

TEACHER AND STUDENT RESOURCE

Trigonometric Ratios as Functions of Angle



NC Math 3 • NC.M3.F-IF.1

Interpreting Functions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.F-IF.1 | NC Math 3 mathematics

This sample contains 20 complete selected pages from the 103-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 62, 81

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 62, 81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

T1 Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.

T2 Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

T3 Use signed circle coordinates and a positive radius to extend angle-to-ratio functions beyond acute triangles.

T4 Justify single-valued angle functions, distinguish repeated outputs from ambiguity and interpret ratios in applications.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M05, M13, M21, M25, M28, O01, O04, O06, O11, O17, O23, O28

T2: M02, M03, M08, M09, M15, M18, M24, M29, O02, O07, O08, O14, O19, O22, O26

T3: M06, M07, M10, M12, M16, M19, M22, M26, O05, O09, O12, O16, O20, O24, O29

T4: M04, M11, M14, M17, M20, M23, M27, M30, O03, O10, O13, O15, O18, O21, O25, O27, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Similarity makes ratios single-valued

Learning target: Prove size invariance.

Learning target: Evaluate degree inputs with exact ratios.

Use Warm-up and I Do. Have students compare two scaled drawings before introducing the circle definition.

Session 2: Circle coordinates extend the input domain

Learning target: Normalize signed coordinates.

Learning target: Explain tangent exclusions and repeated outputs.

Use We Do and Collaborative tasks. Require radius, sign and quotient checks before calculator comparison.

Session 3: Interpret and challenge function statements

Learning target: Distinguish ratios from physical lengths.

Learning target: Diagnose inverse, unit and additivity errors.

Use Independent and selected Small-group items, then collect uncued Exit evidence.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use similarity, right-triangle side roles, the Pythagorean theorem, signed coordinates and function input-output notation. Exact special-angle values may be derived from familiar right triangles.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

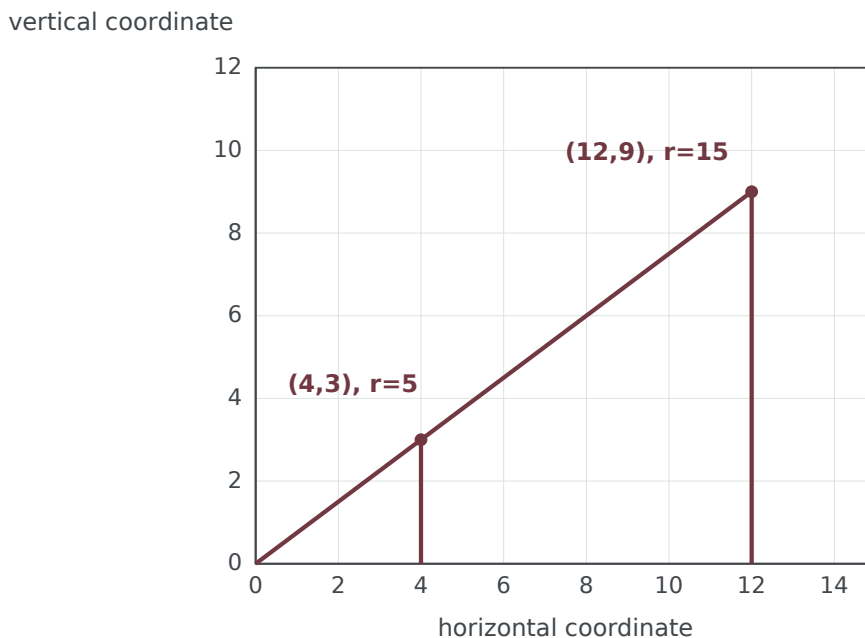
Review convention

Every numerical angle input is in degrees unless explicitly converted to radians. Angles are measured counterclockwise from the positive horizontal axis; negative angles turn clockwise. $\text{Sine} = y/r$, $\text{cosine} = x/r$ and $\text{tangent} = y/x$ where defined, with $r > 0$. Ratio outputs are dimensionless. Circle definitions extend the acute-triangle ratios. sqrt denotes the nonnegative square root. Contexts and measurements are hypothetical instructional models.

