

TEACHER AND STUDENT RESOURCE

Geometric Modeling Density and Design



NC Math 3 • NC.M3.G-MG.1

Modeling with Geometry

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-MG.1 | NC Math 3 mathematics

This sample contains 24 complete selected pages from the 125-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-17, 32-33, 65-66, 92-94

Representative activities

M01: sample PDF page(s) 18; original page(s) 17

M02: sample PDF page(s) 18; original page(s) 17

O01: sample PDF page(s) 19; original page(s) 32

O02: sample PDF page(s) 20; original page(s) 33

Selected teacher prompts and keys: original page(s) 65-66, 92-94

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply geometric concepts in modeling situations

- Use geometric and algebraic concepts to solve problems in modeling situations:
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

T1 Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

T2 Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

T3 Use area and volume density with compatible units, correct region denominators, justified weights and explicit estimation assumptions.

T4 Construct feasible designs, test every constraint, and justify an optimum or the absence of one over the stated design set.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M05, M22, M26, M30, O01, O02, O08, O15, O19

T2: M06, M08, M10, M11, M14, M21, M25, O05, O07, O09, O26, O28

T3: M03, M04, M07, M12, M16, M17, M18, M24, M29, O03, O04, O06, O10, O11, O18, O20, O24, O25

T4: M09, M13, M15, M19, M20, M23, M27, M28, O12, O13, O14, O16, O17, O21, O22, O23, O27, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Choose regions and relationships

Learning target: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Learning target: Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

Use Warm-up and I Do tasks to separate boundaries, surfaces and occupied regions. Collect the first L-shaped plan, then compare the table, graph and algebraic bound in O05.

Session 2 Audit densities and feasible designs

Learning target: Use area and volume density with compatible units, correct region denominators, justified weights and explicit estimation assumptions.

Learning target: Construct feasible designs, test every constraint, and justify an optimum or the absence of one over the stated design set.

Use We Do and Collaborative tasks to compare alternative density denominators and design costs. Require a dimension-based packing check and a proof that a claimed optimum covers its full feasible set.

Session 3 Challenge and transfer models

Learning target: Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

Learning target: Construct feasible designs, test every constraint, and justify an optimum or the absence of one over the stated design set.

Use Independent practice to contrast sampled and global optima, handle measurement uncertainty and construct a bounded grid. Reteach the observed unit or constraint error, then collect the exit designs without teacher keys.

PLANNING

Materials, access and pacing

Materials

Pencil, ruler and graph paper. A calculator is useful for specified decimal estimates; preserve exact pi and radical forms first. Provide separate blank grids when two constructions are requested. Tables must include units and all requested rows.

Prerequisites

Areas and perimeters of polygons and circles; volumes of prisms, cylinders, cones and spheres; the Pythagorean Theorem; ratio, percentage and unit conversions; quadratic expressions, inequalities and integer factor pairs.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

Review convention

All contexts and numerical data are instructional models. Geometric dimensions are positive unless an item explicitly investigates a boundary. State whether a measurement describes an inside region, outside surface, boundary, projection, capacity or material. Density means count or mass per the specified area or volume; averages need not describe every subregion. Do not assume sample uniformity, packing feasibility or equal volume from a drawing. Geometric models may idealize objects; state relevant assumptions. Exact values are preferred; π is not 3.14 unless an approximation is requested. Drawings use stated coordinates or dimension labels. A disk includes its interior, whereas a circle alone is its boundary. Completed teacher constructions are not student templates. Optimization claims apply only to the stated feasible set and require an attained bound or exhaustive finite comparison.

TEACHER MODEL

From a geometric region to a defensible purchase

A rectangular planting bed is 5 m by 4 m. A circular fountain of radius 1 m occupies part of the bed entirely inside it. Mulch covers the remaining interior to a uniform depth 0.08 m. Bags contain 0.25 m^3 . Ignoring waste, what is the least whole-bag purchase? Identify the modeled region and check your integer decision.

The rectangle interior has area 20 m^2 . Exclude the whole fountain disk of area $\pi \text{ m}^2$, not merely its circular boundary. Mulched area is $20 - \pi \text{ m}^2$; layer volume is $0.08(20 - \pi) \text{ m}^3 \approx 1.349 \text{ m}^3$. The bag ratio is $0.08(20 - \pi)/0.25 = 6.4 - 0.32\pi \approx 5.395$. Purchase 6 bags. Five contain 1.25 m^3 , too little, while six contain 1.5 m^3 , enough. The model assumes a uniform layer, an entirely contained fountain and no settling or waste. An area-only calculation cannot determine a volume purchase without depth.

