

TEACHER AND STUDENT RESOURCE

Circle Equations and Completing the Square



NC Math 3 • NC.M3.G-GPE.1

Expressing Geometric Properties with Equations

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-GPE.1 | NC Math 3 mathematics

This sample contains 23 complete selected pages from the 127-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-17, 32-33, 67-68, 93-94

Representative activities

M01: sample PDF page(s) 18; original page(s) 17

M02: sample PDF page(s) 18; original page(s) 17

O01: sample PDF page(s) 19; original page(s) 32

O02: sample PDF page(s) 20; original page(s) 33

Selected teacher prompts and keys: original page(s) 67-68, 93-94

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

T1 Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

T2 Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

T3 Use the Pythagorean circle equation to recover and check circles under point, diameter, center-location, translation, scale and touching constraints.

T4 Compare and critique circle representations, test simultaneous conditions and classify parameter cases as positive-radius circles, single points or empty real loci.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M05, M06, M24, M28, O01, O03, O05, O25

T2: M03, M07, M08, M09, M15, M19, M20, M25, M26, O02, O06, O07, O12, O17, O21, O24, O27

T3: M04, M10, M11, M18, M21, M22, M29, O04, O09, O10, O11, O13, O16, O19, O20, O22, O23, O30

T4: M12, M13, M14, M16, M17, M23, M27, M30, O08, O14, O15, O18, O26, O28, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Derive and reverse the equation

Learning target: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Learning target: Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

Use Warm-up and I Do tasks. Build O01 and O03 with equal coordinate scales, prove the general argument in O05 and reverse expansions in O02 and O06. Require a geometric explanation before giving the standard form.

Session 2 Recover circles from constraints

Learning target: Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

Learning target: Use the Pythagorean circle equation to recover and check circles under point, diameter, center-location, translation, scale and touching constraints.

Use We Do and Collaborative tasks to connect exact square completion, diameter data, point constraints and center restrictions. Give separate grids for O08, O15 and O16. Check every original constraint after solving.

Session 3 Audit parameter cases and transfer

Learning target: Use the Pythagorean circle equation to recover and check circles under point, diameter, center-location, translation, scale and touching constraints.

Learning target: Compare and critique circle representations, test simultaneous conditions and classify parameter cases as positive-radius circles, single points or empty real loci.

Use Independent tasks, selected reteach and Exit tasks. Compare radius, squared radius and expanded constants. Require both roots where appropriate and distinguish a genuine circle from a point or empty locus.

PLANNING

Materials, access and pacing

Materials

Pencil, ruler, compass and equal-scale coordinate grids. Use exact fractions and radicals in calculations; approximate drawings may use a ruler or calculator only after the exact radius is established. Supply all separate workspaces requested by a construction task.

Prerequisites

Use the Pythagorean Theorem, coordinate differences, midpoint and distance reasoning, binomial expansion and exact fraction and radical arithmetic. Solve linear equations and simple squared equations with both square-root branches. A circle is a boundary, not a filled disk.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

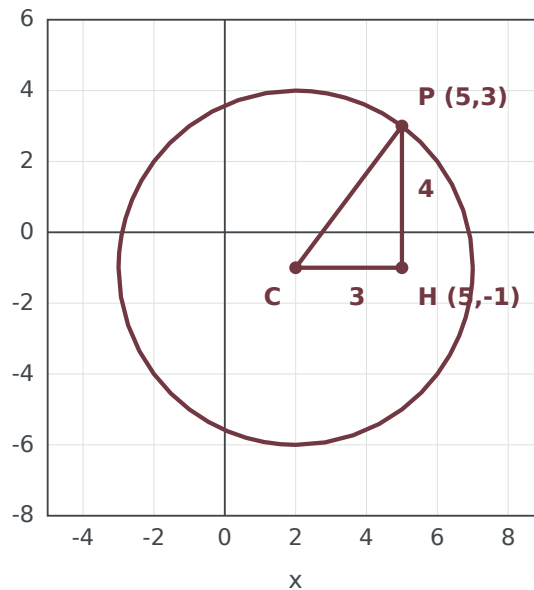
Coordinates are real and axes use the same unit scale. A circle has positive radius; a zero completed squared radius is a one-point degenerate locus and a negative completed squared radius has no real points. Circle equations describe boundaries unless an inequality is explicitly used for a disk. Radius means the positive square root of r^2 . Completing the square must preserve equality; divide every term by a common nonzero leading coefficient first. Exact equivalent expressions and valid alternative constructions are accepted. Drawings do not supply unstated measurements.

