

TEACHER AND STUDENT RESOURCE

Compare Long-Run Function Growth



NC Math 1 • NC.M1.F-LE.3

Building and Comparing Function Models

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.F-LE.3 | NC Math 1 mathematics

This sample contains 22 complete selected pages from the 122-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 35-38, 80, 95-96

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17-18; original page(s) 35-36

O02: sample PDF page(s) 19-20; original page(s) 37-38

Selected teacher prompts and keys: original page(s) 80, 95-96

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

T1 Compare linear and exponential values in tables and graphs.

T2 Compare quadratic and exponential values in tables and graphs.

T3 Explain eventual dominance and distinguish it from early behavior.

T4 Use suitable windows, exact tables and growth-factor reasoning.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M03, M07, O07, M09, M13, O13, M15, M19, O19, M21, M25, O25, M27

T2: M02, O02, M08, O08, M14, O14, M20, O20, M26, O26

T3: O03, M04, O04, M06, O09, M10, O10, M12, O15, M16, O16, M18, O21, M22, O22, M24, O27, M28, O28, M30

T4: M05, O05, O06, M11, O11, O12, M17, O17, O18, M23, O23, O24, M29, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Notice and represent

Learning target: Compare linear and exponential values in tables and graphs.

Use M01–M04 and selected O01–O04 as a diagnostic. Model O05; ask learners to check its rule with the supplied data.

Session 2: Build and justify

Learning target: Compare quadratic and exponential values in tables and graphs.

Learning target: Explain eventual dominance and distinguish it from early behavior.

Choose from M09–M18 and O09–O18. Partners compare representations, check every given condition, and explain disagreements.

Session 3: Apply and revisit

Learning target: Use suitable windows, exact tables and growth-factor reasoning.

Collect independent evidence from M19–M23 and O19–O23. Use M24–M27 and O24–O27 for fresh reteaching examples; select an exit task from each bank.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. For the optional digital comparisons, use a school-approved graphing calculator or spreadsheet; no paid service or account is required. Enter the supplied rules and tables and record your own results.

Prerequisites

Check signed arithmetic, ratios, exponents and ordered pairs. Distinguish a quantity from its change. Review arithmetic and geometric patterns before scoring evidence.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