VISUAL ANCHOR

Model the quantity before optimizing it

Region: distinguish area, boundary, surface, capacity and material.

Relationship: define quantities and units; write constraints and domains.

Density: multiply by matching area or volume; combine totals before rates.

Scaling: length factor k gives area factor k^2 and volume factor k^3 .

Feasibility: test dimensions, exclusions, integers, containment and packing.

Optimization: attain a proved bound or exhaust every allowed candidate.

Interpretation: audit averages, sampled estimates and idealized shapes.

Eliminate dependent dimensions only after stating constraints. Fixed thickness differs from full similarity. Density can describe counts or mass. A numerical answer is evidence only when the model, units, feasible set and conclusion match the physical question.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Have learners identify what is counted before choosing a formula. On O01 distinguish the outside boundary from the artificial partition, and ask which interior region is the garden. On O02 separate capacity from material. Use O03 to show why density weights are areas, not the number of zones; O04 then checks whether all dimensions and density use compatible units. Preserve explanations even when arithmetic is correct.

Core task

An L-shaped garden is the entire interior of the polygon with vertices (0,0), (10,0), (10,4), (6,4), (6,8), (0,8), in meters. Draw and label the plan, partition it into two nonoverlapping rectangles, and find its area and outside perimeter. Explain why your partition segment should not be included when ordering edging for the garden.

Teacher solution and discussion

Join the six vertices in order and close the polygon. Partition along the segment from (6,0) to (6,4). The left rectangle is 6 by 8, area 48 m^2 ; the lower-right rectangle is 4 by 4, area 16 m^2 . Total garden area is 64 m^2 . Outside edge lengths in order are 10, 4, 4, 4, 6, 8 m, summing to 36 m. The shared 4 m partition is inside the garden, not part of its exterior boundary, so it contributes no edging. The completed key depicts the entire polygon interior as garden, not just its outline.

Evidence and response

Require a closed labeled L-shaped plan, an actual nonoverlap partition, both areas and every outside boundary length; reject counting the artificial internal seam as edging. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O05 to connect a perimeter constraint, a five-row table, an open-endpoint graph and a complete-square bound. Explain why a sampled maximum is not automatically a global maximum. In O06 name the unequal volume weights before computing average density. On O07 trace two border widths on each dimension. On O08 contrast the horizontal footprint with the sloping deck, and test each restriction independently.

Core task

Exactly 24 m of edging surrounds a nondegenerate rectangular flower bed. Let x be one side. Derive $A(x)$ and its physical domain. Complete a table for $x=0,3,6,9,12$ with columns x , other side, area, and physical status. Graph the area relationship, distinguishing excluded endpoints. Then use algebra to establish the largest area, not just the largest sampled value.

Teacher solution and discussion

The perimeter gives $2x+2y=24$, so $y=12-x$ and $A=x(12-x)$ on $0<x<12$. The completed table uses x and y in meters, A in m^2 , and B for physical status. Excluded endpoints are degenerate.

Input x	0	3	6	9	12
Output y	12	9	6	3	0
Output A	0	27	36	27	0
Output B	excluded	valid	valid	valid	excluded

The graph is the downward parabola through these points, with open endpoint circles at $(0,0)$ and $(12,0)$. Completing the square gives $A=36-(x-6)^2\leq 36$. Equality at $x=y=6$ proves a maximum of 36 m^2 over the entire physical domain, not merely the five table entries.

Evidence and response

Require all five completed table rows, domain-aware endpoint symbols, a labeled quadratic graph and a nonnegative-square proof of the maximum; a table-only maximum is insufficient. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: O09 • T2: Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

In O09 keep the semicircle centers fixed while extending the boundary outward; compare the exact path area with perimeter times width. On O10 specify the denominator before comparing towns. In O11 collect the incorrect simple mean and explain its implicit equal-volume assumption. For O12 require both complete packet arrangements, not two outlines. In O13 verify side divisibility before treating an area quotient as a purchasable tile count.

Core task

A stadium-shaped lawn consists of a 20 m by 10 m rectangle and two external semicircular ends of radius 5 m. A path 1 m wide runs all the way around the lawn, measured perpendicular to its boundary. Draw labeled inner and outer outlines. Find the lawn perimeter and exact path area. Explain why multiplying the lawn perimeter by 1 m is not the exact path area.

