

TEACHER AND STUDENT RESOURCE

Polynomial Factors Solutions and Zeros



NC Math 3 • NC.M3.A-APR.3

Arithmetic with Polynomial and Rational Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-APR.3 | NC Math 3 mathematics

This sample contains 20 complete selected pages from the 103-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 63, 80

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 63, 80

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand the relationship among factors of a polynomial expression, the solutions of a polynomial equation and the zeros of a polynomial function.

T1 Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

T2 Use polynomial structure to factor expressions and determine complete exact solution sets.

T3 Construct polynomials and infer parameters or transformed zero locations from stated constraints.

T4 Evaluate claims about zeros by checking domains, multiplicity, equivalence and the limits of graphical or partial information.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M05, M11, M17, M22, M25, M28, M29, O01, O03, O08, O19, O23

T2: M04, M06, M07, M10, M15, M18, M30, O02, O06, O09, O11, O15, O27

T3: M13, M14, M16, O04, O05, O07, O10, O14, O16, O21, O22, O24, O25, O28, O30

T4: M03, M08, M09, M12, M19, M20, M21, M23, M24, M26, M27, O12, O13, O17, O18, O20, O26, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session1 Link the representations

Learning target: Translate a root into a factor and graph point.

Learning target: Factor complete expressions and preserve every zero-product branch.

Choose Warm-up and I Do tasks. Ask learners to name the kind of object they are writing: number, expression, equation or point.

Session2 Construct and transform

Learning target: Use degree and extra values to determine a polynomial.

Learning target: Track roots through inner-input transformations and factor combinations.

Use We Do and Collaborative tasks. Verify each constructed example against every condition, and accept valid alternatives where uniqueness is not given.

Session3 Check hypotheses and missing cases

Learning target: Distinguish real, complex and context-restricted solutions.

Learning target: Diagnose hidden multiplicity, cancellation and misleading graph windows.

Use Independent tasks and targeted reteach, then collect fresh Exit evidence without showing completed graphs or factorizations.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use polynomial division and the Remainder Theorem; factor common terms, differences of squares and quadratics; solve linear factor equations; distinguish real and complex numbers. Complex examples explicitly identify their number system, and parameter cases require checking before division.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

A zero is an input c with $p(c)=0$; a factor is an expression such as $x-c$; a solution is a number satisfying the stated equation. Unless otherwise specified, polynomial coefficients and data are exact real numbers. Real graphs show only real inputs. Multiplicity is the exponent of a root factor after the remaining quotient is nonzero at that root. A nonzero constant scale does not change zeros. Restricted physical and rational-function domains must be preserved.

TEACHER MODEL

Three statements tied to the same input

For $p(x)=2x^3-6x^2-8x$, connect a complete factorization to the solutions of $p(x)=0$ and the real graph. Then explain why $p(x)=12$ is a different equation.

Factor out $2x$ and factor the quadratic: $p=2x(x^2-3x-4)=2x(x-4)(x+1)$. Thus the solutions of $p(x)=0$, the zeros of p and the x -intercept inputs are $-1,0,4$. Each factor appears once, so each zero is simple and the graph changes sign there. The scalar 2 is not a zero. To solve $p(x)=12$, form $p(x)-12=0$; the original factors do not make a product equal to zero in that new equation, and the old zeros give output 0 rather than 12 .

VISUAL ANCHOR

Keep the mathematical objects distinct

Number c : a solution of the equation $p(x)=0$

Function statement: $p(c)=0$, so c is a zero

Expression statement: $p(x)=(x-c)q(x)$

Real graph statement, when c is real: $(c,0)$ is an x-intercept

Multiplicity m : $p(x)=(x-c)^m q(x)$, where $q(c) \neq 0$

For a nonzero target $y=k$, solve $p(x)-k=0$, not $p(x)=0$.

A factor, a zero and an equation solution are connected, but a factor expression is not a number and a nonreal zero is not a real-axis intercept.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O01 proves the factor-zero equivalence; require its reverse implication using the Remainder Theorem. O03 sorts root numbers, factor expressions and graph points, then tests whether root data identify a unique function. In O02, keep the factored-out x visible in the zero-product statement. O04 requires every constraint, including scale and multiplicity, before sketching.