TEACHER MODEL

Move from coordinate legs to an equivalent expanded equation

A circle has center $(2,-1)$ and radius 5. Use $P=(5,3)$ to illustrate the coordinate triangle, derive the equation for any boundary point, expand it, and then recover the original geometry by completing the square.

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The horizontal and vertical legs from $C=(2,-1)$ through $H=(5,-1)$ to $P=(5,3)$ have lengths 3 and 4, giving $CP^2=9+16=25$. For any boundary point, use lengths $|x-2|$ and $|y+1|$: $(x-2)^2+(y+1)^2=25$. Expansion gives $x^2+y^2-4x+2y-20=0$. To reverse it, move the constant and add 4+1: $x^2-4x+4+y^2+2y+1=20+4+1$, so $(x-2)^2+(y+1)^2=25$ again. The values making the differences zero locate center $(2,-1)$, and $\sqrt{25}=5$ is the radius. Every algebraic step is reversible; the right side is a squared distance, not a signed radius.

VISUAL ANCHOR

Keep distance geometry and square completion connected

Coordinate legs from (h,k) to (x,y) : $|x-h|$ and $|y-k|$.

Pythagorean boundary equation: $(x-h)^2+(y-k)^2=r^2$, with $r>0$.

A center makes both coordinate differences zero; it is not a boundary point when $r>0$.

Normalize a common nonzero coefficient before completing either square.

$x^2+bx+(b/2)^2=(x+b/2)^2$; add the same constants to both sides.

For $x^2+y^2+ax+by+c=0$: center $(-a/2,-b/2)$, $r^2=(a^2+b^2)/4-c$.

Completed right side positive: circle; zero: one point; negative: no real points.

VISUAL ANCHOR CONTINUED

Keep distance geometry and square completion connected

Recover all square-root branches in inverse constraints, then check every original point and radius condition.

The equation is a fixed-distance statement. Check signs, common scaling, added constants and whether the resulting squared radius is actually positive.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Begin O01 by asking learners to trace the horizontal leg, vertical leg and radius before writing an equation. Ask why the leg lengths must be absolute differences even though their squares need no absolute-value symbols. Use M03 to elicit the expansion $x^2+2ax+a^2$ rather than a rule stated without evidence. In O04 insist that inside a disk is not the same as on a circle. For O03 record all four extreme points and use identical axis scales so the drawing does not introduce a false elliptical appearance.

Core task

Draw a circle centered at the origin with radius 5. Mark $P=(3,4)$, draw its horizontal and vertical coordinate legs, and use the Pythagorean Theorem to show P is on the circle. Then derive an equation that describes every boundary point (x,y) , not just P .

Teacher solution and discussion

Mark $C=(0,0)$, $H=(3,0)$ and $P=(3,4)$, and join CH , HP and PC . The legs have lengths 3 and 4, so $CP^2=3^2+4^2=25$ and $CP=5$. Thus P is on the radius-5 circle. For any boundary point (x,y) , the coordinate leg lengths are $|x|$ and $|y|$, giving $x^2+y^2=25$. Conversely, any real point satisfying that equation has distance $\sqrt{25}=5$ from C , so the equation describes the whole boundary.

Evidence and response

Require the completed circle and labeled perpendicular legs, the numerical membership proof and a general equation with the converse distance interpretation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O05 to name the projection $H=(x,k)$, establish the perpendicular legs and show both implications between equation and distance. Discuss a zero leg separately instead of describing a degenerate triangle as an ordinary right triangle. In O06 write each equality on a new line with the same added constants visible on both sides. In O07 first divide every term by 5, including the constant. Finish O08 by comparing two separately drawn circles: congruent circles need not be the same set of points.