TEACHER MODEL

The same angle gives the same ratio at every scale

A terminal ray contains $(4,3)$ and $(12,9)$. Compare the sine, cosine and tangent values computed at each point. Explain why an angle input determines a ratio even though the points have different lengths.



The radii are 5 and 15. From $(4,3)$, $\text{sine} = 3/5$, $\text{cosine} = 4/5$ and $\text{tangent} = 3/4$. From $(12,9)$, the corresponding quotients are $9/15 = 3/5$, $12/15 = 4/5$ and $9/12 = 3/4$. Multiplying coordinates and radius by the same positive factor changes the triangle size but cancels from each ratio. A fixed ray therefore produces one output for each named trig function. Sine and cosine remain defined at vertical rays; tangent does not because its horizontal-coordinate denominator vanishes.

VISUAL ANCHOR

Angle in ratio out

Fix the input angle and identify the named function.

Acute triangle: sine = opposite/hypotenuse; cosine = adjacent/hypotenuse.

Tangent = opposite/adjacent; on a circle it is y/x when x is nonzero.

Circle extension: sine = y/r and cosine = x/r , with positive radius r .

A common positive scale cancels from every quotient.

One angle must give one output; different angles may share an output.

Angle units belong to the input. Ratios have no length or angle units.

A trig function encodes an angle-dependent ratio, not an unfixed side length. The coordinate definition supplies signs and domain information beyond acute triangles.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

In O01 hold the angle fixed while tripling every length; ask which numbers change and which quotients do not. O02 deliberately includes boundary angles so students must use circle coordinates. O03 separates an angle input, a ratio output and a derived physical length. In O04 keep the two constructions on separate grids and ask students to justify similarity rather than judge appearance.

Core task

A right triangle has opposite 5, adjacent 12 and hypotenuse 13 for angle alpha. A similar copy has hypotenuse 39. Find its other sides, compare all three ratios and explain which quantity is the input to each ratio function.

Teacher solution and discussion

The scale factor is $39/13=3$, so the copy has opposite 15 and adjacent 36. Sine is $5/13=15/39$; cosine is $12/13=36/39$; tangent is $5/12=15/36$. Each function takes the unchanged angle alpha as input. Side lengths vary with scale, but each dimensionless quotient stays fixed.

Evidence and response

Require all three correctly matched quotients and an explicit statement that the angle, not a length, is the input. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T3: Use signed circle coordinates and a positive radius to extend angle-to-ratio functions beyond acute triangles.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 by computing radius before signs; a negative horizontal coordinate does not make radius negative. In O06 write all three cancellations. O07 should expose the difference between a function and a one-to-one function. For O08 write the actual denominator at each angle and never label division by zero as a real output.

Core task

A point on a circle centered at the origin is $P=(-8,15)$. Determine the radius and all three trig ratios of its terminal angle θ . Explain every sign without using an inverse-trig calculator.

Teacher solution and discussion

The radius is $\sqrt{64+225}=17$. $\cos(\theta)=-8/17$, $\sin(\theta)=15/17$ and $\tan(\theta)=15/(-8)=-15/8$. The point is left of the vertical axis and above the horizontal axis, so cosine is negative, sine positive and their quotient tangent negative.

Evidence and response

Check the Pythagorean radius, coordinate ordering and independent sign justification for each ratio. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: O09 • T3: Use signed circle coordinates and a positive radius to extend angle-to-ratio functions beyond acute triangles.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

O09 asks for a circle and radius rather than a decorative triangle. Compare the two sides of O10 before discussing the definition of function. O11 needs the acute-angle restriction to justify a positive sine. O12 checks coordinate normalization at two points on one ray; O13 converts the unitless ratio into wheel height in meters.

Core task

Draw a circle of radius 5 centered at the origin and mark $P=(-3,4)$. Label its radius and terminal angle direction from the positive horizontal axis. Use the coordinates to find cosine, sine and tangent.

Teacher solution and discussion

The circle equation is $x^2+y^2=25$, and P lies on it because $9+16=25$. Draw a radius from the origin to P in quadrant II; the angle is counterclockwise from the positive horizontal axis.