Teacher solution and discussion

The inner straight sections each have length 20 m, and the two inner semicircles total one radius-5 circle. Lawn perimeter is $40+10\pi$ m. The outer straight sections still have length 20 m; the semicircle centers stay fixed, their radii become 6 m, and the outer rectangle has width 12 m. Lawn area is $200+25\pi$ and outer area is $240+36\pi$ m². Their difference is $40+11\pi$ m². Perimeter times width gives $40+10\pi$ m², underestimating by π m² because the rounded ends widen as one moves outward. The region between outlines, not the lawn interior, is the path.

Evidence and response

Require both completed outlines with shared end centers and correct radii, the unchanged straight-section length, an outer-minus-inner computation and the exact π m² correction to the perimeter shortcut. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Construct feasible designs, test every constraint, and justify an optimum or the absence of one over the stated design set.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign modeler, unit checker and skeptic roles after private predictions. O14 uses different fence prices, so separate a cost constraint from a length constraint. O15 requires both a diameter-based fit and a mass calculation. On O16 demand a universal bound and an equality case. On O17 compare total prices, not just one surface area. O18 should end with a reasoned limitation and improved sampling plan, not an exact-census claim.

Core task

A rectangular garden uses an existing wall for one side. Two perpendicular side fences cost \$4 per meter, and the front fence parallel to the wall costs \$2 per meter. The entire \$96 budget is used. Let x be each perpendicular side and y the front length. Determine the area-maximizing positive real dimensions and prove the maximum. Compare the two feasible designs $x=5$ and $x=7$ to explain why checking nearby examples alone is not a proof.

Teacher solution and discussion

The cost constraint is $4(2x)+2y=96$, so $y=48-4x$ and $0<x<12$. Area is $A=x(48-4x)=144-4(x-6)^2$. Thus $A\leq 144$ m², with equality only at $x=6$ m and $y=24$ m; cost is $4\cdot 12+2\cdot 24=\$96$. For $x=5$, $y=28$ and $A=140$; for $x=7$, $y=20$ and $A=140$. These two examples illustrate symmetry but cannot exclude every other feasible x . The nonnegative-square identity supplies the complete proof.

Evidence and response

Model costs separately from physical fence length, derive the full positive domain, verify the optimal cost and use an all-domain algebraic bound rather than sample comparisons alone. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Preserve the learner’s original model in O19 before discussing bag rounding. O20 reverses endpoint order when radius enters a density denominator. O21 intentionally supplies a counterexample to a table-based global maximum; do not ask learners to optimize the entire cubic. O22 needs an interval-wide argument. For O23 check every plotted plant, the inclusive row-count formula and the margin restriction before calculating density.

Core task

A courtyard is the interior of the polygon $(0,0)$, $(12,0)$, $(12,5)$, $(8,9)$, $(0,9)$, with coordinates in meters. It contains a circular pond centered at $(3,4)$, radius 2 m. Draw the plan and identify the region to pave, excluding the pond. Find its exact area. A bag nominally covers 7 m^2 , but 10% of each bag’s coverage is lost to waste. Find the least whole-bag purchase and verify the counts immediately below and at your answer.

Teacher solution and discussion

The courtyard is a 12 by 9 rectangle with a right-triangular corner of legs 4 and 4 removed. Its area is $108 - (1/2)(4)(4) = 100 \text{ m}^2$. The pond lies fully inside the courtyard, so paved area is $100 - 4\pi \approx 87.434 \text{ m}^2$. Effective coverage per bag is $0.9 \cdot 7 = 6.3 \text{ m}^2$. The ratio $(100 - 4\pi)/6.3 \approx 13.878$ requires 14 whole bags. Thirteen cover 81.9 m^2 , insufficient; fourteen cover 88.2 m^2 , sufficient. The completed plan marks the polygon interior outside the pond as the paving region; neither the pond’s filled disk nor the cut-off triangle is paved.

Evidence and response

Require the closed polygon and correctly placed pond, an explicit excluded-disk interpretation, the triangle subtraction and an effective-coverage ceiling checked on both neighboring integer counts. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: Use area and volume density with compatible units, correct region denominators, justified weights and explicit estimation assumptions.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for confusing a rate's dimensions despite a lucky number, O25 for cubic-unit conversion, O26 for treating quadratic growth as linear, or O27 for assuming an optimum exists without a bound. Use one visible model to rebuild the missing relationship, then remove it. Record supported corrections separately from the first response, and ask the learner to state why the repaired reasoning applies.