Core task

For arbitrary real h and k and $r>0$, derive the equation of the circle centered at (h,k) using a coordinate right triangle. Explain why the argument works in every quadrant, include points whose x - or y -coordinate equals the center coordinate, and prove the converse.

Teacher solution and discussion

For $P=(x,y)$, choose $H=(x,k)$. When both displacements are nonzero, CH and HP are perpendicular legs of lengths $|x-h|$ and $|y-k|$, and CP is the hypotenuse. P is on the circle exactly when $CP=r$, so Pythagoras gives $(x-h)^2+(y-k)^2=r^2$. Absolute values make leg lengths nonnegative in every quadrant, and their squares equal the squared differences. If $x=h$ or $y=k$, one leg is zero and the direct horizontal or vertical distance gives the same equation. Conversely, a real solution has $CP=\sqrt{r^2}=r$ because $r>0$, so every solution lies on that circle.

Evidence and response

Require an explicitly named projection point, perpendicular legs, sign reasoning, zero-leg treatment and both directions of the equation-distance equivalence. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Use the Pythagorean circle equation to recover and check circles under point, diameter, center-location, translation, scale and touching constraints.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

For O09 collect inside/on/outside predictions before comparing squared distances with 36. In O10 ask whether the endpoint distance is a radius or diameter, then verify both endpoints after using the midpoint. In O11 distinguish solving for a constant from reading one as a radius. Work O12 with exact sixteenths and expand back after multiplying by 2. For O13 parameterize the center as (a,a) before doing arithmetic; require the learner to explain why the final single value of a exhausts all possibilities.

Core task

On a meter-based coordinate map, a sprinkler is at $(4,3)$ and reaches exactly 6 m in every direction. Derive the equation of the wet region's circular boundary. Classify $A=(9,6)$, $B=(10,3)$ and $D=(11,3)$ as inside, on or outside. State how the equation differs from a description of the entire wet region.

Teacher solution and discussion

For a boundary point, Pythagoras gives $(x-4)^2 + (y-3)^2 = 6^2 = 36$. A has squared distance $5^2 + 3^2 = 34 < 36$, so it is inside. B gives $6^2 + 0^2 = 36$, so it is on the boundary. D gives $7^2 + 0^2 = 49 > 36$, so it is outside. The boundary uses equality; the entire wet disk, including its boundary, is described by $(x-4)^2 + (y-3)^2 \leq 36$. Squared distances have units m^2 even though the coordinate differences are in meters.

Evidence and response

Derive from the specified center and reach, compute all three squared distances and use equality versus inequality accurately for boundary and filled region. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Compare and critique circle representations, test simultaneous conditions and classify parameter cases as positive-radius circles, single points or empty real loci.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign a constraint recorder, algebra checker, constructor and skeptic, with a private first response from each learner. O14 must preserve the center, A and the specified x-coordinate when repairing B; both replacement y-values are valid. O15 needs three separate grids for circle, point and empty locus. O16 should end with the family $(2,k)$, not just two unexplained pictures. On O17 distinguish a balanced addition from the later invalid binomial replacement. O18 requires a checked distinguishing point for each counterexample.

Core task

A proposed circle has center $C=(1,-2)$ and must pass through $A=(4,2)$ and $B=(-2,0)$. Determine whether it exists. If C and A must stay fixed and B must keep $x=-2$, find every possible replacement y-coordinate for B and verify the repaired points.

Teacher solution and discussion

$CA^2=(4-1)^2+(2+2)^2=9+16=25$, but $CB^2=(-2-1)^2+(0+2)^2=9+4=13$. A single radius cannot have both squares, so the proposed circle does not exist. Keeping C and A fixes $(x-1)^2+(y+2)^2=25$. At $x=-2$, $9+(y+2)^2=25$, so $y+2=\pm 4$, giving $y=2$ or -6 . The replacements $(-2,2)$ and $(-2,-6)$ each give $9+16=25$; both satisfy the required fixed x-coordinate.