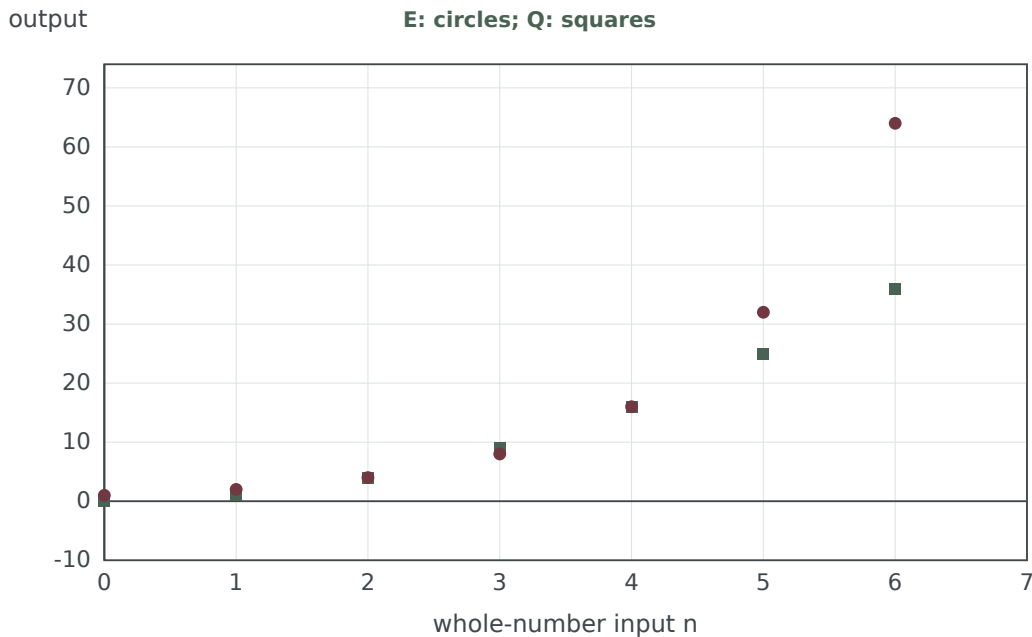
Review convention

Compare end behavior as the input grows without bound, not as it becomes very negative. Positive exponential growth $a \cdot b^x$ ($a > 0, b > 1$) eventually exceeds a fixed linear or quadratic polynomial. A finite table can show a crossing but does not by itself prove all future behavior. In this bank whole-number sampling is stated explicitly; graph points are not connected. Optional digital exploration must use a school-approved tool and retain a table or screenshot as learner-produced evidence.

TEACHER MODEL

A narrow window can hide an overtake

Compare $E(n)=2^n$ and $Q(n)=n^2$ at whole-number inputs 0-6. Does the early leader remain ahead?



E gives 1,2,4,8,16,32,64; Q gives 0,1,4,9,16,25,36. They tie at $n=2$ and 4; Q is larger at $n=3$, while E is larger from $n=5$ in this table. Beyond $n=4$, Q's successive multiplier $((n+1)/n)^2$ is at most $25/16$, less than E's multiplier 2. Since E is already larger at 5 and then grows by a larger factor, it remains larger for all later whole-number inputs. A finite graph alone would not prove that.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Compare linear and exponential values in tables and graphs.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Use $E(n)=2 \cdot 2^n$ and $L(n)=14n+7$ for whole $n=0, \dots, 7$. Fill a table, plot both sets of points on the same axes, and identify the first input in this table where E exceeds L . Explain why the first input must be checked rather than guessed from one point.

Teacher solution and discussion

E values: 2, 4, 8, 16, 32, 64, 128, 256. L values: 7, 21, 35, 49, 63, 77, 91, 105. The first $E > L$ in this table is $n=6$. Check every earlier listed input. Exponential growth eventually dominates, but this finite comparison establishes only the listed orderings.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T4: Use suitable windows, exact tables and growth-factor reasoning.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Use a school-approved graphing calculator or spreadsheet, or exact hand tables, to compare $E(n)=2 \cdot 2^n$ and $Q(n)=7n^2$ for integers 0–10. Record your table and two window choices: 0–4 and 0–10. Sketch both models in the larger window and explain what the smaller window can hide. If using technology, enter n , E and Q in separate columns and record the actual tool and settings.

Teacher solution and discussion

E values for 0–10: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048. Q values: 0, 7, 28, 63, 112, 175, 252, 343, 448, 567, 700. Use horizontal 0–10 and vertical 0–2148 (or another range containing both). The shorter 0–4 window may miss the later overtake. The large exponential values can visually flatten small values; use the exact table to resolve near crossings. No specific software run is asserted here.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Explain eventual dominance and distinguish it from early behavior.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Investigate $E(n)=3 \cdot 2^n$ and $L(n)=6n+30$ at whole $n \geq 0$. Find the first strict overtake with an exact table. Then show it cannot reverse afterward by comparing $E(n+1)-L(n+1)$ with $E(n)-L(n)$.

Teacher solution and discussion

Values through $n=5$: $E=3, 6, 12, 24, 48, 96$; $L=30, 36, 42, 48, 54, 60$. First strict overtake is $n=5$. Let $D(n)=E(n)-L(n)$. $D(n+1)-D(n)=E(n)-6$. At and after the overtake, $E(n)>6n+30>6$, so D increases from a positive value. Thus E stays above L for every later whole-number input.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Compare quadratic and exponential values in tables and graphs.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Compare $E(n)=3 \cdot 2^n$ and $Q(n)=4n^2$ for whole $n=1, \dots, 9$. Make a table and a graph. Find the earliest listed n from which E stays strictly above Q for every remaining listed input. Explain why this differs from asking whether E is ever above Q .

