

MATHEMATICS TEACHING AND ASSESSMENT

Explain the theorem and converse



Grade 8 • NC.8.G.6

Teacher models • Seven instructional sections
30 multiple-choice tasks • 30 open/performance tasks
Worked keys • Student workspaces • Observation tools

Reusable across several lessons. Select evidence purposefully; this is not a 60-item single-session assessment.

Free sample | Contents

NC.8.G.6 | Eighth Grade mathematics

This sample contains 18 complete selected pages from the 91-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-14, 29-30, 59, 74

Representative activities

M01: sample PDF page(s) 15; original page(s) 14

M02: sample PDF page(s) 15; original page(s) 14

O01: sample PDF page(s) 16; original page(s) 29

O02: sample PDF page(s) 17; original page(s) 30

Selected teacher prompts and keys: original page(s) 59, 74

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and scope

Explain the Pythagorean Theorem and its converse.

Current North Carolina Standard Course of Study, adopted June 2017. Source-verified for the 2026-2027 and 2027-2028 school years. Newly adopted standards begin in 2028-2029.

- Explain the theorem through areas and a general rearrangement proof.
- Explain the converse through a right comparison triangle and side correspondence.
- Distinguish assumptions, examples, counterexamples, and general justifications.

The focus is explanation, not only numerical calculation. Student prompts require original drawings, constructions, written arguments, and critique. Paper cutting is optional; a carefully labeled drawn arrangement is an accessible alternative.

PLANNING

Use the bank across sessions

Warm-up: M01-M04 and O01-O04.

I Do: M05-M08 and O05-O08.

We Do: M09-M13 and O09-O13.

Collaborative practice: M14-M18 and O14-O18.

Independent practice: M19-M23 and O19-O23.

Small-group reteach: M24-M27 and O24-O27.

Exit ticket: M28-M30 and O28-O30.

Suggested sequence: launch and model; shared reasoning; collaborative application; independent evidence and responsive reteach. A section may span several meetings. Choose 2-4 tasks for a session, then revisit remaining tasks as needed.

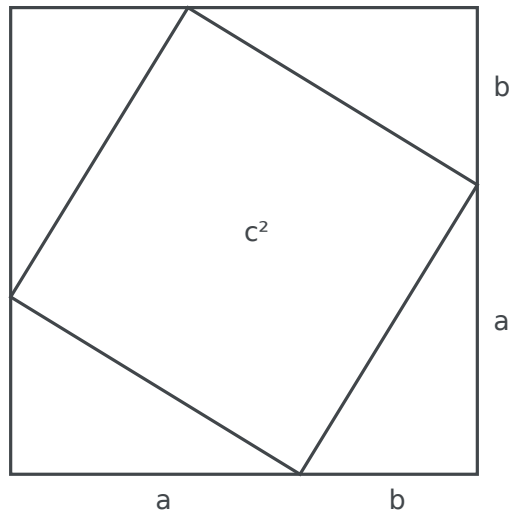
Materials: pencil, eraser, ruler, squared paper, calculator for checking approximations, scissors and plain paper for teacher-supervised models. For volume investigations, use waterproof solids with matching dimensions or the supplied numerical data. No purchased software is required.

Prerequisites: squares and square roots, positive lengths, area of a square/circle, coordinate differences, and consistent units. Check these orally before scored tasks. Read prompts aloud, enlarge diagrams, and offer tactile or high-contrast grids without supplying the relation being assessed.

TEACHER MODEL

An area proof for any right triangle

Let a and b be positive leg lengths and c the hypotenuse. Put four congruent right triangles in the corners of an outer square of side $a + b$. The drawing illustrates the arrangement; the letters represent arbitrary positive lengths.



Four congruent right triangles surround the inner square. Outer side: $a + b$.