Evidence and response

Identify the conflicting distance constraints, hold the designated data fixed during repair and retain both valid square-root branches with explicit checks. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Use the Pythagorean circle equation to recover and check circles under point, diameter, center-location, translation, scale and touching constraints.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O19–O23 without supplying the needed algebraic form. O19 tests map-unit conversion before squaring, not a new distance formula. O20 must retain both center heights and two complete drawings. For O21 ask why the denominator is positive even when A is negative. In O22 assess whether the center and point constraints are both used; one valid substitution is not a uniqueness argument. In O23 the same expanded constant is deliberate: students must compare completed forms rather than decide from a single coefficient.

Core task

A map uses 1 coordinate unit for 100 m. A sensor is at $(-3, 2)$ and has a circular boundary 450 m from it. Derive the boundary equation in map coordinates. Determine whether $P = (0, 5)$ is inside the coverage disk, and give one exact map-coordinate boundary point to check your units.

Teacher solution and discussion

The radius is $450/100 = 9/2$ map units, so the boundary equation is $(x+3)^2 + (y-2)^2 = 81/4$. For P , the squared map distance is $3^2 + 3^2 = 18 = 72/4 < 81/4$, so P is inside. Its physical distance is $100\sqrt{18} = 300\sqrt{2}$ m, less than 450 m. Moving $9/2$ map units horizontally right from the center gives $(3/2, 2)$, which yields $(9/2)^2 + 0^2 = 81/4$ and corresponds to a 450-m radius. Using 450 directly in a map-coordinate equation would mix units.

Evidence and response

Convert physical radius to coordinate units before squaring, compare like-unit squared distances and verify a constructed boundary point in both coordinate and physical interpretations. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O24 for sign and square-root interpretation, O25 for rebuilding an equation from a verified coordinate triangle, O26 for sufficient-versus-necessary conditions, or O27 for exact fractional completion. Preserve the learner’s valid steps, then isolate the first unsupported inference. In O24 do not call the square completion wrong when only the interpretation is wrong. In O26 ask for the $c=0$ boundary case after the positive example. Remove the worked support and use the corresponding MC task as a fresh check of the underlying idea.

Core task

A learner correctly writes $(x+1)^2+(y+6)^2=9$ after starting from $x^2+y^2+2x+12y=-28$, but reports center (1,6) and radius 9. Check the completed equation, correct the geometric interpretation, and give a boundary point that exposes the radius error.

Teacher solution and discussion

The completion is correct: add 1+36 to -28, obtaining 9. The differences vanish at (-1,-6), so that is the center, and $r=\sqrt{9}=3$. A boundary point 3 units to the right is (2,-6), giving $(2+1)^2+0^2=9$. A point 9 units to the right would be (8,-6), giving 81, not 9, so the reported radius is false. Expanding the correct form gives $x^2+y^2+2x+12y+37=9$ and returns the original right side -28.

Evidence and response

Retain the valid square completion, fix the two interpretation errors independently, check an actual boundary point and explicitly disprove the erroneous radius. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Compare and critique circle representations, test simultaneous conditions and classify parameter cases as positive-radius circles, single points or empty real loci.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Collect O28 before revealing which records are valid; ask for a completed form for every record, including the scaled one. O29 requires both irrational parameter values and an explicit explanation of why radius stays fixed. In O30 do not assume the center is above the axis at the start: write $r=|k|$, then let the point condition determine k . Require the contact point and the single $y=0$ intersection to check touching. Decide the next lesson from a specific missing derivation, balancing step or feasibility check rather than a correct final equation alone.

Core task

Three records claim to describe a circle centered at $(2,1)$ with radius 4: A: $x^2+y^2-4x-2y-11=0$; B: $2x^2+2y^2-8x-4y-22=0$; C: $x^2+y^2-4x-2y-16=0$. Audit all three by completing the square. Identify the valid records and explain precisely what is wrong in each invalid record.