Teacher solution and discussion

E values for 1–9: 6, 12, 24, 48, 96, 192, 384, 768, 1536. Q values: 4, 16, 36, 64, 100, 144, 196, 256, 324. The requested starting index is 6. Check all later listed rows, and note any early equality or reversal. For $n \geq 3$, $Q(n+1)/Q(n) = ((n+1)/n)^2 \leq 16/9 < 2$; once E is ahead there, it cannot fall behind at a later integer.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Compare linear and exponential values in tables and graphs.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Use $E(n)=5 \cdot 2^n$ and $L(n)=20n+7$ for whole $n=0, \dots, 7$. Fill a table, plot both sets of points on the same axes, and identify the first input in this table where E exceeds L . Explain why the first input must be checked rather than guessed from one point.

Teacher solution and discussion

E values: 5, 10, 20, 40, 80, 160, 320, 640. L values: 7, 27, 47, 67, 87, 107, 127, 147. The first $E > L$ in this table is $n=5$. Check every earlier listed input. Exponential growth eventually dominates, but this finite comparison establishes only the listed orderings.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Use suitable windows, exact tables and growth-factor reasoning.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

For whole $n=0,2,4,6,8$, compare $E(n)=5 \cdot 3^n$, $L(n)=12n+25$ and $Q(n)=6n^2$. Make a table, identify the largest at each listed input, and explain why a fixed positive multiplier on Q would delay rather than prevent an exponential overtake.

Teacher solution and discussion

E: 5, 45, 405, 3645, 32805. L: 25, 49, 73, 97, 121. Q: 0, 24, 96, 216, 384. Largest at $n=0,2,4,6,8$: L, L, E, E, E. Exponential growth with base 3 eventually exceeds any fixed quadratic coefficient, even if a larger coefficient keeps Q ahead longer. A finite table illustrates but does not prove the general claim.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Explain eventual dominance and distinguish it from early behavior.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Explain why the statement “ $Q(n)=8n^2$ is larger than $E(n)=6 \cdot 2^n$ at one early input, so Q wins forever” is invalid. Supply an exact later input where $E > Q$, and explain why E remains greater after a suitable overtake.

Teacher solution and discussion

One input does not establish end behavior. At $n=10$, $E=6144$ and $Q=800$, so $E > Q$. For $n \geq 3$, $Q(n+1)/Q(n) = ((n+1)/n)^2 \leq 16/9 < 2$, while E doubles. Since E is ahead at 10 and has the larger multiplier thereafter, it remains ahead for all later integer inputs. A table and graph through 10 illustrate the reversal; the multiplier argument extends it.

Evidence and response

Look for the stated initial condition, correct equal-interval change, and a model that matches every supplied observation. Ask learners to test their rule at two known inputs. If a learner adds where the situation multiplies, compare successive differences and ratios using actual data, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

For whole-number n , $E(n)=1 \cdot 2^n$ and $L(n)=9n+4$. At $n=6$, what is $E(n)-L(n)$?

- A. 6
- B. 122
- C. -6
- D. 15

Reasoning: _____

M02 | Warm-up | T2

At $n=8$, compare $E(n)=1 \cdot 2^n$ with $Q(n)=3n^2+2$. Which is $E(n)-Q(n)$?

- A. -62
- B. 107
- C. 450
- D. 62

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Compare linear and exponential values in tables and graphs.

Use $E(n)=2\cdot 2^n$ and $L(n)=14n+7$ for whole $n=0,\dots,7$. Fill a table, plot both sets of points on the same axes, and identify the first input in this table where E exceeds L. Explain why the first input must be checked rather than guessed from one point.

