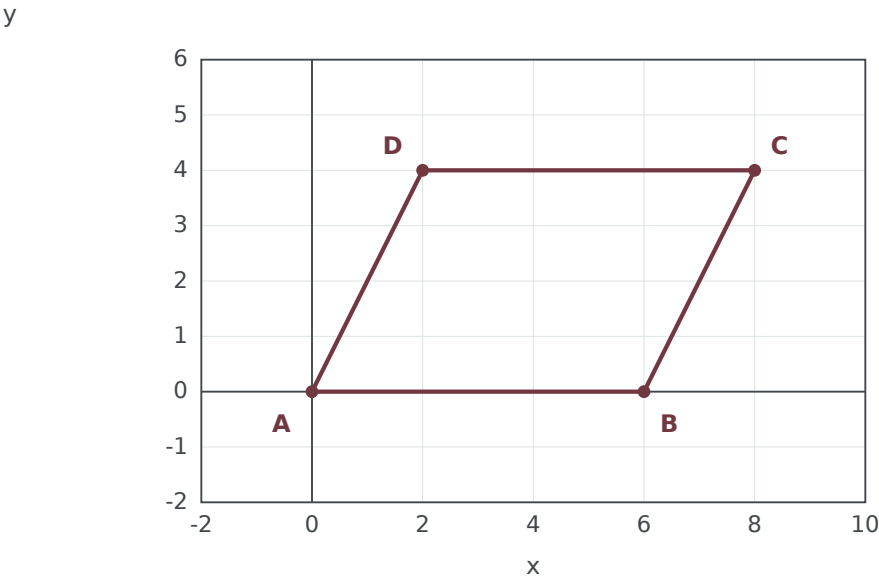
Explanation / check

WARM-UP

002 • Show your reasoning

T2: Prove opposite angles of a parallelogram are congruent, distinguishing opposite equality from consecutive supplementation.

ABCD is a parallelogram. Extend AB past B to a point F, with A, B, F collinear in that order. Draw the extension. Use parallel lines to relate angle FBC to angle DAB, then use the straight angle ABF to prove that the consecutive interior angles DAB and ABC are supplementary. State the reason for each step.



Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Answer: B. AB is parallel to CD and BC is parallel to AD.

The definition specifies two pairs of parallel opposite sides. Congruence conclusions must be proved or supplied separately, not read from the picture.

Check each alternative

- A. One adjacent congruent pair does not establish either opposite-side parallelism.
- B. Correct. The definition specifies two pairs of parallel opposite sides. Congruence conclusions must be proved or supplied separately, not read from the picture.
- C. Equal diagonals alone can occur in a non-parallelogram, such as an isosceles trapezoid.
- D. Equal consecutive angles alone do not define a parallelogram.

Evidence to assess

Identify both correct opposite-side pairs and distinguish a definition from a possible consequence.

M02 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Answer: A. Angle BAC equals angle DCA.

BAC uses ray AB and the transversal AC; DCA uses parallel ray CD and the opposite direction of that transversal. They are alternate interior angles, so they are congruent.

Check each alternative

- A. Correct. BAC uses ray AB and the transversal AC; DCA uses parallel ray CD and the opposite direction of that transversal. They are alternate interior angles, so they are congruent.
- B. Those angles use AB and BC, which are adjacent sides, not the given parallel pair.
- C. This would say that AC bisects angle A, which is not true for every parallelogram.
- D. The angle at B is not formed by transversal AC and the given parallel lines.

