

TEACHER AND STUDENT RESOURCE

Create Relationships and Make Their Graphs Tell the Truth



NC Math 3 • NC.M3.A-CED.2

Creating Equations

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-CED.2 | NC Math 3 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

T1 Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

T2 Create and graph polynomial relationships from geometric and numerical conditions, identifying the contextual domain, intercepts, extrema and nonuniqueness.

T3 Create and graph exponential relationships using initial or later observations, interval clocks, baselines and complements, distinguishing continuous curves from discrete samples.

T4 Create and graph rational relationships for averages, rates, concentrations and calibration, retaining excluded inputs, removable gaps, asymptotes and contextual restrictions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, M09, M14, M20, M28, O01, O05, O10, O19, O24, O28

T2: M02, M05, M06, M10, M13, M15, M19, M23, M29, O02, O06, O09, O13, O18, O22, O23, O27

T3: M03, M07, M11, M16, M21, M25, M27, O03, O07, O11, O14, O20, O25, O29

T4: M04, M08, M12, M17, M18, M22, M24, M26, M30, O04, O08, O12, O15, O16, O17, O21, O26, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: From quantities to equations and exact points

Learning target: Create relationships across the four families.

Learning target: Use units and an exact point to check each model.

Use Warm-up and I Do tasks. Require equation, domain and a labeled graph as three distinct pieces of evidence; do not accept a plausible shape without checked coordinates.

Session 2: Constraints, domains and graph features

Learning target: Recover parameters from observations and geometric constraints.

Learning target: Graph shifted models, holes, implicit relations and physically restricted portions.

Use We Do and Collaborative tasks. Compare one proposed graph with a deliberately wrong domain or baseline, and have students name exactly which original condition the wrong graph violates.

Session 3: Diagnose and construct transferable models

Learning target: Distinguish equivalent expressions from different relationships.

Learning target: Use graph structure to explain nonuniqueness, asymptotic limits and changing rates.

Use Independent practice, selected Small-group tasks and the Exit ticket. Collect individual first attempts and assess model creation separately from graph accuracy and contextual interpretation.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Interpret variables and units, absolute value as distance, areas and volumes as products, positive exponential multipliers and ratios. Use coordinate plotting, factoring and the Pythagorean theorem.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