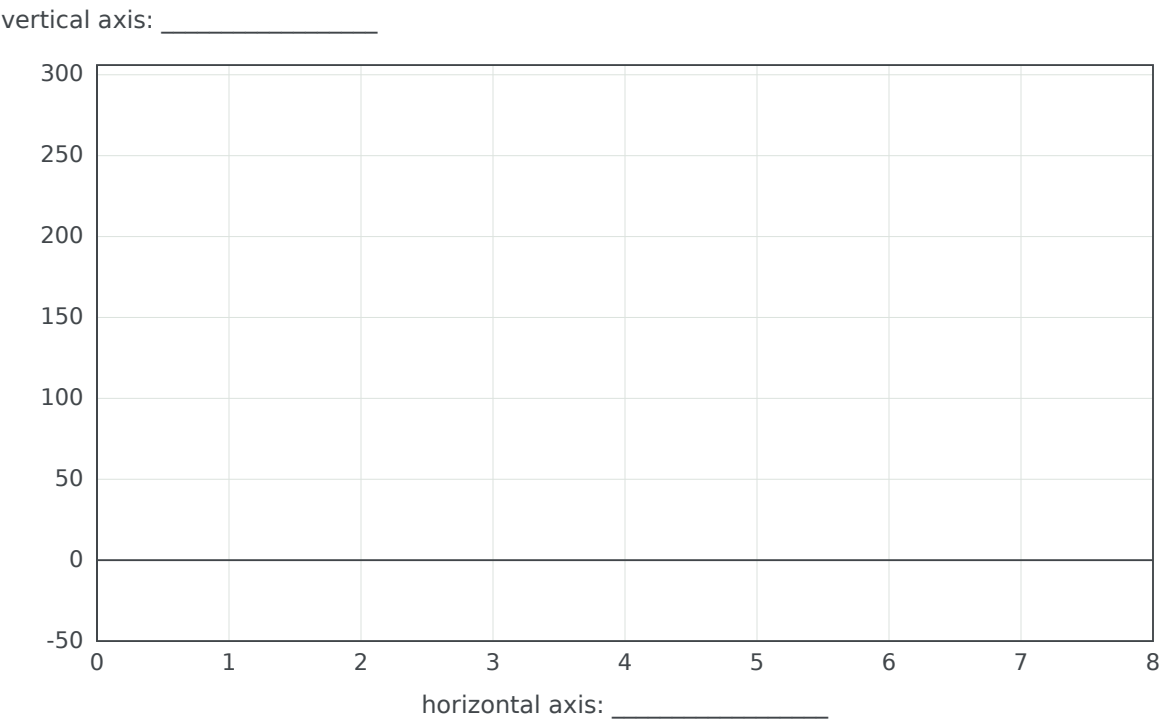
n	E(n)	L(n)

Table observations and comparison

WARM-UP

001 • Graph workspace

Use your table from the preceding page. Label both axes and every model.



Explanation and check

WARM-UP

002 • Show your reasoning

T2: Compare quadratic and exponential values in tables and graphs.

Compare $E(n)=1\cdot 2^n$ and $Q(n)=2n^2$ for whole $n=1,\dots,9$. Make a table and a graph. Find the earliest listed n from which E stays strictly above Q for every remaining listed input. Explain why this differs from asking whether E is ever above Q .

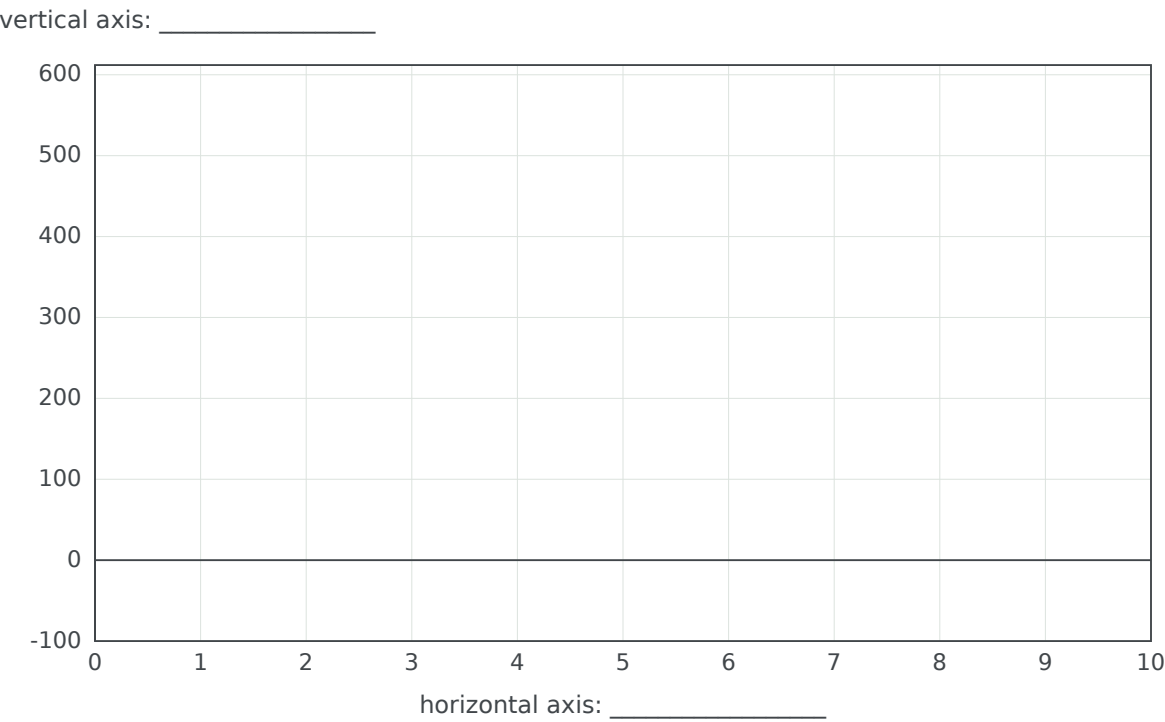
n	E(n)	Q(n)

