

TEACHER AND STUDENT RESOURCE

Vectors: Magnitude, Direction and Resultants



Precalculus • PC.N.3

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.N.3 | North Carolina Precalculus mathematics

This sample contains 22 complete selected pages from the 100-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-17, 32-33, 62, 77-78

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 18; original page(s) 17

O01: sample PDF page(s) 19; original page(s) 32

O02: sample PDF page(s) 20; original page(s) 33

Selected teacher prompts and keys: original page(s) 62, 77-78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand properties and operations with vectors.

T1 PC.N.3.1 Represent vectors using components, magnitude and direction.

T2 PC.N.3.2 Add and subtract vectors algebraically and geometrically.

PC.N.3.1 Represent a vector indicating magnitude and direction.

PC.N.3.2 Execute sum and difference algorithms to combine vectors.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M03, M05, M07, M09, M11, M13, M15, M17, M19, M21, M23, M25, M27, M29, O01, O03, O05, O07, O09, O11, O13, O15, O17, O19, O21, O23, O25, O27, O29

T2: M02, M04, M06, M08, M10, M12, M14, M16, M18, M20, M22, M24, M26, M28, M30, O02, O04, O06, O08, O10, O12, O14, O16, O18, O20, O22, O24, O26, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.N.3.1 Represent vectors using components, magnitude and direction.

Learning target: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Coordinate differences, the Pythagorean theorem, right-triangle trigonometry and angles in degrees.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Write vectors as $\langle \text{horizontal}, \text{vertical} \rangle$. Unless stated otherwise, positive horizontal means east, positive vertical means north, and direction is measured counterclockwise from the positive horizontal axis in $[0^\circ, 360^\circ)$. Magnitude is nonnegative. The zero vector has magnitude 0 and no unique direction angle. A vector may be translated without changing it. Trigonometric approximations use degree mode and retain unrounded values until the final step.

TEACHER REFERENCE

Three representations, one vector

Components

A vector from $A=(-1,2)$ to $B=(2,6)$ is $\langle 2-(-1), 6-2 \rangle = \langle 3, 4 \rangle$. Its position in the plane does not determine its components; its change in position does.

Magnitude and direction

Its magnitude is $\sqrt{3^2+4^2}=5$. Its direction is $\arctan(4/3) \approx 53.13^\circ$, in quadrant I. A vector of length 5 with this direction has components $\langle 5\cos 53.13^\circ, 5\sin 53.13^\circ \rangle$, approximately $\langle 3, 4 \rangle$.

Adding and subtracting

Translate vectors head-to-tail for addition. To form $u-v$, add $-v$, reversing the direction of v while retaining its magnitude. Never subtract lengths as a substitute for subtracting vectors.

Drawing convention

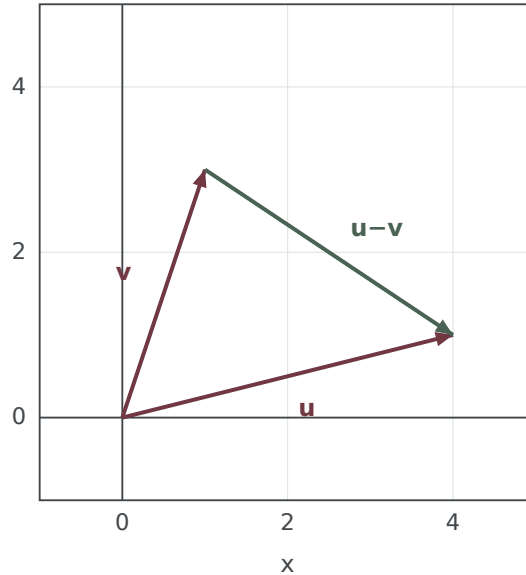
Vector grids in this packet use equal horizontal and vertical unit scales. Arrows show direction; coordinate labels, not ruler measurements, are the authoritative data. Keep worked teacher constructions hidden during independent student work.