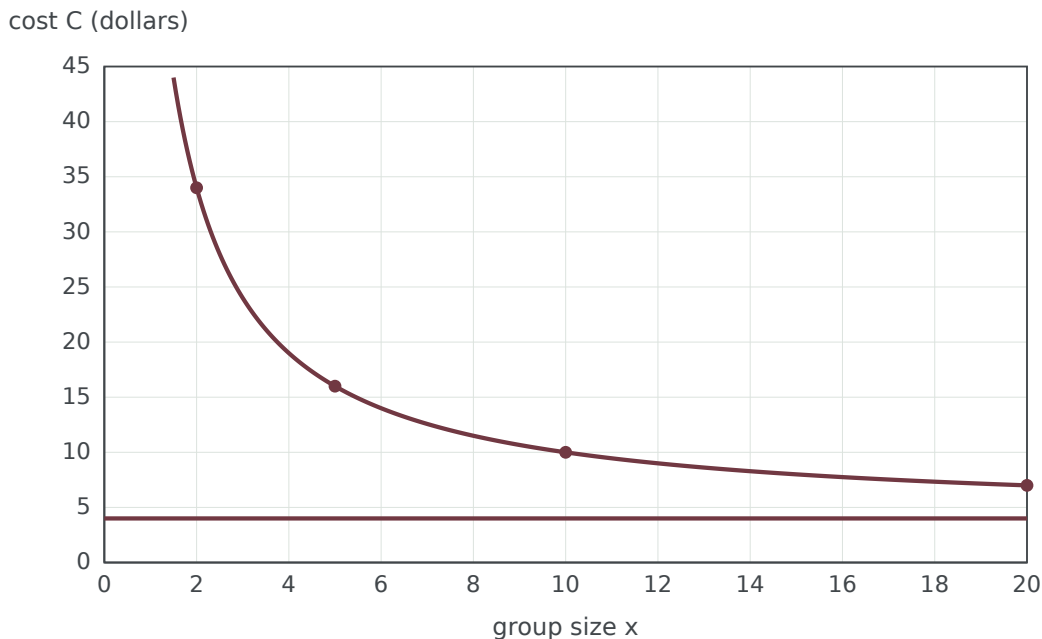
Review convention

All contexts and data are hypothetical instructional models. Graphs use exact marked points and sampled connecting curves. Give axes meaningful labels and units when stated. A requested finite plotting window does not by itself change the mathematical domain. An open marker denotes an excluded endpoint or removable gap; a filled marker denotes an included point. Exponents use $^$ notation; $\sqrt{}$ denotes the nonnegative square root. For physical quantities, retain the stated restrictions even if a formula extends to additional real inputs.

TEACHER MODEL

Two variables, one contextual relationship, a truthful graph

A group shares a fixed \$60 charge, and each participant also pays \$4. Build a continuous planning relationship between positive group size x and cost per person C . Graph it in the window $0 < x \leq 20$, identify a limiting level and explain how actual whole-person groups differ from the planning curve.



Total cost is $60+4x$. Dividing by x gives $C=(60+4x)/x=60/x+4$ for $x>0$. Exact points include $(2,34)$, $(5,16)$, $(10,10)$, $(20,7)$. The cost decreases as the fixed-charge share becomes smaller, approaching horizontal asymptote $C=4$ from above. Zero participants is excluded because the average would be undefined. The smooth curve is the explicitly stated continuous planning model; actual groups use only its positive-integer x samples. Label the horizontal axis group size and the vertical axis dollars per person. A total-cost line $60+4x$ would answer a different question.

VISUAL ANCHOR

Build, restrict, plot and verify

Name both variables, units and which quantity depends on which.

Choose operations from meaning: distance, product, repeated multiplier, or ratio.

Record contextual restrictions and original denominator exclusions before simplifying.

Check initial values, vertices, zeros, exact points and asymptotes appropriate to the family.

Choose a labeled plotting window; use equal coordinate scales for circles.

Keep separate branches separate; show holes and excluded boundary points with open markers.

Validate the graph against the original relationship, not just the simplified formula.

A correct equation with a misleading domain, mislabeled axis or connected gap is not a complete representation. Create the algebra and graph as two mutually checking accounts of the same quantities.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Optional bank: M01–M04; 001–004. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Ask students to name the two variables before using numbers. 001 links a reference position to a V vertex; 002 distinguishes price increase from actual price. In 003 count two-hour intervals. 004 uses positive dimensions to choose only one branch of the area relationship. Check axes, units and one exact point before discussing overall shape.

Core task

A handling service charges \$10 plus \$3 for each kilogram a package's weight differs from 5 kg, for $0 \leq w \leq 10$. Create an equation for charge C in terms of w . Graph the entire allowed relationship, label the vertex and endpoints, and explain why equal deviations produce equal charges.

Teacher solution and discussion

The weight deviation is $|w-5|$, so $C=10+3|w-5|$ on $[0,10]$. At $w=5$ the deviation is zero and $C=10$, giving vertex $(5,10)$. At $w=0$ and 10 the deviation is 5 , so both endpoints have charge 25 . Plot the V through $(0,25), (5,10), (10,25)$, including endpoints. On either side, a deviation d gives $C=10+3d$, so weights $5-d$ and $5+d$ have equal charges whenever both lie in the domain.

Evidence and response

Create the full deviation-charge equation, label the restricted V graph, and justify symmetry using equal distances rather than a visual guess. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Optional bank: M05–M08; 005–008. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

In 005 factor the inside absolute-value expression before deciding its slope. 006 cubes the entire edge, not just its changing term. For 007 draw isolated yearly observations and ask why a smooth connection would add an unstated assumption. 008 treats the average rather than total production cost and retains a nonzero limiting level.

