

TEACHER AND STUDENT RESOURCE

Restrict the sample space



NC Math 2 • NC.M2.S-CP.6

Conditional Probability and the Rules for Probability

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.S-CP.6 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

T1 Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

T2 Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

T3 Construct, compare and critique conditional models, including the equal-likelihood and positive-denominator requirements.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M05, M06, M08, M09, M12, M13, M14, M16, M18, M23, M24, M25, O01, O03, O08, O09, O12, O13, O15, O18, O19, O24

T2: M04, M07, M15, M17, M21, M22, M30, O02, O05, O06, O16, O20, O22, O27, O28, O30

T3: M10, M11, M19, M20, M26, M27, M28, M29, O04, O07, O10, O11, O14, O17, O21, O23, O25, O26, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Learning target: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

Learning target: Construct, compare and critique conditional models, including the equal-likelihood and positive-denominator requirements.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Learning target: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

Learning target: Construct, compare and critique conditional models, including the equal-likelihood and positive-denominator requirements.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Read set membership, fractions and simple tables; distinguish ordered outcomes from categories that contain several outcomes. Use familiar deck facts only where they are explicitly stated.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All finite count models in this packet select individual outcomes uniformly unless an item explicitly critiques an unequal-outcome model. A standard deck has 52 cards, 13 in each of four suits, two red suits and three face-card ranks. Independent fair tosses and dice make ordered elementary outcomes equally likely. The fraction-of-outcomes definition requires a nonempty condition.

TEACHER MODEL

Cross out the impossible keep the weights

A fair die is known to show at least 4. Find the chance of an even result by listing the outcomes that remain.

Original face	Meets known condition?	Even within condition?
1	No	Not retained
2	No	Not retained
3	No	Not retained
4	Yes	Yes
5	Yes	No
6	Yes	Yes

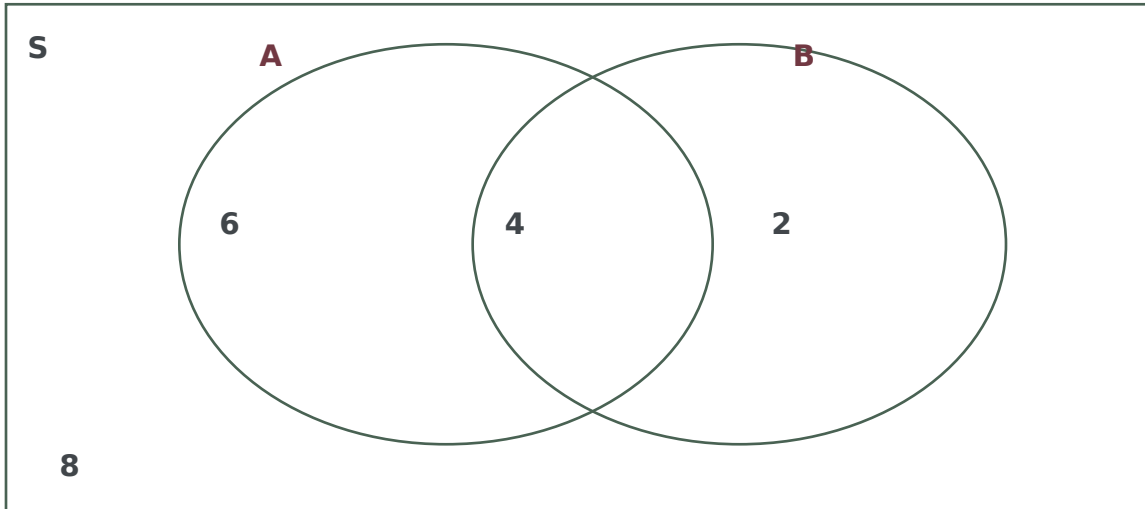
All six faces were equally likely. Restriction retains three equally likely individual outcomes.

The known group is $B = \{4, 5, 6\}$. Its favorable A outcomes are 4 and 6. Hence $P(A|B) = |A \text{ and } B|/|B| = 2/3$. Say: Among die results known to be at least 4, two thirds are even. The $2/6$ all-die fraction describes the joint event, not the requested conditional chance.

VISUAL ANCHOR

The condition becomes the whole group

Suppose a uniform population has A-only 6, both 4, B-only 2 and neither 8. The given event is B.



Numbers are counts of equally likely individual outcomes, not probabilities of entire regions.

Only the 4 shared outcomes and 2 B-only outcomes survive the condition.

$P(A|B) = 4/(4+2) = 2/3$. The A-only outcomes do not count in the numerator because they fail the known condition. The outside outcomes do not count in the denominator.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Sample space, events and known information**Representation and calculation****Justification or check****Conclusion in context**

WARM-UP

Teaching script

Core: 001 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask learners to circle the words after given. Name the surviving individual outcomes before writing a fraction. Check that the condition is not silently reversed.

Core task

A fair die is known to show an odd number. List the conditional sample space, find the chance of a prime number, and compare it with the prime chance before the information. Remember that 1 is not prime.