Core task

Write three equivalent statements expressing that -4 is a zero of a polynomial p : one using evaluation, one using a polynomial factor, and one using a graph point. Prove both directions of the equivalence between the factor and evaluation statements, and explain why the graph statement concerns a real input.

Teacher solution and discussion

The statements are $p(-4)=0$; $x+4$ is a factor of $p(x)$; and the real graph $y=p(x)$ contains $(-4,0)$. If $p=(x+4)q$, evaluating at -4 gives zero. Conversely, division by $x+4$ gives $p=(x+4)q+r$ with $r=p(-4)$; if $p(-4)=0$ then $r=0$, so $x+4$ is a factor. Since -4 is real, the zero appears as an x -intercept on the usual real-coordinate graph.

Evidence and response

Translate accurately among all three representations and justify the reverse implication rather than merely quote the forward product rule. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T3: Construct polynomials and infer parameters or transformed zero locations from stated constraints.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 by writing one factor per root, expanding and checking the coefficient relationships. Use O06 to verify an irrational root exactly. On O07, show how intercepts leave an unknown scalar and how a nonroot point fixes it. O08 is a full identity check: matching zero sets alone would not prove two expressions equal.

Core task

Find the monic cubic whose three distinct zeros are 1, -2 and 3. Expand it and relate its x^2 coefficient and constant term to the three root values. Explain why the word monic is needed for uniqueness.

Teacher solution and discussion

The monic cubic is $p=(x-1)(x+2)(x-3)$. First $(x-1)(x+2)=x^2+x-2$; multiplying by $x-3$ gives x^3-2x^2-5x+6 . The x^2 coefficient is the negative root sum, $-(1-2+3)=-2$. The constant is the negative root product, $-(1 \cdot (-2) \cdot 3)=6$. Any nonzero scalar multiple has the same zeros, but the monic condition fixes that scalar at 1.

Evidence and response

Construct and expand all three linear factors and explain the coefficient relationships and scale ambiguity. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Use polynomial structure to factor expressions and determine complete exact solution sets.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

For O09, put the target height next to the equation and ask which expression must be zero. For O10, state why real coefficients add a conjugate root, and why degree 4 leaves no unspecified factor. O12 distinguishes a candidate-generating theorem from an actual zero test. A checked failed candidate is essential evidence, not an incidental calculator step.

Core task

Let $p(x) = x^3 - 4x$. Solve $p(x) = 0$ and $p(x) = 3$ exactly. For each equation identify the polynomial whose zeros you are finding. Explain why the two solution sets differ and describe the corresponding horizontal graph intersections.

Teacher solution and discussion

For $p=0$, $x(x^2-4)=x(x-2)(x+2)=0$ gives $x=-2, 0, 2$. For $p=3$, the polynomial whose zeros we seek is $p-3=x^3-4x-3$. It factors as $(x+1)(x^2-x-3)$, since expansion gives x^3-4x-3 . Hence $x=-1$ or $x=(1 \pm \sqrt{13})/2$. The first set is where $y=p(x)$ meets $y=0$; the second is where it meets $y=3$. Changing the target height changes the expression to be set equal to zero.

Evidence and response

Factor each correct zero equation, retain all exact solutions and distinguish roots of p from roots of $p-3$. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Construct polynomials and infer parameters or transformed zero locations from stated constraints.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Have each learner produce a first factor or counterexample before sharing. O14 separates the inside transformation from the outside scalar; O15 contrasts a common zero with a zero of a product. In O16, one learner varies the scale while another verifies unchanged root locations. O17 requires repairing the division method through cases, not merely banning cancellation.

Core task

Let $p(u)=u^2-4$ and $h(x)=-3p(2x-1)$. Find the zeros of h by tracking the inner input, then expand and factor h as an independent check. Explain which part of the transformation changes the root locations and which does not.

