

TEACHER AND STUDENT RESOURCE

Counting Permutations and Combinations



DCS.SP.1

Statistics and Probability

Seven instructional sections. Thirty multiple-choice and thirty open or performance tasks, original mathematical visuals, generous workspaces, teaching scripts, and worked keys.

Free sample | Contents

DCS.SP.1 | NC Discrete Mathematics for Computer Science mathematics

This sample contains 20 complete selected pages from the 93-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75-76

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75-76

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply combinatorics concepts to solve problems.

DCS.SP.1.1 Implement the Fundamental Counting Principle to solve problems.

DCS.SP.1.2 Implement procedures to calculate a permutation or combination.

The decimal codes are the official objectives. All are represented in the item bank. These materials support several sessions, not one 60-task lesson.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

DCS.SP.1.1: M01, M02, M03, M04, M05, M06, M16, M25, M27, M29, O01, O02, O03, O04, O05, O06, O07, O08, O09, O10, O26, O28, O30

DCS.SP.1.2: M07, M08, M09, M10, M11, M12, M13, M14, M15, M17, M18, M19, M20, M21, M22, M23, M24, M26, M28, M30, O11, O12, O13, O14, O15, O16, O17, O18, O19, O20, O21, O22, O23, O24, O25, O27, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Build and interpret representations

Learning target: Implement the Fundamental Counting Principle to solve problems. Implement procedures to calculate a permutation or combination.

Use selected Warm-up and I Do tasks; compare an initial prediction with the worked model.

Session 2 Apply and explain methods

Learning target: Implement procedures to calculate a permutation or combination.

Use We Do and collaborative tasks across the objectives. Require each learner to explain one mathematical choice.

Session 3 Transfer and collect independent evidence

Learning target: Apply the listed objectives with accurate representations, justified methods, and contextual checks.

Choose independent tasks covering each objective. Reteach an evidenced gap, then use a fresh exit task without answer-supplying cues.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, printed tasks, and optional scratch paper. A calculator may support arithmetic after the method is stated. Use graph paper or movable labels when useful without supplying the requested construction.

Prerequisites

Whole-number multiplication, factorials, and describing outcomes unambiguously.

Pacing

Select two to four tasks for a session. Preserve first independent evidence, discuss a model, and revisit with a fresh task. The full banks are reusable choices, not a timed requirement.

Neutral access

Read directions aloud without identifying a method or answer. Enlarge diagrams, allow dictated or typed reasoning, and provide accessible tables. Preserve all mathematical conditions.

Keep evidence interpretable

Record neutral supports separately from answer-supplying cues. Retain first responses. A selected distractor is not by itself a diagnosis.

Conventions

$P(n,r)=n!/(n-r)!$ counts ordered selections without replacement. $C(n,r)=n!/[r!(n-r)!]$ counts unordered selections. Repetition, order, and symmetry conventions must be read before choosing a formula.

TEACHER MODEL

Define the outcome before counting

Choose 3 of 6 people and designate one leader. Count in two ways.

Method	First choice	Second choice	Total
Group first	$C(6,3)=20$	3 possible leaders	60
Leader first	6 leaders	$C(5,2)=10$	60

The outcome has one distinguished role and two unordered other members. $C(6,3) \cdot 3=60$ and $6 \cdot C(5,2)=60$ agree. $P(6,3)=120$ would incorrectly distinguish the two nonleader positions.

Think aloud: I ask whether swapping two names changes the outcome. Here swapping the nonleaders does not, but changing the leader does.

REUSABLE REASONING GUIDE

Represent justify interpret

Givens and conditions
Model and calculation
Justification and independent check
Conclusion in context

WARM-UP

Teaching script

Core 001 | DCS.SP.1.1

Collect a short first response before discussion. Ask learners what is given and which condition controls the method.

Display and ask

A club offers 3 workshops, 4 start times for each, and 2 room choices for every workshop-time pair. Count the possible bookings using a decision tree structure. Explain what independence means here.

Work and discuss

The tree has 3 first branches, 4 branches from each, and 2 from each of those, giving $3 \cdot 4 \cdot 2 = 24$ leaves. Independence here means the stated number of choices remains available after every earlier choice; it is not a claim about random events.

Listen for evidence

A labeled stage structure and interpretation of multiplication.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M01 for contrast after discussion. Select another task from M01-M04 or O01-O04 to check transfer. Record support and the new evidence separately.

I DO

Teaching script

Core 005 | DCS.SP.1.1

Make a prediction before modeling. Think aloud through the core task, making each calculation and check visible.

Display and ask

How many length 6 binary strings contain at least one 1? Count by complement and explain why directly adding counts for each position containing 1 would overcount.

Work and discuss

All strings $2^6=64$. Only 000000 has no 1, so the answer 63. Adding six separate position counts counts a string with several 1s several times; those cases are not disjoint.

