

TEACHER AND STUDENT RESOURCE

Intersections of Square-Root and Inverse-Variation Graphs



NC Math 2 • NC.M2.A-REI.11

Reasoning with Equations and Inequalities

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-REI.11 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Extend the understanding that the x-coordinates of the points where the graphs of two square root and/or inverse variation equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values.

T1 Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

T2 Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

T3 Use successive output tables, graphing estimates and valid brackets to approximate inputs, justify rounding and distinguish exact from displayed equality.

T4 Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M04, M06, O11, M14, M17, O17, M18, O18, M23, O23, O24, M29, O29

T2: M02, O02, O04, M08, O08, M11, M13, O13, O15, M16, M19, O20, O21, M24, M25, O25, M26, O27

T3: M03, O03, O06, M07, O07, M10, O14, O19, M22, O26, M28, M30, O30

T4: M05, O05, M09, O09, O10, M12, O12, M15, O16, M20, M21, O22, M27, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Learning target: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

Learning target: Use successive output tables, graphing estimates and valid brackets to approximate inputs, justify rounding and distinguish exact from displayed equality.

Learning target: Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Learning target: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

Learning target: Use successive output tables, graphing estimates and valid brackets to approximate inputs, justify rounding and distinguish exact from displayed equality.

Learning target: Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Real square roots and domains, inverse variation $y=k/x$, ordered pairs, evaluating and comparing function outputs, exact versus approximate values, simple algebraic transformations and graph windows.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Use principal real square roots. Inverse-variation expressions retain the original denominator exclusions, including when a coefficient is zero. A graph intersection has one common input and output; $f(x)=g(x)$ asks for the input. Tables display rounded outputs unless marked exact. Bracketing by opposite signs requires continuity on the whole retained interval. A touching zero need not change sign. Graph colors distinguish original curves only; student workspaces never contain answer curves.

TEACHER MODEL

The input solves the equation; the point connects the graphs

Compare $f(x)=\sqrt{x}$ and $g(x)=3/x$ on $x>0$. Find a positive-input bracket, refine it and report an approximate intersection while explaining why it is unique.

At $x=2$, $f \approx 1.414214 < 1.5 = g$; at $x=3$, $f \approx 1.732051 > 1 = g$. Both functions are continuous on $[2, 3]$, so an intersection lies between them. At 2.08 , $f \approx 1.442221 < 1.442308 = g$; at 2.09 , $f \approx 1.445683 > 1.435407 = g$. Thus $2.08 < r < 2.09$. The equation is equivalent on $x > 0$ to $x^3 = 9$, so $r = \sqrt[3]{9} \approx 2.080084$ and the common output is $\sqrt{r} \approx 1.442250$. The approximate intersection is $(2.080084, 1.442250)$, while the equation's input solution is r . The increasing root and decreasing positive reciprocal make the meeting unique.

VISUAL ANCHOR

A trustworthy intersection search

Use the original functions, not just an equation produced by squaring or a rounded table display.

Checkpoint	What to establish
Common input	Both functions are real and defined at the same x .
Original equality	The two original outputs agree, exactly or to stated precision.
Valid numerical bracket	A continuous interval contains the meeting; no denominator gap lies inside.
Precision and completeness	Refine before rounding; explain why other windows, branches or touching points cannot be missed.

Same rounded output does not mean exact equality. A sign change across an asymptote does not mean a crossing. A table without a matching column does not prove there is no solution.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask students to distinguish input solutions from intersection points and to check both original functions. Establish the common domain before any transformation.

Core task

Find the intersection of $y=\sqrt{x+8}$ and $y=\sqrt{3x+2}$. Check both original functions and state separately the solution of $\sqrt{x+8}=\sqrt{3x+2}$ and the full graph point.

Teacher solution and discussion

The common domain is $x \geq -2/3$. Equating the nonnegative square roots is equivalent to $x+8=3x+2$, so $x=3$. Both outputs are $\sqrt{11}$. Thus the equation solution is $x=3$, while the intersection point is $(3, \sqrt{11})$. The output $\sqrt{11}$ is not another input solution.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T4: Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Optional bank: M05–M08; 005–008. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 005, including its visual when present. Model an exact original check and a successively refined table. Mark approximations explicitly. Explain why the rounding midpoint, not a bracket endpoint alone, determines nearest-hundredth reporting.

Core task

Graph $f(x)=\sqrt{x}$ and $g(x)=8/x$ for $1 \leq x \leq 9$. Find and check their exact intersection, identify the equation solution, and explain which function is greater on each side of the meeting within this interval.