Teacher solution and discussion

A completes to $(x-2)^2+(y-1)^2=11+4+1=16$, so it has the required center and radius. Dividing B entirely by 2 gives A, so B is the same valid circle. C completes to $(x-2)^2+(y-1)^2=16+4+1=21$, so its center is correct but radius is $\sqrt{21}$, not 4. C appears to use $-r^2$ as the expanded constant without including h^2+k^2 . The correct expanded constant is $2^2+1^2-4^2=5-16=-11$. Thus exactly A and B meet all claims.

Evidence and response

Audit each full record, recognize harmless common scaling, retain the correct center in the invalid case and identify the missing center-square contribution to its constant. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Which center and radius belong to the circle $(x+5)^2+(y-2)^2=36$?**

- A. Center $(-5,2)$; radius 6.
- B. Center $(-5,2)$; radius 36.
- C. Center $(5,-2)$; radius 6.
- D. Center $(5,2)$; radius 6.

Reasoning: _____

M02 | Warm-up | T1**A circle is centered at the origin and has radius $\sqrt{7}$ units. Which equation describes its boundary?**

- A. $x+y=\sqrt{7}$.
- B. $x^2+y^2=\sqrt{7}$.
- C. $x^2+y^2=49$.
- D. $x^2+y^2=7$.

Reasoning: _____

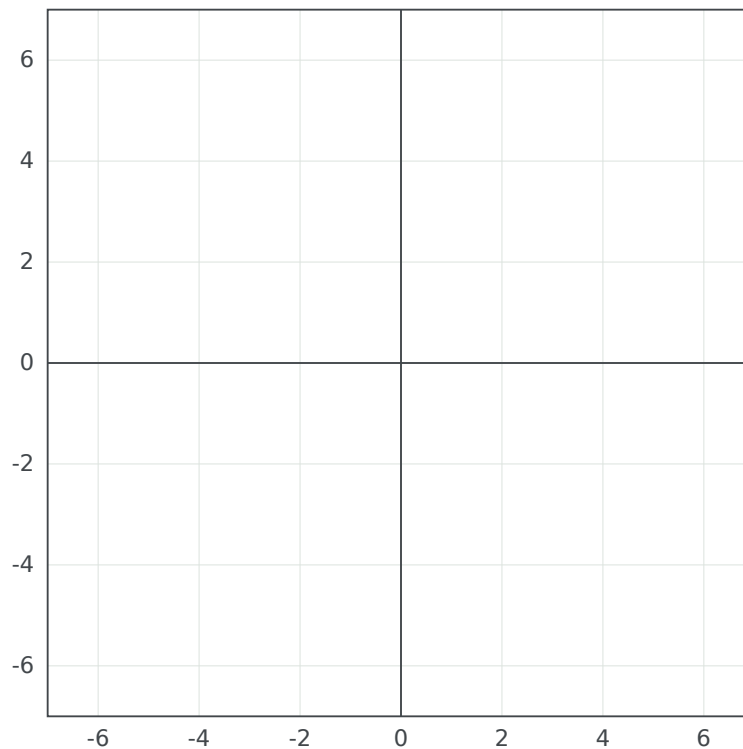
WARM-UP

001 • Show your reasoning

T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Draw a circle centered at the origin with radius 5. Mark $P=(3,4)$, draw its horizontal and vertical coordinate legs, and use the Pythagorean Theorem to show P is on the circle. Then derive an equation that describes every boundary point (x,y) , not just P .

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

T2: Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

Complete the square in $x^2+y^2+8x-2y=8$. State the center and radius, and expand your completed equation to check that no solutions were added or lost.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Answer: A. Center (-5,2); radius 6.

In $(x-h)^2+(y-k)^2=r^2$, the differences vanish at the center. Thus $h=-5$, $k=2$ and $r^2=36$. A radius is a positive length, so $r=6$.