Core task

A rectangular wall is 9 m wide and 3 m tall, with two nonoverlapping unpainted windows, each 1.5 m by 1 m. Paint is applied at 0.2 L/m^2 per coat, and two coats are required. Find paintable area and liters needed, ignoring waste. A learner uses the wall perimeter instead of its area. Explain the dimensional mistake and calculate the amount their method would incorrectly report if they multiplied perimeter by 0.2 and by 2.

Teacher solution and discussion

Wall area is $9 \cdot 3 = 27 \text{ m}^2$, and the two windows total $2 \cdot 1.5 \cdot 1 = 3 \text{ m}^2$. Paintable area is 24 m^2 . Two coats use $24 \cdot 0.2 \cdot 2 = 9.6 \text{ L}$. The wall perimeter is $2(9+3) = 24 \text{ m}$, numerically equal to the paintable area but dimensionally different. Multiplying that perimeter by 0.2 L/m^2 and by 2 gives 9.6 L/m , not liters. Reporting 9.6 L from this method would be an accidental numerical match, not a valid model; it ignores the window geometry and lacks area units.

Evidence and response

Subtract both window areas and apply the areal rate twice, then recognize the deliberate numerical coincidence and use units to distinguish valid evidence from a lucky answer. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Build and interpret geometric formulas and algebraic relationships, solve for quantities, and justify physical domains and scaling.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 distinguishes fixed thickness from full similarity and requires both symbolic ratios. O29 must enumerate every integer factor pair and justify completeness before selecting the least area. In O30 collect two separate completed constructions and the two-sided optimality proof. A visually plausible disk or a successful candidate alone is insufficient. Decide the next task from the missing constraint, unit, model or proof evidence.

Core task

A square metal panel has side s and uniform thickness t , with both lengths positive. A new panel doubles each side length. Compare its mass to the original in two cases: thickness remains t , or thickness also doubles. Assume the same uniform volume density and negligible surface coatings. Derive both ratios symbolically and explain why a phrase such as “twice as large” is insufficient for predicting mass.

Teacher solution and discussion

Original volume is s^2t and original mass is ρs^2t . With side lengths doubled but thickness fixed, new mass is $\rho(2s)^2t=4\rho s^2t$, a ratio of 4. If thickness also doubles, mass is $\rho(2s)^2(2t)=8\rho s^2t$, a ratio of 8. The same material density cancels. “Twice as large” does not specify whether it concerns one length, both planar lengths, every dimension, area or volume, and these interpretations give different mass changes.

Evidence and response

Derive both mass ratios from all three dimensions, explicitly cancel the same positive density and identify the ambiguity in an unspecified size description. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A straight cable is modeled as a solid circular cylinder of length 80 cm and diameter 0.6 cm. Which expression models the volume of cable material, ignoring small surface grooves?

- A. $\pi(0.6)^2(80) \text{ cm}^3$.
- B. $\pi(0.3)^2(80) \text{ cm}^3$.
- C. $\pi(0.3)^2 \text{ cm}^3$.
- D. $2\pi(0.3)(80) \text{ cm}^3$.

Reasoning: _____

M02 | Warm-up | T1

A rectangular roof slopes from a ridge to an eave. One roof panel is 12 m long along the ridge and 5 m down the slope; its horizontal projection is 12 m by 4 m. Which area should be used to estimate coating for that panel, ignoring waste?

- A. 34 m^2 , twice the sum of 12 and 5.
- B. 48 m^2 , the horizontal footprint.
- C. 30 m^2 , half the panel rectangle.
- D. 60 m^2 , the actual sloping panel area.