Teacher solution and discussion

The zeros of p are $u=2$ and $u=-2$. Thus $2x-1=2$ or -2 , giving $x=3/2$ or $-1/2$. Expansion gives $h=-3[(2x-1)^2-4]=-12x^2+12x+9=-3(2x-3)(2x+1)$, which has the same two roots. The inside change $2x-1$ moves the input locations; the nonzero outside factor -3 changes vertical scale and orientation but not the zeros.

Evidence and response

Solve both inner-input equations and independently confirm the resulting factors and transformation roles. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 asks for factor-sign evidence behind the sketch. O20 requires a numerical scale appropriate to the two exact close roots; a generic parabola without the negative interval is incomplete. O21 distinguishes repeated roots from a degree drop, while O22 requires exact exceptional parameter cases. Keep first responses and neutral access supports separate from hints about which factor must vanish.

Core task

Use factors, not a graphing calculator, to make a sign chart for $p(x)=(x+1)(x-2)(x-4)$. State its zeros and the sign on every interval between them. Sketch the graph's crossing behavior and justify each crossing from the change in factor signs.

Teacher solution and discussion

The zeros are $-1, 2, 4$, each simple. For $x < -1$ all three factors are negative, so $p < 0$. On $(-1, 2)$, the signs are $+, -, -$, so $p > 0$. On $(2, 4)$, they are $+, +, -$, so $p < 0$. For $x > 4$ all are positive, so $p > 0$. At each root exactly one linear factor changes sign while the other two remain nonzero, so the graph crosses the axis each time. A positive cubic leading term gives the compatible down-left/up-right ends.

Evidence and response

Record all interval signs from factor signs and connect each root's odd multiplicity to an actual local sign change. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: Construct polynomials and infer parameters or transformed zero locations from stated constraints.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for nonlinear input changes that alter multiplicity, O25 for dividing by a parameter that might be zero, O26 for transferring a product-zero rule to sums, or O27 for dropping a square-equation branch. Rebuild only the missing connection and then give another task without the previous cue. Do not present the completed key graph as a construction template.

Core task

Let $p(u)=u(u-4)$ and $F(x)=p(x^2)$. A learner claims that F has exactly the same zeros and multiplicities as p . Find and factor F , determine its real zeros with multiplicity, and explain how the inner square changes both root locations and the multiplicity at zero.

Teacher solution and discussion

Substitution gives $F(x)=x^2(x^2-4)=x^2(x-2)(x+2)=x^4-4x^2$. Its zeros are 0 with multiplicity 2, and 2 and -2 each with multiplicity 1. The outer zero $u=4$ requires $x^2=4$ and therefore produces two real inputs ± 2 . The outer simple factor u becomes x^2 under substitution, producing a double zero at 0. Thus a nonlinear input change need not preserve the original root list or multiplicities.

Evidence and response

Track each outer factor through the nonlinear substitution and justify every new root and exponent. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Construct polynomials and infer parameters or transformed zero locations from stated constraints.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 checks multiplicity, leading sign and interval reasoning; O29 checks whether the apparent factorization hides another copy of a root factor; O30 integrates construction, expansion and the meaning of a zero. Require a verification tied to the actual conditions and credit all valid scales or constructions where the prompt permits them.

Core task

Construct a cubic with a double zero at 0, a simple zero at 2 and a negative leading coefficient. Choose a specific scale, sketch the graph, and justify its signs on the intervals $(-\infty, 0)$, $(0, 2)$ and $(2, \infty)$. Explain which features are fixed by the requirements and which are not.

Teacher solution and discussion

Choose $p(x) = -x^2(x-2) = x^2(2-x)$. For $x < 0$, $x^2 > 0$ and $2-x > 0$, so $p > 0$. The same signs hold on $(0, 2)$, while for $x > 2$ the second factor is negative and $p < 0$. Thus the graph touches the axis from above at 0 and crosses downward at 2. The roots, multiplicities and negative leading sign are fixed, but any negative scalar a in a $x^2(x-2)$ satisfies the conditions and changes vertical scale.

Evidence and response

Satisfy the requested factor multiplicities and leading sign, support the sketch with a full sign argument and describe the remaining scale freedom. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A nonzero polynomial $p(x)$ has factor $x+5$. Which conclusion follows?