$\cos(\theta)=-3/5$, $\sin(\theta)=4/5$ and $\tan(\theta)=-4/3$. The radius remains positive even though x is negative.

Evidence and response

Check the circle and terminal ray, correct quadrant and all three signed ratios. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign one partner to locate the angle, one to normalize coordinates and one to challenge the inference. O14 distinguishes repeated output from ambiguity. O15 reverses that relationship and explains why principal inverse notation does not enumerate every angle. O16 isolates the zero-denominator restriction. O17 and O18 revisit complementary angles in different physical and geometric settings.

Core task

Create an exact-value table for $C(\theta) = \cos(\theta)$ at 0, 60, 120, 180, 240, 300, 360 degrees. Describe how the repeated outputs do and do not affect its status as a function.

Teacher solution and discussion

The table is:

Input x	0	60	120	180	240	300	360
Output y	1	1/2	−1/2	−1	−1/2	1/2	1

Each angle has one horizontal unit-circle coordinate, so C is a function. Repeated outputs, such as $C(60) = C(300) = 1/2$, show it is not one-to-one on this input set.

Evidence and response

Verify every sign and boundary value, and distinguish a function from an invertible one-to-one table. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 is a derived function and can exceed the bounds of an individual sine or cosine. O20 compares a half-turn using both coordinates. O21 requires both roots before the quadrant interpretation. O22 checks calculator units explicitly. O23 tests both Pythagoras and the ratio, so a plausible larger triangle cannot pass by visual appearance alone.

Core task

For $F(\theta) = 2\sin(\theta) - \cos(\theta)$, evaluate $F(30)$, $F(90)$, $F(180)$ exactly in degree measure. Explain why the values of F can exceed 1 even though sine and cosine individually cannot.

Teacher solution and discussion

$F(30) = 2(1/2) - \sqrt{3}/2 = 1 - \sqrt{3}/2$. $F(90) = 2(1) - 0 = 2$. $F(180) = 2(0) - (-1) = 1$. F is a combination of two ratio functions, not itself a single sine or cosine ratio; the factor 2 and subtraction allow values outside $[-1, 1]$.

Evidence and response

Assess independent evaluations before combination and distinguish a derived function from a bounded basic ratio. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: Use signed circle coordinates and a positive radius to extend angle-to-ratio functions beyond acute triangles.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

O24 reconstructs a quadrant-consistent ray from tangent. O25 contrasts reflection with a full turn. In O26 retain the undefined entry visibly instead of skipping it or inserting zero. O27 requests only isolated samples; ask what additional information the full cosine definition supplies between them.

Core task

A terminal ray lies in quadrant IV and has $\tan(\theta) = -2$. Choose one coordinate point on it, find its radius, and compute sine and cosine. Explain why your choice of point does not affect the outputs.

Teacher solution and discussion

Choose $(1, -2)$, whose slope y/x is -2 and which lies in quadrant IV. Its radius is $\sqrt{5}$, so $\sin(\theta) = -2/\sqrt{5}$ and $\cos(\theta) = 1/\sqrt{5}$. Every other point on this ray is $(k, -2k)$, $k > 0$, with radius $k\sqrt{5}$; the common k cancels in both ratios.

Evidence and response

Require a quadrant-consistent coordinate choice, normalized ratios and an explanation of invariance for every positive scale. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 repairs an ambiguous length rule by fixing scale. O29 uses the quadrant to select the sign after taking a square root. O30 requires a precise function-rule rebuttal supported by two reduced ratios. Score angle convention, ratio identification and single-valued reasoning separately.

Core task

Explain why the rule “input an acute angle, output its opposite side” is ambiguous unless another length is fixed. Then repair it for hypotenuse 12 and evaluate your repaired function at 30 and 45 degrees.

Teacher solution and discussion

Similar triangles can share one acute angle but have different opposite lengths, so the original rule is not single-valued. Fixing hypotenuse 12 makes the opposite-length function $L(\theta) = 12\sin(\theta)$, $0 < \theta < 90$. Then $L(30) = 6$ and $L(45) = 6\sqrt{2}$, in the same length units as 12. The fixed scale turns a ratio into a unique length.