TEACHER MODEL

Subtract vectors, not their lengths

From the same origin, draw $u = \langle 4, 1 \rangle$ and $v = \langle 1, 3 \rangle$. Find $u - v$ and show it as a directed tip-to-tip connector.

y



$u-v = \langle 3, -2 \rangle$. The connector begins at the tip of v , (1, 3), and ends at the tip of u , (4, 1); endpoint minus start gives $\langle 3, -2 \rangle$. Translating that connector to the origin does not change the vector. Its magnitude is $\sqrt{13}$, not $|u| - |v| = \sqrt{17} - \sqrt{10}$. Vector subtraction is addition of the opposite, not subtraction of lengths.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

A vector starts at $A=(-2, 5)$ and ends at $B=(4, -3)$. Find its components, magnitude and direction angle. Draw it at its original position and translate an equal copy to the origin.

Teacher solution and discussion

$B-A=\langle 6, -8 \rangle$. Its magnitude is $\sqrt{36+64}=10$. It lies in quadrant IV with reference angle $\arctan(8/6)\approx 53.13^\circ$, so its direction is about 306.87° . An equal copy runs from $(0, 0)$ to $(6, -8)$; translation changes location but not components, length or direction.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Find a unit vector with the same direction as $\langle 8, -15 \rangle$, and one with the opposite direction. Verify the magnitude of each and explain why the normalization divisor is positive.

Teacher solution and discussion

The magnitude is $\sqrt{64+225}=17$. The same-direction unit vector is $\langle 8/17, -15/17 \rangle$; its opposite is $\langle -8/17, 15/17 \rangle$. Each squared magnitude is $(64+225)/289=1$. Dividing by positive 17 preserves direction; multiplying the resulting unit vector by -1 reverses it.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

For $v = \langle -7, -7 \rangle$, find exact magnitude and direction angle. A learner reports 45° using $\arctan(y/x)$. Diagnose the missing information and explain a reliable correction.

Teacher solution and discussion

$|v| = \sqrt{98} = 7\sqrt{2}$. Both components are negative, so v is in quadrant III. The reference angle is 45° , giving direction 225° . The ratio y/x is positive in both quadrants I and III; $\arctan$ of the ratio alone loses quadrant information. Inspect both component signs or use a quadrant-aware angle function.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

A triangular trip starts at the origin, follows $u = \langle 4, 1 \rangle$, then $v = \langle -2, 3 \rangle$, then returns directly to the origin. Find the return vector w and draw the complete directed path. Verify $u + v + w = 0$.

Teacher solution and discussion

The first two legs end at $\langle 2, 4 \rangle$, so $w = \langle -2, -4 \rangle$. The vertices are $(0, 0)$, $(4, 1)$, $(2, 4)$, and $(0, 0)$. Summing components gives $\langle 4 - 2 - 2, 1 + 3 - 4 \rangle = \langle 0, 0 \rangle$.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

A vector points along the ray through $(3, -4)$ from the origin. Find a vector of magnitude 2 on this ray and a vector of magnitude 2 on the opposite ray. Give the direction angle of each.

Teacher solution and discussion

The ray vector $\langle 3, -4 \rangle$ has length 5. Scale its unit vector by 2 to get $\langle 6/5, -8/5 \rangle$. The opposite is $\langle -6/5, 8/5 \rangle$. Their directions are about 306.87° and 126.87° respectively; each magnitude is 2.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Give one pair of nonzero vectors for which the magnitude of the sum equals the sum of the magnitudes, and another pair for which it is strictly smaller. Verify both cases and explain the role of direction.

Teacher solution and discussion

For $u = \langle 1, 2 \rangle$ and $v = \langle 2, 4 \rangle$, the sum is $\langle 3, 6 \rangle$ of length $3\sqrt{5}$, equal to $\sqrt{5} + 2\sqrt{5}$; they point in the same direction. For $p = \langle 3, 0 \rangle$ and $q = \langle 0, 4 \rangle$, the sum has length 5, less than $3 + 4 = 7$; their directions differ. These examples show why adding magnitudes is not a general vector-addition rule.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Prove componentwise that $(u-v)+v=u$ for arbitrary planar vectors. Then show why the analogous-looking magnitude statement $|u-v|+|v|=|u|$ is not generally true using a numerical counterexample.

Teacher solution and discussion

With $u=\langle a, b \rangle$, $v=\langle c, d \rangle$, $(u-v)+v=\langle a-c, b-d \rangle+\langle c, d \rangle=\langle a, b \rangle=u$. For $u=\langle 1, 0 \rangle$ and $v=\langle 0, 1 \rangle$, $|u-v|+|v|=\sqrt{2}+1$, while $|u|=1$. Vector cancellation does not imply cancellation after taking nonnegative lengths.