Core task

A device outputs 3 less than the magnitude of $2x+4$. Create its equation, rewrite it to expose the graph's vertex, and draw it with both x-intercepts and the y-intercept. Explain why the output can be negative even though a magnitude appears in the rule.

Teacher solution and discussion

The equation is $y=|2x+4|-3=2|x+2|-3$. Its vertex is $(-2,-3)$ and it opens upward with branch slopes -2 and 2 . To find zeros, solve $2|x+2|=3$, giving $x=-2\pm3/2$, namely $-7/2$ and $-1/2$. At $x=0$, $y=4-3=1$, so the y-intercept is $(0,1)$. The magnitude itself is nonnegative, but subtracting 3 shifts the graph downward; outputs range from -3 upward.

Evidence and response

Translate the magnitude of an entire expression, factor its internal scale correctly, and verify a completed graph with all three intercepts and a negative minimum. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Create and graph polynomial relationships from geometric and numerical conditions, identifying the contextual domain, intercepts, extrema and nonuniqueness.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

O09 uses zeros plus a nonzero value to recover scale. In O10 derive all three distance regions and notice the flat minimum interval. O11 models excess above a baseline; O12 keeps solute fixed during dilution. O13 distinguishes a mathematically valid negative root from the physical flight domain. Require students to justify every restriction they add to a graph.

Core task

A cubic response has zeros 0, 2, 5 and has output 8 at input 1. Create its equation, justify the scale factor, and graph it over $-1 \leq x \leq 6$. Label all zeros and the scale-setting point, and explain the sign of the response between consecutive zeros.

Teacher solution and discussion

A cubic with those three distinct roots has form $y = a x(x-2)(x-5)$. At $x=1$ the product without a is $1 \cdot (-1) \cdot (-4) = 4$, so $4a=8$ and $a=2$. Thus $y=2x(x-2)(x-5)$. Plot the cubic through $(0,0), (1,8), (2,0), (5,0)$, with values -36 at -1 and 48 at 6 . On $(0,2)$, the factor signs are $+, -, -$, giving positive y . On $(2,5)$, they are $+, +, -$, giving negative y . The simple roots give crossings rather than tangencies.

Evidence and response

Construct a scaled polynomial from root and value data, plot its real shape and use factor signs to explain the graph between roots. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Create and graph exponential relationships using initial or later observations, interval clocks, baselines and complements, distinguishing continuous curves from discrete samples.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O14 separates a two-period ratio from its one-period multiplier. O15 preserves a removable gap after cancellation. O16 shows that a rational function need not have a real vertical asymptote. O17 contrasts inverse-square scaling with inverse scaling. O18 compares an unrestricted vertex with a contextual minimum. Assign one partner to model, one to verify points and one to challenge the domain, then rotate roles.

Core task

A positive continuous exponential relationship $y=A \cdot b^t$ has values 18 at $t=2$ and 162 at $t=4$, with $A>0$ and $b>0$. Recover both parameters, graph the model for $0 \leq t \leq 4$ and verify the two observations plus its initial value. Explain why 9 is not the one-unit multiplier.

Teacher solution and discussion

The observations give $Ab^2=18$ and $Ab^4=162$. Dividing yields $b^2=9$, so $b=3$ because the base is positive. Then $A=18/9=2$. The model is $y=2 \cdot 3^t$, with points $(0,2),(1,6),(2,18),(3,54),(4,162)$. Draw its increasing curve through these values. The observed ratio $162/18=9$ spans two time units; it equals b^2 , while the one-unit multiplier is 3.

Evidence and response

Recover an exponential model from nonzero-time observations, distinguish a multi-period ratio from a one-period base and verify the initial value on the completed graph. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 constructs rather than reads the absolute-value scale. O20 repairs interval units and uses a fractional time check. O21 derives an average from total distance and total time. O22 is a two-variable polynomial equation whose feasible graph is a restricted circle, not a polynomial function. O23 requires an honest normalized vertical axis and conversion back to actual volume.