Teacher solution and discussion

Given odd, the space is $\{1, 3, 5\}$. Prime outcomes are 3 and 5, so $P(\text{prime}|\text{odd})=2/3$. Before conditioning, primes are 2, 3, 5 among six faces, giving $1/2$. The information increases the favorable proportion even though fewer outcomes remain.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model crossing out outcomes incompatible with the information. State why retained elementary outcomes remain equally likely. Contrast the conditional fraction with the original joint fraction.

Core task

A card is drawn uniformly from a standard 52-card deck. Find queen given red and red given queen. Name the favorable cards and interpret both fractions.

Teacher solution and discussion

The shared cards are the queen of hearts and queen of diamonds. Among 26 red cards, 2 are queens, so $P(\text{queen}|\text{red})=2/26=1/13$. Among 4 queens, 2 are red, so $P(\text{red}|\text{queen})=2/4=1/2$. The reference groups are red cards and queens, respectively.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Build lists, grids and membership tables together. Ask whether every listed outcome meets the condition and whether any has been counted twice. Require an interpretation sentence.

Core task

A code is chosen uniformly from all two-letter strings made with A, B, C, allowing repeats. Given that its two letters are the same, find the probability it contains A. Construct an outcome grid and identify the retained cells.

Teacher solution and discussion

The 9 codes are AA, AB, AC, BA, BB, BC, CA, CB, CC. The same-letter condition retains AA, BB, CC. Only AA contains A, so the requested chance is $\frac{1}{3}$. The completed grid marks retained cells with an asterisk.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Construct, compare and critique conditional models, including the equal-likelihood and positive-denominator requirements.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Use a constructor, denominator checker, uniformity checker and interpreter. Compare two conditions on the same experiment and two different models sharing the same marginal data.

Core task

A group B contains 13 equally likely individual records. Can $P(A|B)$ equal exactly $2/5$? If not, find the attainable fraction closest to $2/5$ and explain why.

Teacher solution and discussion

An exact $2/5$ would require $(2/5) \times 13 = 26/5 = 5.2$ favorable records, not an integer. The nearest integer numerator is 5, giving $5/13$. Its distance from $2/5$ is $1/65$; $6/13$ is $4/65$ away, and other integer numerators are farther. Thus $5/13$ is closest, but not equal.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect a complete model and explanation before prompting. Look for transfer to reversed time order, complements, multiple categories and practical choices.

Core task

Tokens R1, R2, B1, B2 are drawn uniformly in order without replacement. This time you are told the second token was blue. List the retained ordered outcomes and find the probability the first token was red. Why need the condition not be earlier in time?

Teacher solution and discussion

Retained draws are R1B1, R2B1, B2B1, R1B2, R2B2, B1B2. Four of these six start red, so $P(\text{first red}|\text{second blue}) = 4/6 = 2/3$. Conditioning uses known information to restrict possible histories; it does not require the known event to happen first.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use a small physical set and visibly remove outcomes that fail the condition. Contrast no favorable outcomes in a nonempty group with an impossible, empty condition.

Core task

A uniformly selected card is numbered 1 through 6. Given that its number exceeds 2, a learner says the prime chance is $\frac{2}{6}$. Correct the fraction by enumerating outcomes and interpret both the correct answer and $\frac{2}{6}$.

Teacher solution and discussion

The known group is $\{3, 4, 5, 6\}$; primes are 3 and 5. Thus the conditional chance is $\frac{2}{4} = \frac{1}{2}$. The fraction $\frac{2}{6} = \frac{1}{3}$ is the all-card chance of both exceeding 2 and being prime. It retains the original denominator and therefore is joint rather than conditional.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Require a new contextual interpretation, a reason for the denominator and a critique of a tempting joint or reversed fraction. Use the evidence to choose targeted follow-up.

Core task

A student must choose one of two eligible-coupon pools, then select uniformly within that pool. Pool A contains 4 eligible coupons, 3 usable today. Pool B contains 10 eligible coupons, 5 usable today. Which gives the larger chance of a usable coupon, and why is comparing 3 with 5 alone misleading?

Teacher solution and discussion

Within eligible pool A, the usable fraction is $\frac{3}{4}=75\%$; within B it is $\frac{5}{10}=50\%$. Choose A for the larger conditional chance. Comparing favorable counts alone ignores that pool B also has a much larger denominator. The selection is specifically within each eligible pool.

Evidence and response

Look for a named restricted set, a matching favorable subset and an interpreted fraction. Do not accept a correct number with the wrong denominator story. Compare different information statements carefully; construction tasks must show valid nonnegative integer counts and complete totals.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A fair die is known to show a number greater than 3. What fraction of the remaining outcomes are even?

- A. $\frac{2}{3}$
- B. $\frac{1}{6}$
- C. $\frac{1}{2}$
- D. $\frac{1}{3}$

Reasoning: _____

M02 | Warm-up | T1

Twenty equally likely tickets include 8 in event B and 3 in both A and B. What is $P(A|B)$?