Reasoning: _____

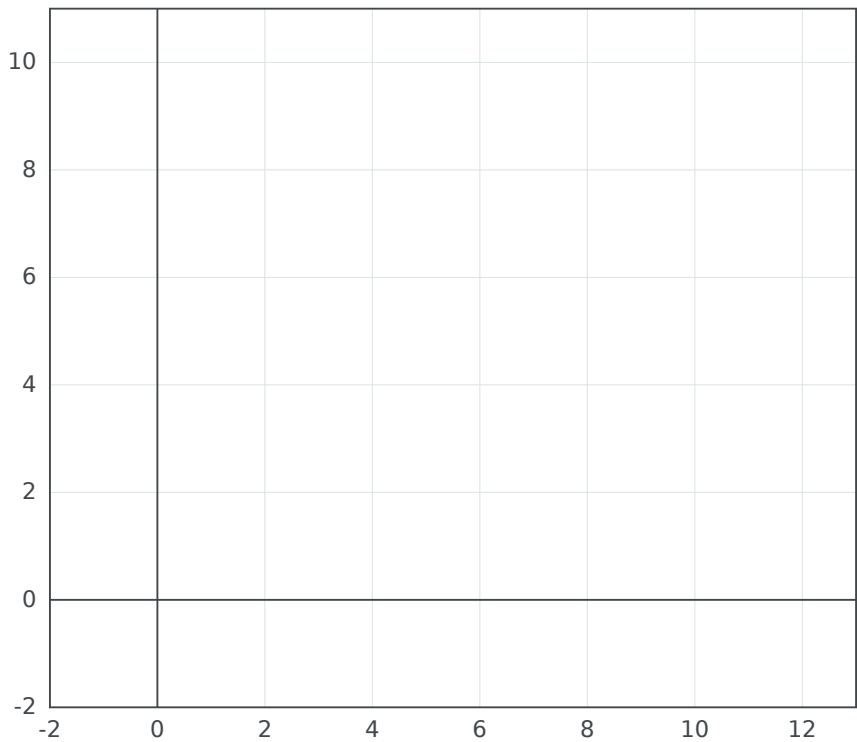
WARM-UP

001 • Show your reasoning

T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

An L-shaped garden is the entire interior of the polygon with vertices (0,0), (10,0), (10,4), (6,4), (6,8), (0,8), in meters. Draw and label the plan, partition it into two nonoverlapping rectangles, and find its area and outside perimeter. Explain why your partition segment should not be included when ordering edging for the garden.

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

A hollow bowl is approximated by concentric hemispheres with outer radius 6 cm and inner radius 5 cm, whose flat rims lie in the same plane. There is no lid. Find its modeled material volume and brim-full capacity. Explain which volume should be multiplied by material density to estimate the bowl's mass, and state one limitation of the idealized model.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Answer: B. $\pi(0.3)^2(80)$ cm³.

The diameter is 0.6 cm, so $r=0.3$ cm. A constant circular cross-section gives material volume $V=\pi r^2L=\pi(0.3)^2(80)=7.2\pi$ cm³. The model assumes a solid cable with constant radius and ignores grooves; a hollow cable would require subtracting a cavity.

Check each alternative

- A. This treats diameter as radius, making the modeled volume four times too large.
- B. Correct. The diameter is 0.6 cm, so $r=0.3$ cm. A constant circular cross-section gives material volume $V=\pi r^2L=\pi(0.3)^2(80)=7.2\pi$ cm³. The model assumes a solid cable with constant radius and ignores grooves; a hollow cable would require subtracting a cavity.
- C. This gives the circular cross-sectional area and omits the cable length.
- D. This is the lateral surface-area expression; its units are cm², not cm³.

Evidence to assess

Identify a solid constant-cross-section model, halve the diameter, and connect cross-sectional area times length to cubic units.

WORKED TEACHER KEYS

M02

M02 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Answer: D. 60 m^2 , the actual sloping panel area.

The coating covers the sloping rectangle, not its projection onto the ground. Its perpendicular in-panel dimensions are 12 m and 5 m, giving $12 \cdot 5 = 60 \text{ m}^2$. The horizontal 4 m measurement would be appropriate for a footprint question, not this surface treatment.

Check each alternative

- A. Twice the sum is the perimeter 34 m, not a coating area.
- B. The footprint is smaller than the sloping surface that receives the coating.
- C. The described panel is rectangular; there is no triangular half-area factor.
- D. Correct. The coating covers the sloping rectangle, not its projection onto the ground. Its perpendicular in-panel dimensions are 12 m and 5 m, giving $12 \cdot 5 = 60 \text{ m}^2$. The horizontal 4 m measurement would be appropriate for a footprint question, not this surface treatment.

Evidence to assess

Choose the requested physical surface and distinguish actual panel area from projected area and boundary length.