Core task

Create the unique equation in the family $y=a|x-h|+k$ whose vertex is $(2,-1)$ and whose graph contains $(-1,5)$. Graph it for $-1 \leq x \leq 5$, label both zeros, and explain how the extra point fixes the direction as well as the vertical scale.

Teacher solution and discussion

The vertex gives $h=2$ and $k=-1$, so $y=a|x-2|-1$. At $(-1,5)$, $5=3a-1$, hence $a=2$. Thus $y=2|x-2|-1$, opening upward because $a>0$. Solving $y=0$ gives $|x-2|=1/2$, so zeros are $(3/2,0)$ and $(5/2,0)$. The V passes through $(-1,5), (2,-1), (5,5)$. Once the vertex is fixed, the nonvertex point determines the only possible a , including its positive sign.

Evidence and response

Recover all parameters from geometric constraints, establish uniqueness and graph the two rational zeros rather than omit the narrow part below the axis. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O24 for students who subtract distances in the wrong order; ask for a test position beyond each station. O25 targets confusion between a new time anchor and a new curve. O26 separates an asymptotic baseline from an inverse-proportional excess. O27 uses two exact boxes to show a graph can repeat an output without being invalid. Collect an uncued explanation after the focused discussion.

Core task

Stations lie at -1 and 3 . Define y as distance from x to station 3 minus distance from x to station -1 . Create the equation, graph it on $-4 \leq x \leq 6$ and derive its three-piece rule. Explain why y is signed even though each individual distance is nonnegative.

Teacher solution and discussion

The equation is $y = |x - 3| - |x + 1|$. If $x \leq -1$, $y = (3 - x) - (-x - 1) = 4$. If $-1 \leq x \leq 3$, $y = (3 - x) - (x + 1) = 2 - 2x$. If $x \geq 3$, $y = (x - 3) - (x + 1) = -4$. Draw horizontal $y = 4$ to $x = -1$, the segment from $(-1, 4)$ to $(3, -4)$, and horizontal $y = -4$ afterward. The graph crosses at $(1, 0)$, where the distances agree. Subtraction compares two nonnegative distances, so it is negative when station 3 is closer; its full range is $[-4, 4]$.

Evidence and response

Preserve the order of the distance subtraction, derive every sign region and explain both constant outer sections and the signed output. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 asks for a sign justified by a maximum rather than guessed from a V shape. O29 models the requested complement and explains why its direction reverses. O30 uses a horizontal asymptote to fix a parameter and demands both rational branches. Score the equation, domain, completed graph and interpretation separately; a selected option alone is not evidence of independent graph construction.

Core task

Create a vertically scaled and shifted absolute-value function whose maximum is 4 at $x=2$ and whose zeros are $x=-2$ and $x=6$. Prove the scale from one zero, graph it for $-4 \leq x \leq 8$ and explain why an upward-opening V cannot satisfy the maximum condition.

Teacher solution and discussion

The vertex fixes $y=a|x-2|+4$. At $x=-2$, zero output gives $0=4a+4$, so $a=-1$. Thus $y=4-|x-2|$, which also gives zero at $x=6$. Draw the inverted V through $(-4,-2), (-2,0), (2,4), (6,0), (8,-2)$. Since $|x-2| \geq 0$, subtracting it gives $y \leq 4$ with equality only at $x=2$. A positive scale would give a minimum and outputs above 4, contradicting the required maximum.

Evidence and response

Construct the sign and magnitude of a scale from zero and vertex constraints, verify both zeros and justify the global maximum from absolute-value nonnegativity. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

An inspection charge C is \$4 plus \$2 for each kilometer from track position

3. Which equation and graph vertex model charge against position x ?

- A. $C=4+2(x-3)$; a straight line.
- B. $C=4+2|x+3|$; vertex $(-3,4)$.
- C. $C=2|x-4|+3$; vertex $(4,3)$.
- D. $C=4+2|x-3|$; vertex $(3,4)$.

