

TEACHER AND STUDENT RESOURCE

Rational Exponents and Radical Meaning



NC Math 2 • NC.M2.N-RN.1

The Real Number System

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.N-RN.1 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Explain how expressions with rational exponents can be rewritten as radical expressions.

T1 Translate rational exponents into roots and powers.

T2 Evaluate rational-exponent expressions and justify the values.

T3 Explain allowed real inputs and principal-root values for even and odd roots.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M03, O03, M07, O07, M08, O08, M11, O11, M12, O12, M16, O16, M19, O19, M23, O23, M24, O24, M27, O27, M30, O30

T2: M02, O02, M04, O04, M09, O09, M10, O10, M14, O14, M15, O15, M18, O18, M20, O20, M22, O22, M26, O26, M28, O28

T3: M05, O05, M06, O06, M13, O13, M17, O17, M21, O21, M25, O25, M29, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Translate rational exponents into roots and powers.

Learning target: Evaluate rational-exponent expressions and justify the values.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Evaluate rational-exponent expressions and justify the values.

Learning target: Explain allowed real inputs and principal-root values for even and odd roots.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Translate rational exponents into roots and powers.

Learning target: Evaluate rational-exponent expressions and justify the values.

Learning target: Explain allowed real inputs and principal-root values for even and odd roots.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Integer powers, factors, positive and negative numbers, reciprocals, and the nonnegative meaning of a principal square root.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

In this packet, positive bases permit all stated rational-exponent rules. Negative bases occur only with explicit odd-root interpretations. An even principal root is nonnegative; an equation such as $x^2=9$ has two solutions even though $\sqrt{9}=3$.

TEACHER MODEL

Why two thirds means cube root then square

Explain $64^{(2/3)}$ using the meaning of a cube root, not only a calculator.

Let $r=64^{(1/3)}$. The exponent law gives $r^3=64$, so $r=4$. Then $64^{(2/3)}=(64^{(1/3)})^2=4^2=16$. The denominator selects the root; the numerator selects the power. Checking $16^3=64^2$ verifies the relationship.

VISUAL ANCHOR

Read the exponent in two steps

For positive a, $a^{(m/n)} = (\sqrt[n]{a})^m$. The two integers do different jobs.

Expression	Root step	Power step
$64^{(2/3)}$	$\sqrt[3]{64} = 4$	$4^2 = 16$
$81^{(3/4)}$	$\sqrt[4]{81} = 3$	$3^3 = 27$
$25^{(-1/2)}$	$\sqrt{25} = 5$	Take reciprocal: $1/5$

Check by reversing the operation. An nth root is a number whose nth power returns the radicand. Negative exponents add a reciprocal and require a nonzero base.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Translate rational exponents into roots and powers.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Collect the first response. Ask learners to locate the numerator and denominator, then show which integer power checks a proposed root. Do not say that an exponent is multiplication by a fraction.

Core task

Let $a > 0$. Rewrite $a^{4/5}$ with a radical in two equivalent ways, and explain why the denominator determines the root index.

Teacher solution and discussion

$a^{4/5} = (\text{root of index 5 of } a)^4 = (\text{root of index 5 of } a^4)$. Raising the first-root factor to power 5 recovers a ; the remaining numerator raises that factor to power 4. Positivity makes both forms real and unambiguous.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T3: Explain allowed real inputs and principal-root values for even and odd roots.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model a root-and-power chain aloud. Name why odd roots can accept negative inputs and why even principal roots must be nonnegative. Contrast a negative exponent with a negative base.

Core task

Evaluate $(-64)^{1/3}$. Then decide whether $(-16)^{1/2}$ is a real number. Justify the difference using odd and even powers.

Teacher solution and discussion

$(-64)^{1/3} = -4$, because $(-4)^3 = -64$. But $(-16)^{1/2}$ has no real value: the square of every real number is nonnegative. Odd powers preserve a negative sign; even powers do not.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Evaluate rational-exponent expressions and justify the values.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Ask learners to choose a useful root before calculating. For a missing exponent, rewrite both sides using a common positive base and justify the exponent comparison. Invite a second check by reversing the powers.

Core task

Evaluate $(64/125)^{(2/3)}$ by taking a root first. Explain why a fractional base causes no difficulty here.

Teacher solution and discussion

First take the cube root: $\sqrt[3]{(64/125)}=4/5$. Squaring gives $16/25$. The positive denominator is nonzero, so the quotient and its root are defined.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Evaluate rational-exponent expressions and justify the values.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign one partner to calculate and another to challenge the domain or general claim. Each learner first records an independent prediction. Require a numerical example and a general reason when the task asks for both.

Core task

Order $(1/16)^{(1/4)}$, $(1/16)^{(1/2)}$, and $(1/16)^{(3/4)}$ from least to greatest. Explain why this reverses the order in the corresponding three powers of 16.

Teacher solution and discussion

The fourth root of $1/16$ is $1/2$. The three values are $1/2, 1/4, 1/8$, so $(1/16)^{(3/4)} < (1/16)^{(1/2)} < (1/16)^{(1/4)}$. Repeated multiplication by $1/2$ decreases a positive value, while repeated multiplication by 2 increases it.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: 019 • T1: Translate rational exponents into roots and powers.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Give only neutral reading or recording support. Require the radical meaning, stated domain, and a check when requested. Preserve the first response before discussing a missing numerator or sign restriction.