Table observations and comparison

WARM-UP

002 • Graph workspace

Use your table from the preceding page. Label both axes and every model.



Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Compare linear and exponential values in tables and graphs.****Answer: A. 6** $E(6)=64$; $L(6)=58$. Difference= $64-58=6$. At this input E is larger.**Check each alternative**

- A. Evaluate both models at the requested input, then subtract.
- B. This adds the outputs instead of subtracting.
- C. This reverses the subtraction.
- D. This uses mismatched inputs: $E(6) - L(5) = 64 - 49 = 15$.

M02 • T2: Compare quadratic and exponential values in tables and graphs.**Answer: D. 62** $E(8)=1 \cdot 256=256$; $Q(8)=3 \cdot 64+2=194$. $E-Q=62$. A larger exponential value at this input is consistent with its long-run dominance.**Check each alternative**

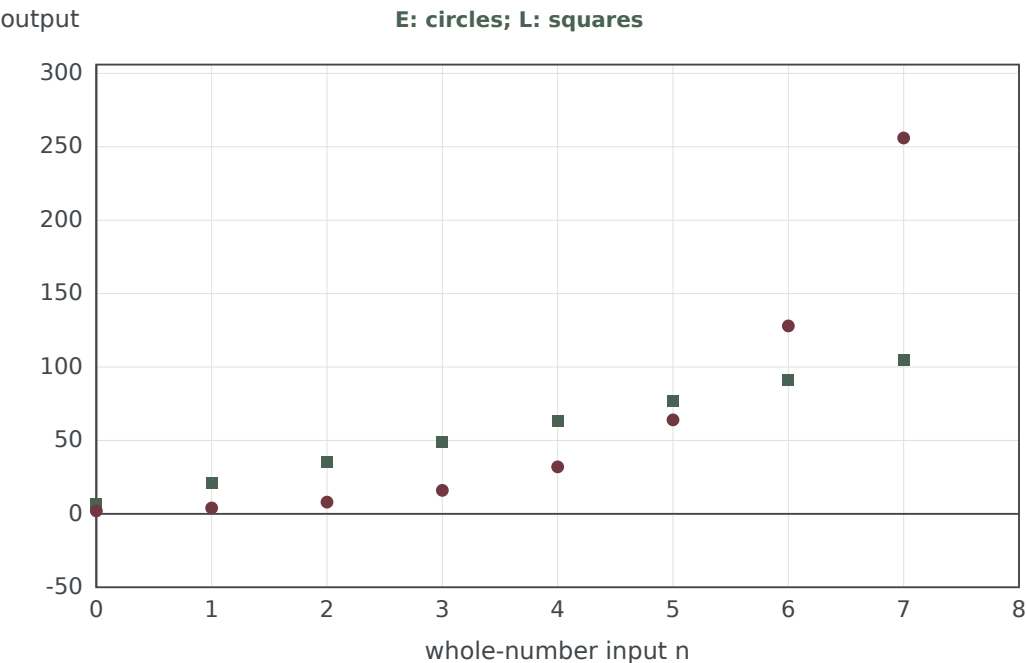
- A. This reverses the subtraction.
- B. This uses mismatched inputs: $E(8) - Q(7) = 256 - 149 = 107$.
- C. This adds the outputs instead of subtracting.
- D. Evaluate both models at the requested input, then subtract.