Reasoning: _____

M02 | Warm-up | T2

A rectangle is w meters wide and 4 meters longer than its width. Which equation and domain represent area A against width?

- A. $A=w^2+4$, with $w>0$.
- B. $A=2w+2(w+4)$, with $w>0$.
- C. $A=w(w+4)$, with $w>0$.
- D. $A=w(w+4)$, for every real w as physical widths.

Reasoning: _____

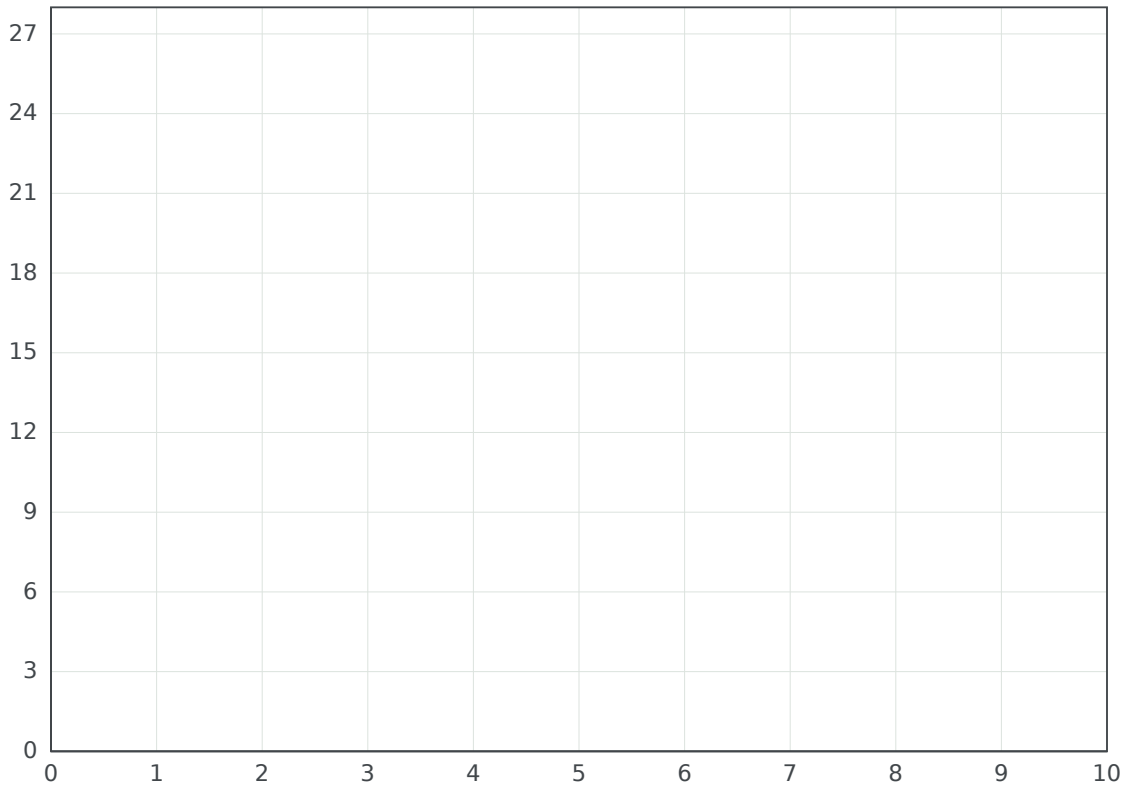
WARM-UP

001 • Show your reasoning

T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

A handling service charges \$10 plus \$3 for each kilogram a package’s weight differs from 5 kg, for $0 \leq w \leq 10$. Create an equation for charge C in terms of w. Graph the entire allowed relationship, label the vertex and endpoints, and explain why equal deviations produce equal charges.