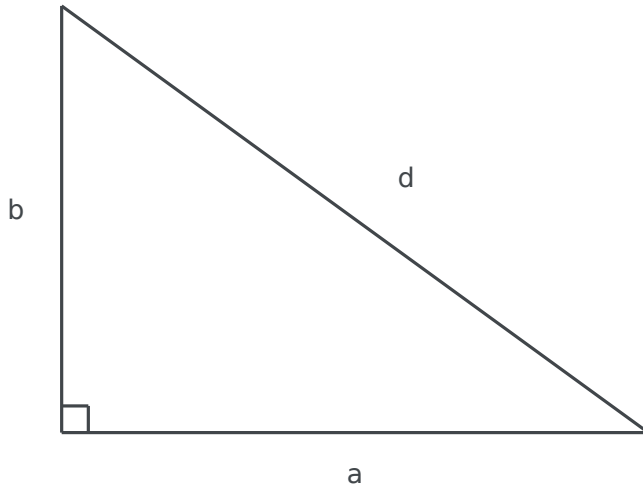
Each inner side is c . If the triangle's acute angles are α and β , then $\alpha + \beta = 90^\circ$. Each inner angle is $180^\circ - \alpha - \beta = 90^\circ$, so the center is a square. Outer area: $(a + b)^2$. Pieces: $4(ab/2) + c^2$. Therefore $a^2 + 2ab + b^2 = 2ab + c^2$. Subtract $2ab$ from both sides: $a^2 + b^2 = c^2$.

TEACHER MODEL

Why the converse also holds

Begin with a triangle of positive side lengths a , b , c satisfying $a^2 + b^2 = c^2$. Do not assume a right angle in that triangle. Construct a second triangle whose legs a and b are perpendicular; call its hypotenuse d .

Schematic; use labels, not measured lengths.



The theorem applies to the constructed right triangle: $d^2 = a^2 + b^2 = c^2$. Since c and d are positive, $d = c$. The two triangles have three matching side lengths, so SSS congruence makes corresponding angles equal. The angle opposite c in the original is therefore right. Anchor: theorem = right angle implies squared equality; converse = squared equality implies right angle. Numerical checks illustrate these statements but do not replace a general explanation.

REPRODUCIBLE ORGANIZER

Make reasoning visible

Name: _____ Date: _____

Givens and what I need to find**Representation and calculations****Justification or independent check****Conclusion with units and meaning**

TEACHING SEQUENCE

Warm-up

Optional bank: M01-M04; O01-O04. Core: O01. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Launch

Ask learners to read the givens and sketch the situation without calculating. Invite two explanations of what the unknown measures.

Teacher language

“Which quantities in this equation are areas, and which are lengths?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

A correct model has perpendicular 3- and 4-unit legs and a 5-unit hypotenuse opposite the right angle. Squares on these sides have areas 9, 16, and 25 square units; $9 + 16 = 25$. The equation concerns areas, not a sum of side lengths.

TEACHING SEQUENCE**I Do**

Optional bank: M05-M08; O05-O08. Core: O05. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Model aloud

Work through the core task, pausing before each equation. Say why the relation applies, what each symbol means, and how the conclusion answers the question.

Teacher language

“Where do the two cross terms in the outer-square area come from?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Place right-angle vertices at the four outer corners. Along each outer side, segments 3 and 4 sum to 7. Inner sides are hypotenuses, each 5. Adjacent acute angles total 90° , so each inner angle is 90° . Outer area 49 minus four triangle areas $4(3 \times 4/2) = 24$ leaves 25. Algebraically $49 = 9 + 24 + 16$, so subtracting 24 yields $9 + 16 = 25$.

TEACHING SEQUENCE

We Do

Optional bank: M09-M13; O09-O13. Core: O09. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Share the work

Ask partners to propose the first representation, then compare equations. Record both a valid approach and a plausible error; have learners explain the difference.

Teacher language

“How can we build a right triangle without assuming the original triangle is right?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

The constructed triangle is right by construction, so the theorem gives $d^2 = a^2 + b^2$. The given equality says the same sum is c^2 . Thus $d^2 = c^2$, and positivity implies $d = c$. The first triangle's angle is not used as a premise.

TEACHING SEQUENCE

Collaborative practice

Optional bank: M14-M18; O14-O18. Core: O14. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Partner roles

One learner constructs or labels the model; the other checks the equation and units. Switch roles on the next task. Both write an individual explanation.

Teacher language

“What must remain unchanged when pieces are rearranged?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

$4+5>6$ and the other two triangle inequalities hold, so these lengths form a triangle. $4^2+5^2=41 \neq 36=6^2$. The equality fails, so the triangle is not right. Accept other valid nondegenerate counterexamples with complete checks.

TEACHING SEQUENCE

Independent practice

Optional bank: M19-M23; O19-O23. Core: O19. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Collect independent evidence

Give the core prompt without an example visible. Preserve the first response. Ask only neutral clarifications such as “What does this label represent?”

Teacher language

“Does this establish every case or merely illustrate one?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Scaled sides are 7.5,10,12.5. Squares are 56.25,100,156.25, and $56.25+100=156.25$. Generally $(ka)^2+(kb)^2=k^2(a^2+b^2)=k^2c^2=(kc)^2$.