Evidence and response

Check whether the learner distinguishes an endpoint, a displacement and a magnitude. Ask for endpoint minus starting point, a Pythagorean length and quadrant-aware direction. For sums or differences, compare component work with a head-to-tail path, parallelogram or addition of an opposite. Record angle conventions and units explicitly.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 • Select and justify

M01 | Warm-up | T1

The displayed vector starts at the origin and ends at $(-3, 4)$. Which describes it?

y



- A. Magnitude 7, direction in quadrant II
- B. Magnitude 5, direction in quadrant IV
- C. Magnitude -5 , direction in quadrant II
- D. Magnitude 5, direction in quadrant II

Reasoning: _____

MULTIPLE-CHOICE BANK

M02 / M03 • Select and justify**M02 | Warm-up | T2****Find $\langle 4, -1 \rangle + \langle -2, 5 \rangle$.**

- A. $\langle 6, -6 \rangle$
- B. $\langle 2, 4 \rangle$
- C. $\langle -8, -5 \rangle$
- D. $\langle 2, -4 \rangle$

Reasoning: _____

M03 | Warm-up | T1**A vector goes from $(5, -2)$ to $(1, 1)$. What are its components?**

- A. $\langle 4, -3 \rangle$
- B. $\langle 1, 1 \rangle$
- C. $\langle 6, -1 \rangle$
- D. $\langle -4, 3 \rangle$

Reasoning: _____

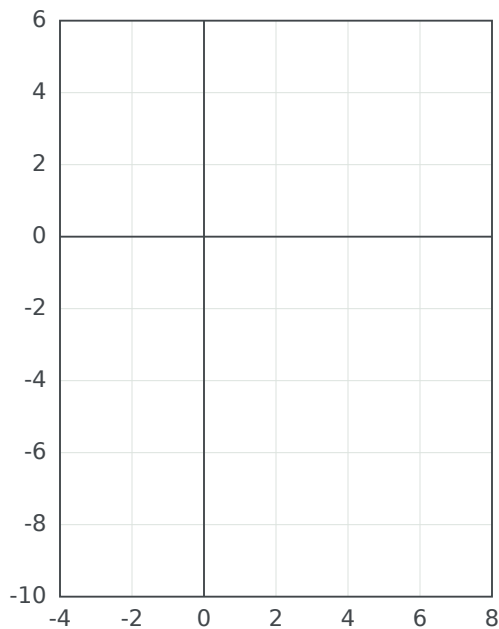
WARM-UP

001 • Show your reasoning

T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

A vector starts at $A=(-2, 5)$ and ends at $B=(4, -3)$. Find its components, magnitude and direction angle. Draw it at its original position and translate an equal copy to the origin.

vertical axis: _____



horizontal axis: _____

Explanation and check

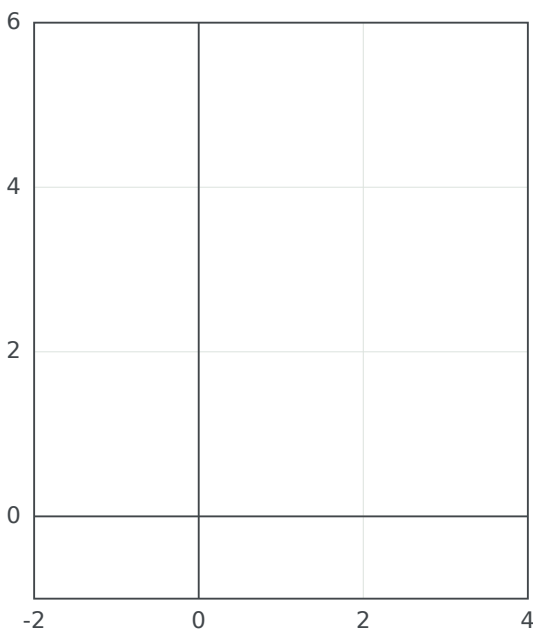
WARM-UP

002 • Show your reasoning

T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

For $u = \langle 3, 1 \rangle$ and $v = \langle -1, 4 \rangle$, compute $u + v$ and construct it head-to-tail. Also show both vectors from a common origin and explain the parallelogram interpretation.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.****Answer: D. Magnitude 5, direction in quadrant II**

The length is $\sqrt{9+16}=5$; the signs place the direction in quadrant II.