WORKED TEACHER KEYS

001

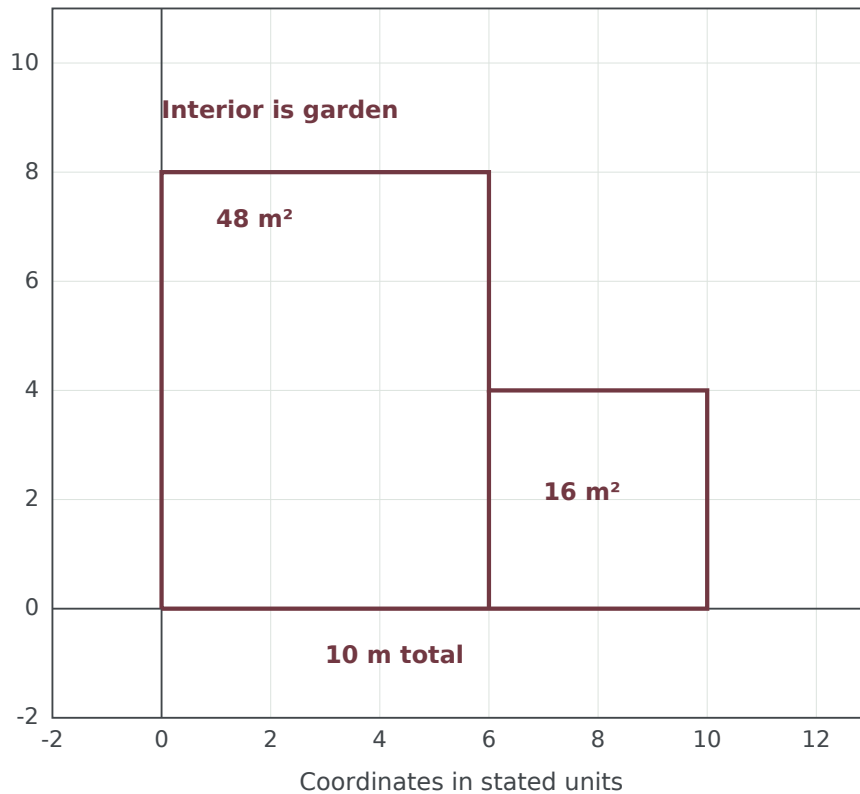
001 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.

Join the six vertices in order and close the polygon. Partition along the segment from (6,0) to (6,4). The left rectangle is 6 by 8, area 48 m^2 ; the lower-right rectangle is 4 by 4, area 16 m^2 . Total garden area is 64 m^2 . Outside edge lengths in order are 10,4,4,4,6,8 m, summing to 36 m. The shared 4 m partition is inside the garden, not part of its exterior boundary, so it contributes no edging. The completed key depicts the entire polygon interior as garden, not just its outline.

WORKED CONSTRUCTION CONTINUATION

001 • Construction 1**Completed plan and nonoverlap partition; all polygon interior is garden**

Plan view

**Evidence to assess**

Require a closed labeled L-shaped plan, an actual nonoverlap partition, both areas and every outside boundary length; reject counting the artificial internal seam as edging.

WORKED TEACHER KEYS

002 / 003**002 • T1: Choose and audit geometric models of real objects, distinguish measured regions and units, and state assumptions or limitations.**

A hemisphere has half a sphere's volume, or $(2/3)\pi r^3$. The exterior hemisphere encloses $144\pi \text{ cm}^3$ and the interior cavity holds $250\pi/3 \text{ cm}^3$. Material volume is their difference: $(2/3)\pi(216-125)=182\pi/3 \text{ cm}^3$. Brim-full capacity is $250\pi/3 \text{ cm}^3$, not the exterior volume. Material density multiplies $182\pi/3$ because the empty cavity contributes no bowl material. The model assumes truly hemispherical concentric surfaces with no feet or handles; a real flattened base or varying wall geometry could change the result.

Evidence to assess

Distinguish exterior enclosure, empty cavity and occupied material, apply the hemisphere factor consistently, and name a relevant geometric limitation rather than treating the idealization as exact observation.

003 • T3: Use area and volume density with compatible units, correct region denominators, justified weights and explicit estimation assumptions.

Areas are 600 m^2 and 300 m^2 . Estimated counts are $600 \cdot 1.2 = 720$ and $300 \cdot 0.8 = 240$, totaling 960 people. Total occupied area is 900 m^2 , so combined average density is $960/900 = 16/15 \approx 1.067$ people/ m^2 . The unweighted average 1.0 gives equal weight to unequal areas and would imply only 900 people. The area-weighted expression is $(600 \cdot 1.2 + 300 \cdot 0.8)/900$. Estimates assume each supplied rate represents its entire zone and the zones do not overlap; a local crowd hotspot need not have the overall average density.

Evidence to assess

Calculate both areas and estimated counts, form the weighted overall rate, quantify the error in an unweighted approach and identify a spatial-representativeness assumption.