Evidence and response

Require a same-input ambiguity, a properly restricted repaired function and two exact length evaluations. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Two right triangles have the same acute angle θ . Their opposite/hypotenuse pairs are $3/5$ and $12/20$. What makes $\sin(\theta)$ a function of θ rather than triangle size?

- A. Sine takes the opposite side alone as its output.
- B. The ratios agree only because both hypotenuses are integers.
- C. The ratios agree because corresponding sides scale equally.
- D. The longer triangle has the larger sine.

Reasoning: _____

M02 | Warm-up | T2

Define $c(\theta) = \cos(\theta)$, with θ measured in degrees. What is $c(60)$?

- A. $\sqrt{3}/2$.
- B. 60.
- C. 2.
- D. $1/2$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.

A right triangle has opposite 5, adjacent 12 and hypotenuse 13 for angle alpha. A similar copy has hypotenuse 39. Find its other sides, compare all three ratios and explain which quantity is the input to each ratio function.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

Define $S(\theta) = \sin(\theta)$ and $C(\theta) = \cos(\theta)$ for degree inputs. Evaluate $S(0)$, $C(0)$, $S(90)$, $C(90)$. Explain why these values are valid even though no nondegenerate right triangle has an acute angle of 0 or 90 degrees.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.

Answer: C. The ratios agree because corresponding sides scale equally.

The two quotients are $\frac{3}{5}$ and $\frac{12}{20} = \frac{3}{5}$. Equal acute angles in right triangles imply similarity, so every corresponding length has the same positive scale factor. That factor cancels from opposite/hypotenuse.

Check each alternative

- A. That length varies with triangle size; the quotient is the sine.
- B. Integrality is irrelevant; similarity preserves ratios for any positive scale.
- C. Correct. The two quotients are $\frac{3}{5}$ and $\frac{12}{20} = \frac{3}{5}$. Equal acute angles in right triangles imply similarity, so every corresponding length has the same positive scale factor. That factor cancels from opposite/hypotenuse.
- D. Sine compares lengths by division, so an equal scale factor cancels.

Evidence to assess

Connect equal angles to similarity and cancellation, not merely the observed numerical equality.

M02 • T2: Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

Answer: D. $\frac{1}{2}$.

For a 60-degree angle in a 30-60-90 right triangle, adjacent:hypotenuse=1:2. Hence $\cos(60) = \frac{1}{2}$. The input has degree units; the ratio is dimensionless.

Check each alternative

- A. That is $\sin(60)$, not $\cos(60)$.
- B. The input angle is 60 degrees; it is not the ratio output.
- C. This reverses adjacent/hypotenuse to hypotenuse/adjacent.
- D. Correct. For a 60-degree angle in a 30-60-90 right triangle, adjacent:hypotenuse=1:2. Hence $\cos(60) = \frac{1}{2}$. The input has degree units; the ratio is dimensionless.

Evidence to assess

Read the function name and angle input, and identify the adjacent-to-hypotenuse quotient.

WORKED TEACHER KEYS

001 / 002**001 • T1: Explain by similarity why a fixed acute angle determines scale-independent sine, cosine and tangent ratios.**

The scale factor is $39/13=3$, so the copy has opposite 15 and adjacent 36. Sine is $5/13=15/39$; cosine is $12/13=36/39$; tangent is $5/12=15/36$. Each function takes the unchanged angle α as input. Side lengths vary with scale, but each dimensionless quotient stays fixed.

Evidence to assess

Require all three correctly matched quotients and an explicit statement that the angle, not a length, is the input.

002 • T2: Evaluate and interpret named trig functions, retaining degree conventions, dimensionless outputs and tangent domain exclusions.

At 0 degrees the unit-circle point is $(1,0)$, so $S(0)=0$ and $C(0)=1$. At 90 degrees it is $(0,1)$, so $S(90)=1$ and $C(90)=0$. The circle definition uses signed coordinates divided by a nonzero radius, extending the acute-triangle ratios to boundary angles.

Evidence to assess

Check all four values and the coordinate definition that justifies the extension beyond acute triangles.