Evidence to assess

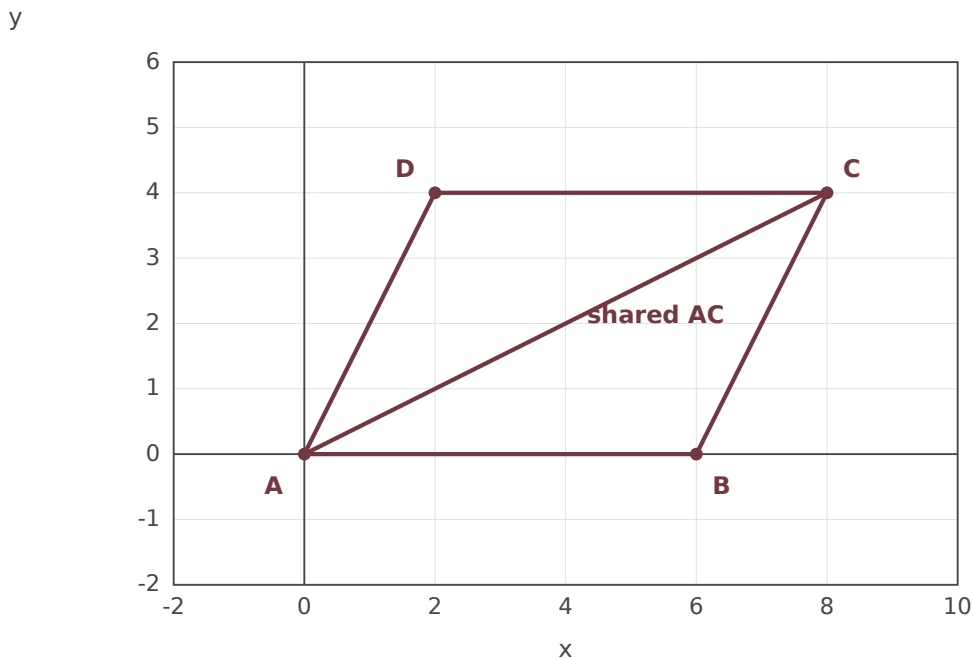
Trace the two parallel sides and transversal and preserve the middle-letter vertex in each angle name.

WORKED TEACHER KEYS

001

001 • T1: Prove opposite sides of a parallelogram are congruent using parallel-line angles, triangle congruence, transformations or exact coordinates.

Draw AC. AB parallel to CD gives angle $BAC = \text{angle } DCA$. BC parallel to AD gives angle $BCA = \text{angle } DAC$. $AC = CA$ by the reflexive property. The correspondence is $A \leftrightarrow C$, $B \leftrightarrow D$, $C \leftrightarrow A$, so triangle ABC corresponds to triangle CDA. In each triangle the shared side lies between the two stated angles. These facts would support ASA without assuming the opposite-side theorem.



Evidence to assess

Draw the requested diagonal; provide two correctly named transversal angle pairs, the reflexive side and a consistent ordered vertex map.

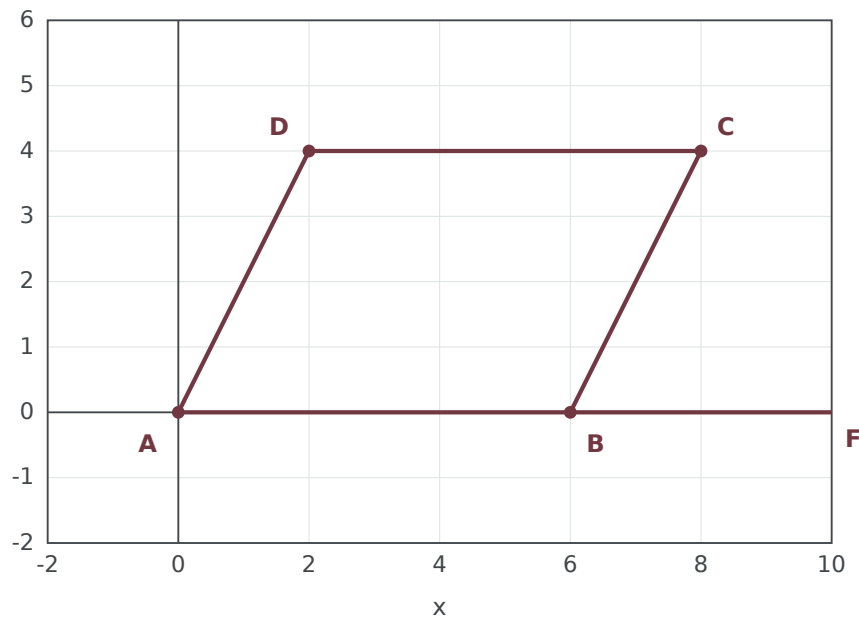
WORKED TEACHER KEYS

002

002 • T2: Prove opposite angles of a parallelogram are congruent, distinguishing opposite equality from consecutive supplementation.

Draw BF on the continuation of AB. Because BC is parallel to AD and BF points in the same direction as AB, angle FBC equals angle DAB by corresponding angles. Rays BA and BF are opposite rays, so angle ABC and angle CBF form a linear pair and sum to 180 degrees. Angle CBF and angle FBC name the same angle. Substituting angle DAB for angle FBC gives $m(\text{angle DAB}) + m(\text{angle ABC}) = 180$. No right-angle or equal-side assumption is needed.

y

**Evidence to assess**

Construct the correctly directed extension and link corresponding angles, a linear pair and substitution into a complete supplementary-angle proof.