vertical axis: _____



horizontal axis: _____

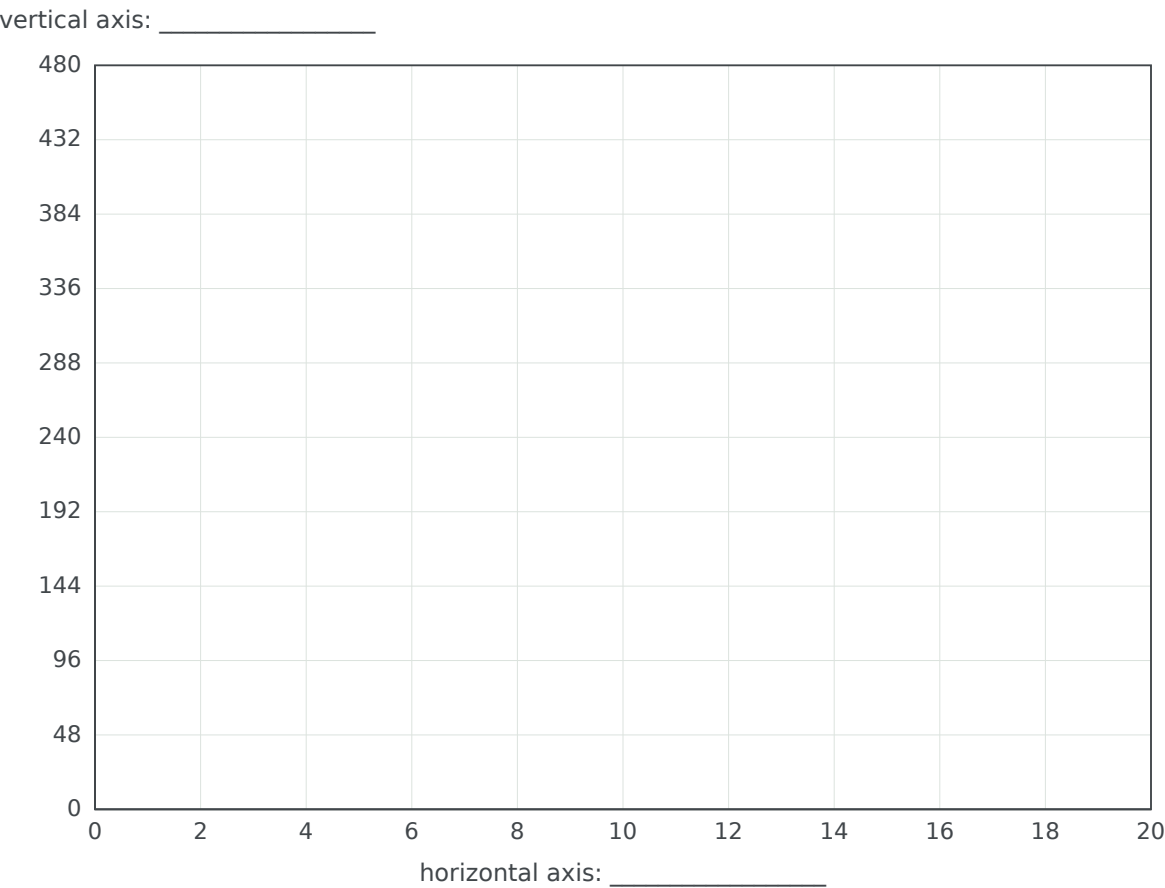
Explanation and check

WARM-UP

002 • Show your reasoning

T2: Create and graph polynomial relationships from geometric and numerical conditions, identifying the contextual domain, intercepts, extrema and nonuniqueness.

A continuous planning model sells 40 units at a price of \$10. Raising the price by x dollars reduces predicted sales by $2x$ units, where $0 \leq x \leq 20$. Create revenue R as a polynomial in x . Graph it, identify the maximum in this domain, and explain why the horizontal variable is not the total price.



Explanation and check

WORKED TEACHER KEYS

M01

M01 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

Answer: D. $C=4+2|x-3|$; vertex (3,4).

Distance from position 3 is $|x-3|$. Multiplying by \$2 per km and adding \$4 gives $C=4+2|x-3|$. At $x=3$ the distance is zero, so the graph has minimum (3,4); moving one kilometer either way increases C by 2.