TEACHING SEQUENCE

Small-group reteach

Optional bank: M24-M27; O24-O27. Core: O24. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Diagnose and reteach

Use one previously observed error to select a model. Reconnect the diagram to each term, then remove the model and offer a different item in this range.

Teacher language

“Can your chart distinguish the starting assumption from the conclusion?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Theorem panel begins with a right angle and finds a squared relation, e.g. $5^2 + 12^2 = 13^2$; warn that c is the hypotenuse. Converse panel begins with side lengths and verifies a right angle, e.g. $8^2 + 15^2 = 17^2$; warn to test the longest side and not assume a right angle from the picture.

TEACHING SEQUENCE

Exit ticket

Optional bank: M28-M30; O28-O30. Core: O28. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Close and decide

Ask learners to complete the core task independently and explain one check. Use the remaining exit tasks later to distinguish a one-time slip from a persistent gap.

Teacher language

“Show the full logical chain, including the geometry behind each equality.”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Four congruent right triangles form an outer square of side $a+b$ and inner square of side c . Each inner angle is $180^\circ - (\alpha + \beta) = 90^\circ$ since $\alpha + \beta = 90^\circ$. Areas give $(a+b)^2 = 4(ab/2) + c^2 = 2ab + c^2$. Expand to $a^2 + 2ab + b^2 = 2ab + c^2$ and subtract $2ab$. Thus $a^2 + b^2 = c^2$.

MULTIPLE-CHOICE BANK

M01 - M02**M01 | Warm-up**

Target: State the theorem precisely.

Which statement gives the Pythagorean Theorem for a right triangle with legs a , b and hypotenuse c ?

- A. $a^2 + c^2 = b^2$
 - B. $a + b = c$
 - C. $a^2 + b^2 = c^2$
 - D. $a^2 - b^2 = c^2$
-

M02 | Warm-up

Target: Identify the hypotenuse.

Triangle PQR has a right angle at Q. Which side is the hypotenuse?

- A. PR
 - B. No side can be identified
 - C. QR
 - D. PQ
-

WARM-UP

001 | Show your reasoning

Target: Explain the theorem with a constructed model.

Draw a right triangle with legs 3 and 4 units. Mark the right angle, determine the hypotenuse, and draw a square on each side. Label each square's area and explain the area relationship.

Model, construction, or calculations

Explanation, check, and conclusion

WARM-UP

002 | Show your reasoning

Target: State theorem and converse in your own words.

Write the theorem and its converse in two separate if-then sentences. Include the role of the longest side and identify which sentence starts with a right angle.

Model, construction, or calculations

Explanation, check, and conclusion

WORKED KEYS

M01 - M02**M01 | Warm-up**

Target: State the theorem precisely.

Answer: C • $a^2 + b^2 = c^2$

The square on the hypotenuse has area equal to the sum of the squares on the two legs.

A: This treats b as hypotenuse even though c is specified.

B: This would make a degenerate straight segment, not a triangle.

D: This subtracts squared legs rather than adding them.

A selection alone does not establish a misconception; ask for the reasoning.

M02 | Warm-up

Target: Identify the hypotenuse.

Answer: A • PR

PR is opposite Q, the right angle. The hypotenuse is also the longest side.

B: The side opposite the stated right angle is determined.

C: QR touches the right angle and is a leg.

D: PQ touches the right angle and is a leg.

A selection alone does not establish a misconception; ask for the reasoning.

WORKED KEYS

001 - 002**001 | Warm-up**

Target: Explain the theorem with a constructed model.

Result: 5 units; $9 + 16 = 25$

A correct model has perpendicular 3- and 4-unit legs and a 5-unit hypotenuse opposite the right angle. Squares on these sides have areas 9, 16, and 25 square units; $9 + 16 = 25$. The equation concerns areas, not a sum of side lengths.

Assess the requested representation, reasoning, accuracy, and interpretation separately. Equivalent exact forms and valid alternative constructions are acceptable.

002 | Warm-up

Target: State theorem and converse in your own words.

Result: Two valid implications

Theorem: If a triangle is right with legs a, b and hypotenuse c , then $a^2 + b^2 = c^2$. Converse: If a triangle has positive sides a, b, c with c longest and $a^2 + b^2 = c^2$, then the angle opposite c is right. The theorem starts with the right-angle assumption.

Assess the requested representation, reasoning, accuracy, and interpretation separately. Equivalent exact forms and valid alternative constructions are acceptable.