Teacher solution and discussion

On $x > 0$, equality $\sqrt{x}=8/x$ gives $x\sqrt{x}=8$ and then $x^3=64$, so $x=4$ and the common output is 2. The point is (4, 2), while the equation solution is 4. The root function increases and the positive reciprocal decreases; consequently $f < g$ for $1 \leq x < 4$ and $f > g$ for $4 < x \leq 9$. At the meeting, both original outputs equal 2.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Contrast a continuous bracket, a sign change across a denominator gap, and a touching intersection. Use algebra or monotonicity to decide what the graph and finite table cannot prove alone.

Core task

Investigate the claim that $y=2/x$ and $y=5/x$ intersect because their difference changes sign from $x=-1$ to $x=1$. Make a small table, sketch both graphs with the domain gap visible, and use algebra to determine the true solution set.

Teacher solution and discussion

At -1 , $f=-2$ and $g=-5$, so $f-g=3$. At 1 , $f=2$ and $g=5$, so $f-g=-3$. Neither function is defined at 0 , so a sign change across that gap does not certify a crossing. For nonzero x , $2/x=5/x$ would imply $2=5$, which is impossible. The solution set is empty. Draw each reciprocal with separate negative and positive branches, never a line joining through $x=0$.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Use successive output tables, graphing estimates and valid brackets to approximate inputs, justify rounding and distinguish exact from displayed equality.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign domain checker, table builder, graph-window reviewer and original-equation verifier. Use reverse constructions, two increasing curves, scaling and time-unit conversion to challenge shortcut rules.

Core task

A two-decimal table lists both $\sqrt{1.59}$ and $2/1.59$ as 1.26. A learner declares that $x=1.59$ exactly solves $\sqrt{x}=2/x$. Test the claim with more precision, identify the exact input, and use $x=1.587$ and 1.588 to bracket it.

Teacher solution and discussion

At 1.59 the unrounded values are approximately 1.260952 and 1.257862, so they are not equal. On $x>0$, equality is equivalent to $x^3=4$, giving exact input $\sqrt[3]{4}\approx 1.587401$. At 1.587 the difference is negative; at 1.588 it is positive, so the input lies strictly between them. Rounded matching cells indicate closeness at the display precision, not exact equality.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Use successive output tables, graphing estimates and valid brackets to approximate inputs, justify rounding and distinguish exact from displayed equality.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require new tasks involving a discrete domain, relative-position intervals, constructing multiple meetings, precision guarantees and a full parameter classification. Ask for evidence, not a guessed decimal.

Core task

A positive-domain bisection search for $\sqrt{x}=2/x$ starts with $[1, 2]$. If the reported estimate is always the midpoint of the retained bracket, how many bracket halvings are needed to guarantee input error at most 0.01? Treat this as a worst-case guarantee, not an early exact-midpoint hit, and justify the interval assumptions.

Teacher solution and discussion

At 1, $f=1<2=g$; at 2, $f=\sqrt{2}>1=g$. Both functions are continuous on $[1, 2]$, with f increasing and g decreasing, so one crossing is retained by each valid bisection. After n halvings the width is $1/2^n$ and midpoint error is at most $1/2^{n+1}$. Five halvings give bound $1/64=0.015625$, too large. Six give $1/128=0.0078125 \leq 0.01$, so six is the minimum worst-case guarantee.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Match instruction to the observed issue: swapped coordinates, an excluded denominator, a square-root domain, an unjustified rounding step or a false sign-based candidate. Recheck the repair in the original functions.

Core task

For $f(x)=\sqrt{3x+1}$ and $g(x)=\sqrt{x+9}$, a learner correctly finds the shared input 4 but reports the equation solution as $\sqrt{13}$. Repair the response by giving the input solution, the full intersection and checks of both original functions.

Teacher solution and discussion

Equating radicands gives $3x+1=x+9$, so $x=4$. Both original outputs are $\sqrt{13}$, and the full point is $(4, \sqrt{13})$. The equation solution is the input 4, not the output $\sqrt{13}$. The common domain is $x \geq -1/3$, so the input is allowed.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Interpret original graphs, window limitations, relative values, coincident graphs and touching intersections; justify completeness without crossing a domain gap.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Require a completed table or a justified parameter argument, distinguish equation inputs from graph points, and state the scope and precision of every conclusion. Reject unsupported claims of exactness from a rounded display.