Check each alternative

- A. Signed displacement would lower the fee on one side instead of charge for distance.
- B. The distance from position 3 is $|x-3|$, not $|x+3|$.
- C. This interchanges the reference position, fixed charge and resulting vertex.
- D. Correct. Distance from position 3 is $|x-3|$. Multiplying by \$2 per km and adding \$4 gives $C=4+2|x-3|$. At $x=3$ the distance is zero, so the graph has minimum (3,4); moving one kilometer either way increases C by 2.

Evidence to assess

Build the charge from unsigned distance and connect reference position and fixed charge to the two coordinates of the vertex.

WORKED TEACHER KEYS

M02

M02 • T2: Create and graph polynomial relationships from geometric and numerical conditions, identifying the contextual domain, intercepts, extrema and nonuniqueness.

Answer: C. $A=w(w+4)$, with $w>0$.

Length is $w+4$, so area is width times length: $A=w(w+4)=w^2+4w$. The physical domain $w>0$ also makes the length positive. The appropriate graph is only the positive-width portion of the polynomial, not its whole algebraic curve.

Check each alternative

- A. The extra length 4 must multiply the entire width w .
- B. This gives perimeter in meters, not area in square meters.
- C. Correct. Length is $w+4$, so area is width times length: $A=w(w+4)=w^2+4w$. The physical domain $w>0$ also makes the length positive. The appropriate graph is only the positive-width portion of the polynomial, not its whole algebraic curve.
- D. The polynomial exists at negative inputs, but a physical width must be positive.

Evidence to assess

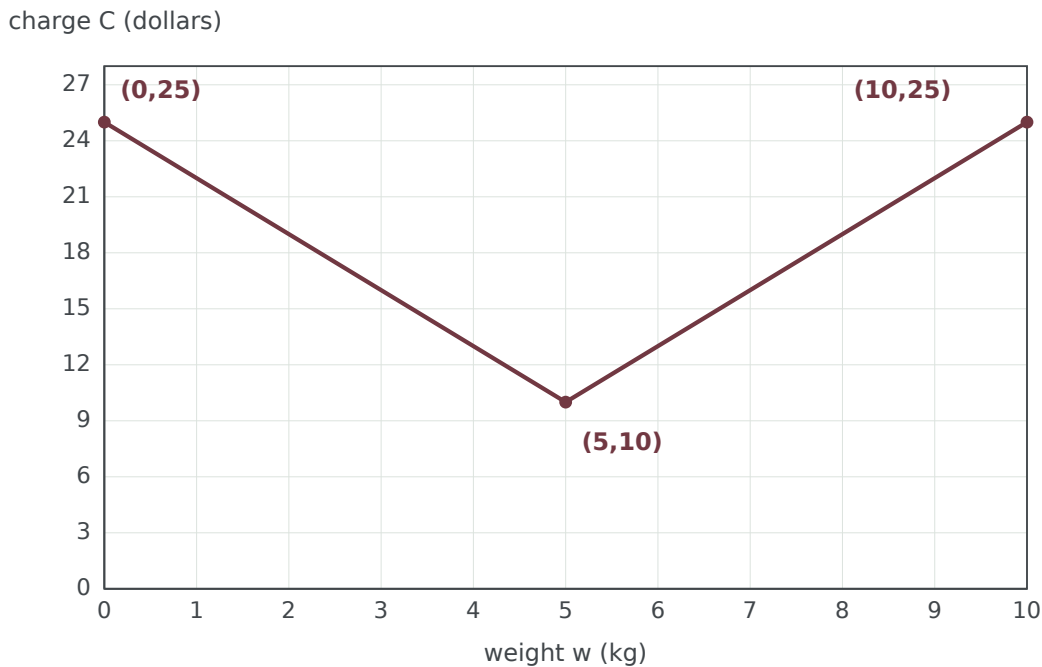
Translate both dimensions before multiplying and distinguish the mathematical formula's domain from the physical graph.