Check each alternative

- A. Correct. In $(x-h)^2+(y-k)^2=r^2$, the differences vanish at the center. Thus $h=-5$, $k=2$ and $r^2=36$. A radius is a positive length, so $r=6$.
- B. The right side is the squared radius, so the radius is $\sqrt{36}=6$, not 36.
- C. This copies the signs inside the parentheses instead of solving $x+5=0$ and $y-2=0$ for the center.
- D. The y-coordinate is correct, but $x+5$ means $x-(-5)$, so the center x-coordinate is -5.

Evidence to assess

Read both center coordinates as the values that make the differences zero and distinguish the radius from its square.

WORKED TEACHER KEYS

M02

M02 • T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Answer: D. $x^2+y^2=7$.

For a boundary point (x,y) , a right triangle has leg lengths $|x|$ and $|y|$ and hypotenuse $\sqrt{7}$. The Pythagorean Theorem gives $x^2+y^2=(\sqrt{7})^2=7$.

Check each alternative

- A. Adding signed coordinate differences gives a line, not the sum of squared leg lengths for a fixed distance.
- B. This uses the radius rather than its square as the right side of the Pythagorean equation.
- C. This squares 7 instead of squaring the given radius $\sqrt{7}$.
- D. Correct. For a boundary point (x,y) , a right triangle has leg lengths $|x|$ and $|y|$ and hypotenuse $\sqrt{7}$. The Pythagorean Theorem gives $x^2+y^2=(\sqrt{7})^2=7$.

Evidence to assess

Connect the sum of squared coordinate legs to the square of the stated radius rather than match an unsquared distance expression.

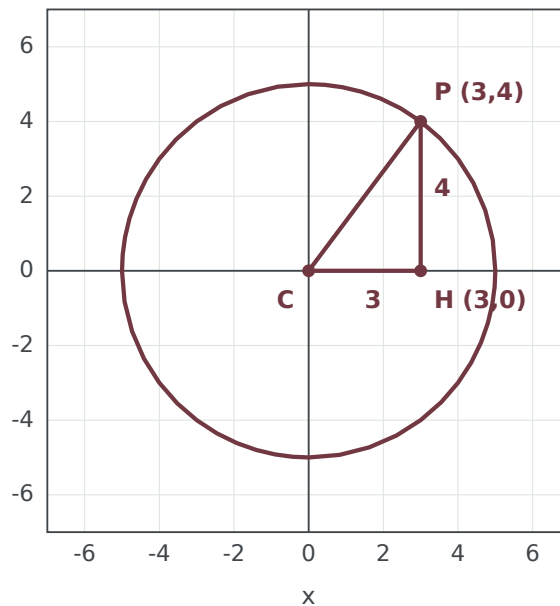
WORKED TEACHER KEYS

001

001 • T1: Derive the circle equation from perpendicular coordinate differences and the Pythagorean Theorem, explain both directions, and construct circles from center-radius data.

Mark $C=(0,0)$, $H=(3,0)$ and $P=(3,4)$, and join CH , HP and PC . The legs have lengths 3 and 4, so $CP^2=3^2+4^2=25$ and $CP=5$. Thus P is on the radius-5 circle. For any boundary point (x,y) , the coordinate leg lengths are $|x|$ and $|y|$, giving $x^2+y^2=25$. Conversely, any real point satisfying that equation has distance $\sqrt{25}=5$ from C , so the equation describes the whole boundary.

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**Evidence to assess**

Require the completed circle and labeled perpendicular legs, the numerical membership proof and a general equation with the converse distance interpretation.

WORKED TEACHER KEYS

002

002 • T2: Complete both squares in a circle equation, normalizing a common nonzero coefficient first, to recover the exact center and positive radius and verify algebraic equivalence.

Add 16+1 to both sides: $(x^2+8x+16)+(y^2-2y+1)=8+16+1$. Thus $(x+4)^2+(y-1)^2=25$, with center $(-4,1)$ and radius 5. Expanding gives $x^2+8x+16+y^2-2y+1=25$; subtracting 17 returns $x^2+y^2+8x-2y=8$ exactly. Every operation is reversible, so the solution set is unchanged.

Evidence to assess

Show both balancing additions, read the center signs and positive radius correctly, and reverse the algebra to the original full equation.