Check each alternative

- A. Adding absolute component values gives 7, not the Euclidean magnitude.
- B. The endpoint has negative horizontal and positive vertical components.
- C. A magnitude cannot be negative.
- D. Correct. The length is $\sqrt{9+16}=5$; the signs place the direction in quadrant II.

M02 • T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.**Answer: B. $\langle 2, 4 \rangle$**

Add like components: $\langle 4-2, -1+5 \rangle = \langle 2, 4 \rangle$.

Check each alternative

- A. This is the first vector minus the second.
- B. Correct. Add like components: $\langle 4-2, -1+5 \rangle = \langle 2, 4 \rangle$.
- C. This multiplies corresponding components.
- D. The vertical component is $-1+5=4$, not -4 .

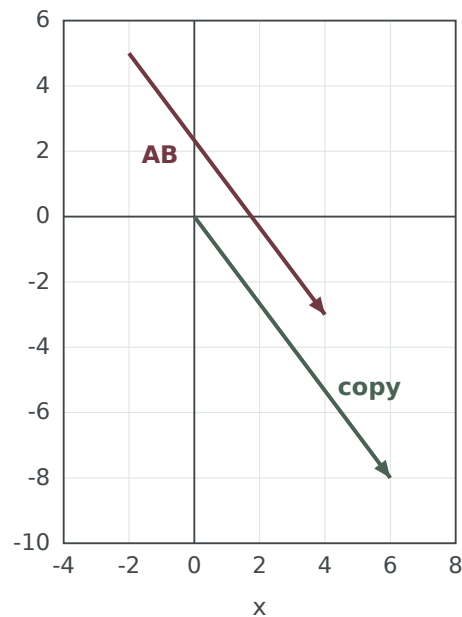
WORKED TEACHER KEYS

001

001 • T1: PC.N.3.1 Represent vectors using components, magnitude and direction.

$B-A=\langle 6, -8 \rangle$. Its magnitude is $\sqrt{(36+64)}=10$. It lies in quadrant IV with reference angle $\arctan(8/6)\approx 53.13^\circ$, so its direction is about 306.87° . An equal copy runs from $(0, 0)$ to $(6, -8)$; translation changes location but not components, length or direction.

y

**Evidence to assess**

Full evidence: Endpoint-minus-start, magnitude 10, quadrant-correct angle, and two equal directed arrows. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

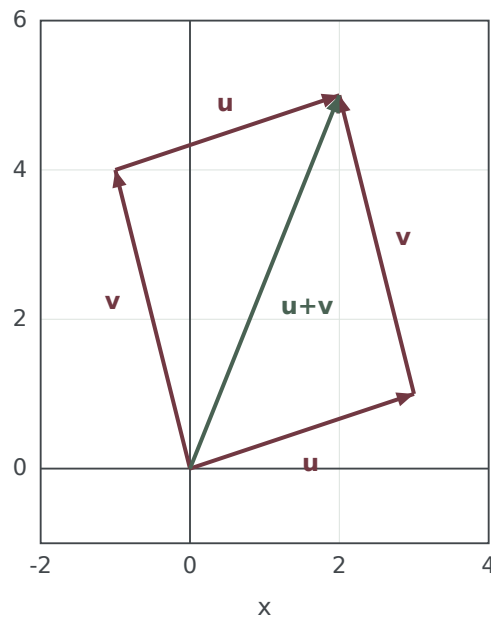
WORKED TEACHER KEYS

002

002 • T2: PC.N.3.2 Add and subtract vectors algebraically and geometrically.

$u+v=\langle 2, 5 \rangle$. One head-to-tail path goes $(0, 0) \rightarrow (3, 1) \rightarrow (2, 5)$; the resultant goes directly $(0, 0) \rightarrow (2, 5)$. With tails at the origin, u ends at $(3, 1)$ and v at $(-1, 4)$; translating u from the tip of v reaches the same point $(2, 5)$.

y

**Evidence to assess**

Full evidence: Correct component sum, directed head-to-tail path and resultant, and a coherent common-tail parallelogram. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.