Core task

For $f(x)=\sqrt{x+5}$ and $g(x)=12/x$, make a table at $x=2, 3, 4, 5$ and graph both functions on a suitable positive-input window. Find the exact equation solution and full intersection. Explain why there are no other intersections anywhere in the common real domain.

Teacher solution and discussion

The table matches at $x=4$, where both original outputs equal 3. Thus the input solution is 4 and the point is (4, 3). The common domain is $x \geq -5$ with $x \neq 0$. Negative inputs give $g < 0$ while $f \geq 0$, so no negative-input intersection occurs. On positive inputs f increases and g decreases, making the verified meeting unique. The graph and table locate it; the original check and domain/monotonic argument establish exactness and completeness.

Evidence and response

Assess input/output interpretation, domain control, original-equation checks, table and graph construction, approximation precision and completeness arguments separately. A calculator screenshot is evidence of a displayed approximation, not automatic proof of exactness, uniqueness or absence of other roots. Require completed original-function tables when the task asks for a table.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

The graphs $y=\sqrt{x}$ and $y=8/x$ meet at (4, 2). What is the solution of $\sqrt{x}=8/x$?

- A. $y=4$
- B. $x=4$
- C. $x=4$ and $x=2$
- D. $x=2$

Reasoning: _____

M02 | Warm-up | T2

What is the common real domain for $y=\sqrt{x-2}$ and $y=6/x$?

- A. All real x except 0
- B. $x \geq 0$
- C. $x \geq 2$
- D. $x > 0$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Find the intersection of $y=\sqrt{x+8}$ and $y=\sqrt{3x+2}$. Check both original functions and state separately the solution of $\sqrt{x+8}=\sqrt{3x+2}$ and the full graph point.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

For $f(x)=\sqrt{x-1}$ and $g(x)=10/x$, state the common domain. Test each proposed intersection (5, 2), (1, 10) and (0, 0) in both original functions, and identify every valid proposed point.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

Answer: B. $x=4$

The equation asks for a shared input, the x-coordinate 4. Both original outputs at that input are 2: $\sqrt{4}=2$ and $8/4=2$.

Check each alternative

- A. This reverses the roles of input and output.
- B. The equation asks for a shared input, the x-coordinate 4. Both original outputs at that input are 2: $\sqrt{4}=2$ and $8/4=2$.
- C. The two coordinates are not two different input solutions.
- D. This is the common output, not the shared input.

Evidence to assess

Require the input 4, not output 2, with both original values checked.

WORKED TEACHER KEYS

M02

M02 • T2: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

Answer: C. $x \geq 2$

The root requires $x-2 \geq 0$, so $x \geq 2$. The reciprocal excludes $x=0$, which is already outside that interval. The endpoint $x=2$ is allowed in both functions.

Check each alternative

- A. This checks the denominator but ignores the radicand.
- B. Inputs from 0 through values below 2 fail the root domain, and the reciprocal also excludes 0.
- C. The root requires $x-2 \geq 0$, so $x \geq 2$. The reciprocal excludes $x=0$, which is already outside that interval. The endpoint $x=2$ is allowed in both functions.
- D. Positive inputs below 2 still give a negative radicand.

Evidence to assess

Require the root inequality and denominator exclusion combined correctly, including the allowed endpoint 2.

WORKED TEACHER KEYS

001

001 • T1: Interpret an intersection input as a solution of $f(x)=g(x)$, distinguish it from the common output and verify original ordered pairs.

The common domain is $x \geq -2/3$. Equating the nonnegative square roots is equivalent to $x+8=3x+2$, so $x=3$. Both outputs are $\sqrt{11}$. Thus the equation solution is $x=3$, while the intersection point is $(3, \sqrt{11})$. The output $\sqrt{11}$ is not another input solution.

Evidence to assess

Require the valid domain, equality of radicands, checked input 3, full pair and a clear distinction between input solution and common output.

WORKED TEACHER KEYS

002

002 • T2: Use the common domain and original signs to control equation transformations, exclude extraneous candidates and classify solution sets.

Both functions are defined when $x \geq 1$. At 5, $f = \sqrt{4} = 2$ and $g = 10/5 = 2$, so (5, 2) is valid. At 1, $f = 0$ while $g = 10$, so (1, 10) lies only on g. At 0, the root is not real and the reciprocal is undefined. Of the listed points, only (5, 2) is an intersection.

Evidence to assess

Require the common domain and both-function checks for all three proposals; distinguish a point on one graph from a shared point.