- A. $\frac{3}{8}$
- B. $\frac{3}{20}$
- C. $\frac{8}{20}$
- D. $\frac{8}{3}$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

A fair die is known to show an odd number. List the conditional sample space, find the chance of a prime number, and compare it with the prime chance before the information. Remember that 1 is not prime.

Representation / calculation

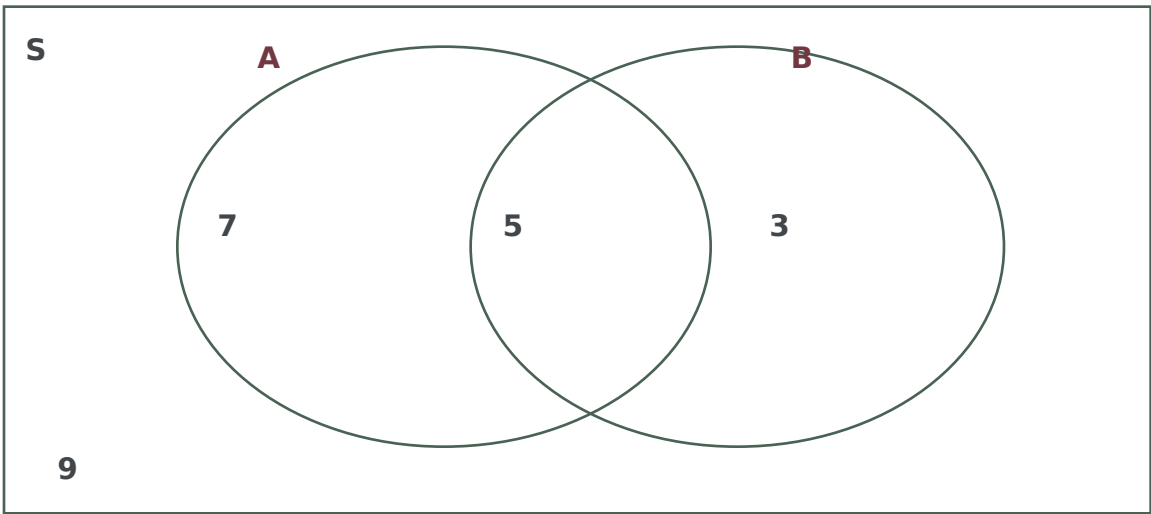
Explanation / check

WARM-UP

002 • Show your reasoning

T2: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

An outcome is selected uniformly from the population in the diagram. Compute $P(A|B)$ and $P(B|A)$, and explain why the shared numerator does not make the answers equal.



Numbers are counts of equally likely individual outcomes, not probabilities of entire regions.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Answer: A. $\frac{2}{3}$

The known outcomes are $\{4, 5, 6\}$. Of those three, 4 and 6 are even, so the conditional chance is $\frac{2}{3}$.

Check each alternative

- A. The known outcomes are $\{4, 5, 6\}$. Of those three, 4 and 6 are even, so the conditional chance is $\frac{2}{3}$.
- B. $\frac{1}{6}$ is the chance of one particular face in the unrestricted space.
- C. $\frac{1}{2}$ uses the whole die before the information.
- D. One of the three is odd, not even.

Evidence to assess

Require the restricted three-outcome set and two favorable outcomes.

WORKED TEACHER KEYS

M02

M02 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Answer: A. $\frac{3}{8}$

Given B, only its 8 tickets remain possible. Three also belong to A, so $P(A|B)=\frac{3}{8}$.

Check each alternative

- A. Given B, only its 8 tickets remain possible. Three also belong to A, so $P(A|B)=\frac{3}{8}$.
- B. $\frac{3}{20}$ is the joint probability among all tickets.
- C. $\frac{8}{20}$ is the probability of the condition B.
- D. $\frac{8}{3}$ reverses favorable and possible counts and exceeds 1.

Evidence to assess

Require overlap divided by the B count.

WORKED TEACHER KEYS

001

001 • T1: Identify the nonempty set of outcomes satisfying the condition and calculate its favorable fraction in a uniform model.

Given odd, the space is $\{1, 3, 5\}$. Prime outcomes are 3 and 5, so $P(\text{prime}|\text{odd})=2/3$. Before conditioning, primes are 2, 3, 5 among six faces, giving $1/2$. The information increases the favorable proportion even though fewer outcomes remain.

Evidence to assess

Item evidence: restricted set; two correct primes; $2/3$ and $1/2$; contextual comparison. Award one point per named requirement, up to 4; accept equivalent exact fractions and correctly labeled alternative representations.

WORKED TEACHER KEYS

002

002 • T2: Interpret a conditional fraction in context and distinguish it from joint, marginal and reversed probabilities.

B contains $5+3=8$ outcomes, so $P(A|B)=5/8$. A contains $7+5=12$ outcomes, so $P(B|A)=5/12$. Both use the overlap 5, but compare it with different known groups.

Evidence to assess

Item evidence: B count 8; A count 12; two exact fractions; denominator interpretation. Award one point per named requirement, up to 4; accept equivalent exact fractions and correctly labeled alternative representations.