- A. $p(0)=5$.
- B. $p(-5)=0$.
- C. The only zero is -5 .
- D. $p(5)=0$.

Reasoning: _____

M02 | Warm-up | T1

For a polynomial p , which statement is equivalent to $p(3)=0$?

- A. 3 is a factor of every coefficient of p .
- B. $x-3$ is a factor of $p(x)$.
- C. $x+3$ is a factor of $p(x)$.
- D. The graph contains $(0,3)$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

Write three equivalent statements expressing that -4 is a zero of a polynomial p : one using evaluation, one using a polynomial factor, and one using a graph point. Prove both directions of the equivalence between the factor and evaluation statements, and explain why the graph statement concerns a real input.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Use polynomial structure to factor expressions and determine complete exact solution sets.

Factor $p(x)=x^4-x^3-4x^2+4x$ completely over the real numbers. Give all distinct zeros, identify their multiplicities, and verify the factorization by regrouping or multiplication. Explain why factoring out x does not allow you to discard the zero $x=0$.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

Answer: B. $p(-5)=0$.

There is a polynomial q with $p(x)=(x+5)q(x)$. At $x=-5$ the factor becomes 0, so $p(-5)=0$ regardless of $q(-5)$. This establishes a zero but does not exclude others.

Check each alternative

- A. A factor does not specify this value of the full product.
- B. Correct. There is a polynomial q with $p(x)=(x+5)q(x)$. At $x=-5$ the factor becomes 0, so $p(-5)=0$ regardless of $q(-5)$. This establishes a zero but does not exclude others.
- C. Other factors may provide additional zeros.
- D. The factor $x+5$ vanishes at -5 , not $+5$.

Evidence to assess

Link a displayed factor to its zero without claiming the full zero set.

M02 • T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.

Answer: B. $x-3$ is a factor of $p(x)$.

Polynomial division by $x-3$ has remainder $p(3)$. Thus $p(3)=0$ exactly when $p(x)=(x-3)q(x)$ for a polynomial q , which is the definition of $x-3$ being a factor.

Check each alternative

- A. Numerical coefficient divisibility is unrelated to the stated input-output condition.
- B. Correct. Polynomial division by $x-3$ has remainder $p(3)$. Thus $p(3)=0$ exactly when $p(x)=(x-3)q(x)$ for a polynomial q , which is the definition of $x-3$ being a factor.
- C. That would require $p(-3)=0$, a different value condition.
- D. The condition $p(3)=0$ describes $(3,0)$, not $(0,3)$.

Evidence to assess

Express the two-way factor-zero relationship rather than only one implication.

WORKED TEACHER KEYS

001 / 002**001 • T1: Explain the equivalence among polynomial factors, equation solutions, function zeros and real graph intercepts.**

The statements are $p(-4)=0$; $x+4$ is a factor of $p(x)$; and the real graph $y=p(x)$ contains $(-4,0)$. If $p=(x+4)q$, evaluating at -4 gives zero. Conversely, division by $x+4$ gives $p=(x+4)q+r$ with $r=p(-4)$; if $p(-4)=0$ then $r=0$, so $x+4$ is a factor. Since -4 is real, the zero appears as an x -intercept on the usual real-coordinate graph.

Evidence to assess

Translate accurately among all three representations and justify the reverse implication rather than merely quote the forward product rule.

002 • T2: Use polynomial structure to factor expressions and determine complete exact solution sets.

First $p=x(x^3-x^2-4x+4)$. Group the cubic as $x^2(x-1)-4(x-1)=(x-1)(x^2-4)$. Hence $p=x(x-1)(x-2)(x+2)$. The distinct zeros are $0,1,2,-2$, each with multiplicity 1. Multiplying $(x-1)(x^2-4)$ gives x^3-x^2-4x+4 , and multiplying by x recovers p . Factoring x retains the branch $x=0$ in the zero-product statement; division by x would require treating that branch separately.

Evidence to assess

Use a complete structural factorization, retain the common-factor zero and verify the exact original expression.