WORKED TEACHER KEYS

001

001 • T1: Create and graph absolute-value relationships from distance, vertex, zero and comparison constraints; explain piecewise behavior and signed differences.

The weight deviation is $|w-5|$, so $C=10+3|w-5|$ on $[0,10]$. At $w=5$ the deviation is zero and $C=10$, giving vertex $(5,10)$. At $w=0$ and 10 the deviation is 5 , so both endpoints have charge 25 . Plot the V through $(0,25), (5,10), (10,25)$, including endpoints. On either side, a deviation d gives $C=10+3d$, so weights $5-d$ and $5+d$ have equal charges whenever both lie in the domain.



Evidence to assess

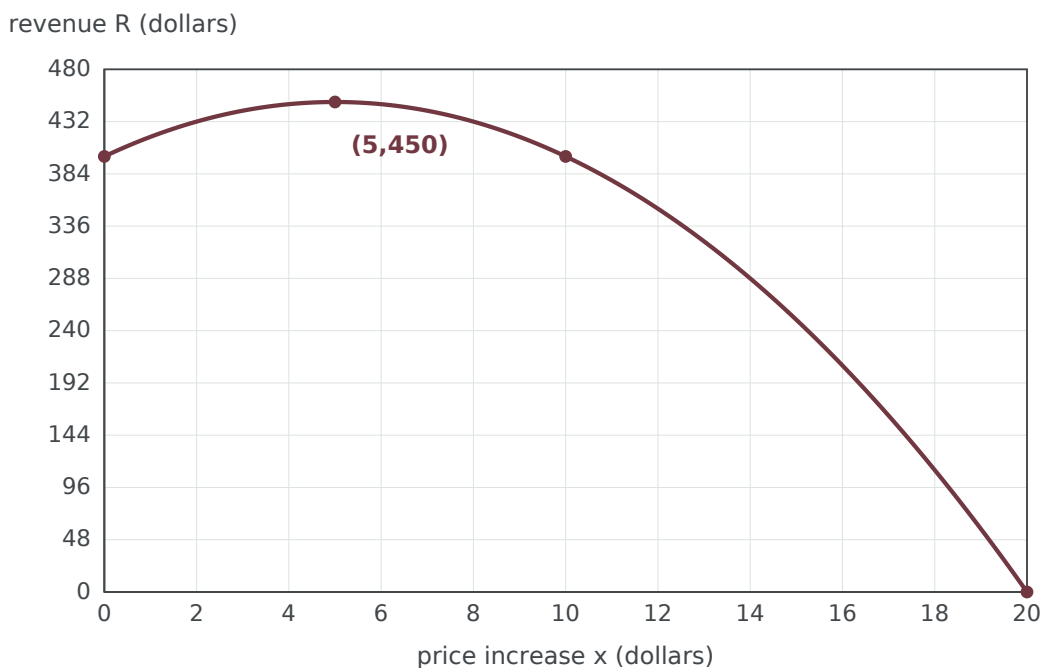
Create the full deviation-charge equation, label the restricted V graph, and justify symmetry using equal distances rather than a visual guess.

WORKED TEACHER KEYS

002

002 • T2: Create and graph polynomial relationships from geometric and numerical conditions, identifying the contextual domain, intercepts, extrema and nonuniqueness.

Price is $10+x$ and predicted quantity is $40-2x$, so $R=(10+x)(40-2x)=-2x^2+20x+400$. Completing the square gives $R=-2(x-5)^2+450$. The domain $[0,20]$ ensures nonnegative predicted sales. The graph includes $(0,400)$, $(5,450)$, $(10,400)$, $(20,0)$ and is concave down. Its maximum is \$450 at $x=5$, corresponding to total price \$15 and 30 predicted sales. The horizontal coordinate records the increase x , so labeling $x=5$ as a \$5 selling price would misinterpret the model.

**Evidence to assess**

Build revenue from changed price and changed demand, complete the square or locate the vertex, and interpret graph coordinates in the originally defined variable.