Core task

Write a verbal sequence of operations for $a^{-3/4}$, where $a > 0$. Then choose a positive integer $a > 1$ that makes the value exactly $1/8$, and prove that it works.

Teacher solution and discussion

Take the principal fourth root, cube it, then take the reciprocal. For $a=16$, the fourth root is 2, its cube is 8, and the reciprocal is $1/8$. Thus $16^{-3/4}=1/8$. Equivalently $a^{3/4}=8$ forces the fourth root of a to be 2.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Translate rational exponents into roots and powers.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Diagnose whether the gap is root index, power, reciprocal, or allowed inputs. Rebuild only the missing step with a small perfect-power example, then use a new item without answer-supplying cues.

Core task

Sort these three expressions into equivalent-value groups: $27^{(1/3)}$, $9^{(1/2)}$, and $81^{(1/4)}$. Explain each value, then write a fourth different rational-power expression belonging to the same group.

Teacher solution and discussion

All three equal 3: $3^3=27$, $3^2=9$, and $3^4=81$. One additional example is $243^{(1/5)}=3$ because $3^5=243$. Other correctly justified expressions equal to 3 are acceptable.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Evaluate rational-exponent expressions and justify the values.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Collect a complete transfer response: an exact value, a justified contextual domain, or an explanation from the root definition. Do not show the worked example or answer choices while the learner writes.

Core task

Evaluate $(32/243)^{-2/5}$ exactly. Show the root, power, and reciprocal steps, then verify your answer by reversing them.

Teacher solution and discussion

The fifth root of $32/243$ is $2/3$. Squaring gives $4/9$, and reciprocating gives $9/4$. To reverse: the reciprocal of $9/4$ is $4/9$, its positive square root is $2/3$, and $(2/3)^5 = 32/243$.

Evidence and response

Listen for denominator-as-root and numerator-as-power reasoning. If those roles are reversed, have the learner test the proposed root by raising it to its index. Separate a principal-root value from solving an even-power equation.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****For $x > 0$, which radical equals $x^{(3/4)}$?**

- A. (root of index 3 of x)⁴
- B. $4x^3$
- C. x^7
- D. (root of index 4 of x)³

Reasoning: _____

M02 | Warm-up | T2**What is $8^{(2/3)}$, and why?**

- A. 64, from ignoring the denominator
- B. $16/3$, from multiplying by $2/3$
- C. 2, from ignoring the numerator
- D. 4, because (cube root of 8)² = 2^2

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Translate rational exponents into roots and powers.

Let $a > 0$. Rewrite $a^{4/5}$ with a radical in two equivalent ways, and explain why the denominator determines the root index.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Evaluate rational-exponent expressions and justify the values.

Evaluate $64^{(2/3)}$ without a calculator. Write the radical form and verify your answer by using integer powers.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01**M01 • T1: Translate rational exponents into roots and powers.****Answer: D. (root of index 4 of x)³**

The denominator 4 gives the root index and the numerator 3 gives the power. If r is the positive root of index 4 of x , $r^4=x$, so $x^{(3/4)}=r^3$.

Check each alternative

- A. This reverses root index and power.
- B. A root is not multiplication by its index.
- C. The numerator and denominator are not added.
- D. The denominator 4 gives the root index and the numerator 3 gives the power. If r is the positive root of index 4 of x , $r^4=x$, so $x^{(3/4)}=r^3$.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02**M02 • T2: Evaluate rational-exponent expressions and justify the values.****Answer: D. 4, because $(\text{cube root of } 8)^2 = 2^2$** Take the cube root first: $\sqrt[3]{8}=2$. Then square: $2^2=4$. Thus $8^{(2/3)}=4$.**Check each alternative**

- A. The denominator 3 requires a cube root.
- B. A rational exponent specifies a root and power, not a fraction of the base.
- C. This completes the root step but not the square.
- D. Take the cube root first: $\sqrt[3]{8}=2$. Then square: $2^2=4$. Thus $8^{(2/3)}=4$.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001**001 • T1: Translate rational exponents into roots and powers.**

$a^{4/5} = (\text{root of index 5 of } a)^4 = (\text{root of index 5 of } a^4)$. Raising the first-root factor to power 5 recovers a ; the remaining numerator raises that factor to power 4. Positivity makes both forms real and unambiguous.

Evidence to assess

Look for a correct radical/exponent correspondence, a justified real-number restriction when relevant, and a checked value or contextual unit. A bare value is partial evidence, not an explanation.

WORKED TEACHER KEYS

002**002 • T2: Evaluate rational-exponent expressions and justify the values.**

$64^{(2/3)} = (\sqrt[3]{64})^2 = 4^2 = 16$. Verification: $(16)^3 = 4096 = 64^2$, matching the exponent identity.

Evidence to assess

Look for a correct radical/exponent correspondence, a justified real-number restriction when relevant, and a checked value or contextual unit. A bare value is partial evidence, not an explanation.