WORKED TEACHER KEYS

001

001 • T1: Compare linear and exponential values in tables and graphs.

E values: 2, 4, 8, 16, 32, 64, 128, 256. L values: 7, 21, 35, 49, 63, 77, 91, 105. The first $E > L$ in this table is $n=6$. Check every earlier listed input. Exponential growth eventually dominates, but this finite comparison establishes only the listed orderings.



Evidence to assess

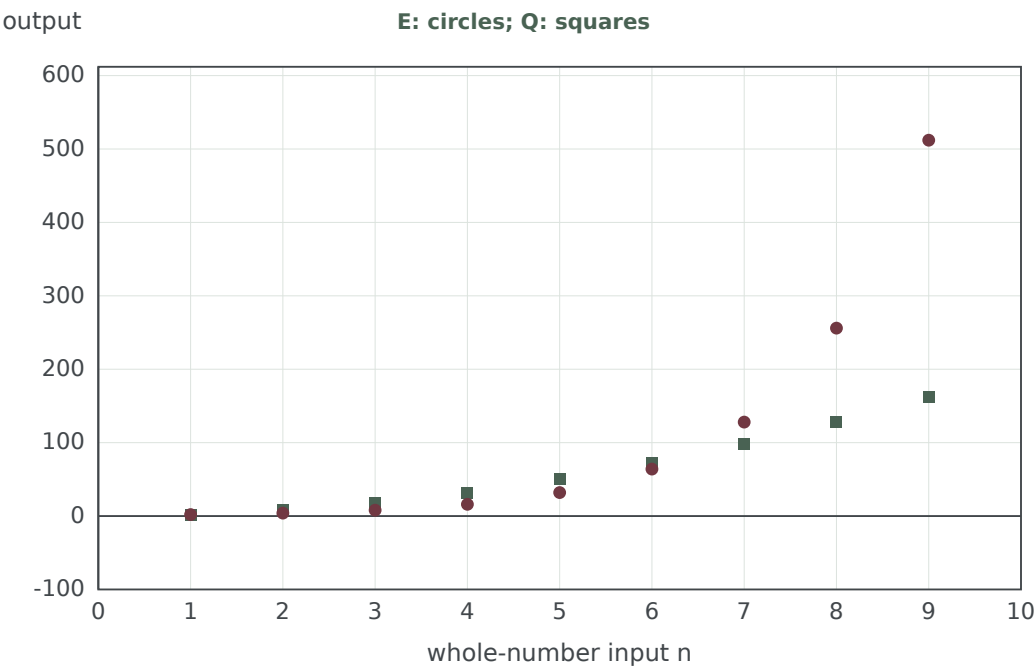
Score model/representation, calculation, reasoning and contextual interpretation separately. Require the stated initial condition, index or domain, and units where relevant. Accept equivalent formulas and valid alternative checks.

WORKED TEACHER KEYS

002

002 • T2: Compare quadratic and exponential values in tables and graphs.

E values for 1-9: 2, 4, 8, 16, 32, 64, 128, 256, 512. Q values: 2, 8, 18, 32, 50, 72, 98, 128, 162. The requested starting index is 7. Check all later listed rows, and note any early equality or reversal. For $n \geq 3$, $Q(n+1)/Q(n) = ((n+1)/n)^2 \leq 16/9 < 2$; once E is ahead there, it cannot fall behind at a later integer.



Evidence to assess

Score model/representation, calculation, reasoning and contextual interpretation separately. Require the stated initial condition, index or domain, and units where relevant. Accept equivalent formulas and valid alternative checks.