Listen for evidence

Complementary counting and an overlap-based critique of a flawed method.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M05 for contrast after discussion. Select another task from M05-M08 or O05-O08 to check transfer. Record support and the new evidence separately.

WE DO

Teaching script

Core 009 | DCS.SP.1.1

Ask learners to propose the next step and have a partner verify it against the original conditions.

Display and ask

A three-digit number must end in an even digit and may repeat digits. Count such numbers. Then explain how the count changes if it is instead a three-character digit code that may start with 0.

Work and discuss

Numbers: $9 \cdot 10 \cdot 5 = 450$. Codes: $10 \cdot 10 \cdot 5 = 500$. The even choices are 0,2,4,6,8 in either case; only the first-position restriction changes, adding 50 strings beginning 0.

Listen for evidence

Two related sample spaces and a precise explanation of the difference.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M09 for contrast after discussion. Select another task from M09-M13 or O09-O13 to check transfer. Record support and the new evidence separately.

COLLABORATIVE PRACTICE

Teaching script

Core O14 | DCS.SP.1.2

Assign reader, modeler, checker, and explainer roles. Require a private first attempt before comparing approaches.

Display and ask

How many distinct arrangements of the letters in LEVEL exist? Explain each repeated-letter correction and verify with a position-selection method.

Work and discuss

LEVEL has 5 letters with L twice, E twice, V once. Count $5!/(2!2!)=30$. Alternatively choose 2 of 5 L positions, then 2 of 3 remaining E positions: $C(5,2)C(3,2)=10 \cdot 3=30$; V takes the last position.

Listen for evidence

All repeated groups and an independent position-based check.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M14 for contrast after discussion. Select another task from M14-M18 or O14-O18 to check transfer. Record support and the new evidence separately.

INDEPENDENT PRACTICE

Teaching script

Core O19 | DCS.SP.1.2

Provide only neutral access supports. Keep the first response and ask for justification without selecting a method.

Display and ask

Ten volunteers include Alex and Blair. Choose a 4-person committee that includes Alex but excludes Blair. Count and explain both reductions in the selection pool.

Work and discuss

Alex is fixed in the group and Blair is removed. Choose 3 remaining members from the other 8 people: $C(8,3)=56$. Neither Alex nor Blair remains available in the pool of 8.

Listen for evidence

Mandatory and forbidden members handled before the combination count.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M19 for contrast after discussion. Select another task from M19-M23 or O19-O23 to check transfer. Record support and the new evidence separately.

SMALL-GROUP RETEACH

Teaching script

Core O24 | DCS.SP.1.2

Use the earlier work to identify a specific gap. Model that connection, then remove the cue on a fresh item.

Display and ask

Explain combinatorially why $C(n,r)=C(n,n-r)$. Illustrate with selecting 2 versus excluding 2 from a 7-person group, and evaluate both counts.

Work and discuss

Each r -subset uniquely determines its $(n-r)$ -element complement, and complementing twice returns the original subset. For $n=7$, selecting 2 or selecting the 5 remaining people both give $C(7,2)=C(7,5)=21$.

Listen for evidence

A reversible correspondence and correct numerical example.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M24 for contrast after discussion. Select another task from M24-M27 or O24-O27 to check transfer. Record support and the new evidence separately.

EXIT TICKET

Teaching script

Core O28 | DCS.SP.1.1

Collect a concise claim and supporting computation before showing the key. Use the evidence to select a next task.

Display and ask

A test contains 4 true-false questions followed by 2 multiple-choice questions with 3 options each. Count complete answer sheets with exactly the final multiple-choice answer fixed to a given option.

Work and discuss

The four true-false responses give $2^4=16$ choices. The first multiple-choice answer has 3 choices; the final fixed answer has 1. Total $16 \cdot 3 \cdot 1=48$. Fixing one answer removes a factor 3 from the unrestricted 144 sheets.

Listen for evidence

Independent response positions and a fixed-choice factor of 1.

Probe and respond

Ask whether swapping two choices changes the outcome. If a formula is unsupported, list a tiny analogous sample space and identify duplicates before returning to the original scale.

Next: use M28 for contrast after discussion. Select another task from M28-M30 or O28-O30 to check transfer. Record support and the new evidence separately.

MULTIPLE-CHOICE BANK

M01 / M02 Select and justify

M01 | Warm-up | DCS.SP.1.1

A meal chooses one of 3 mains, one of 4 sides, and one of 2 drinks. Every combination is offered. How many meals are possible?

- A. 9
- B. 12
- C. 24
- D. 8

Reasoning: _____

M02 | Warm-up | DCS.SP.1.1

A code has 4 positions, each chosen from A,B,C. Repetition is allowed. How many codes exist?

- A. 24
- B. 81
- C. 12
- D. 6

Reasoning: _____

WARM-UP

001 Show your reasoning

DCS.SP.1.1 | Implement the Fundamental Counting Principle to solve problems.

A club offers 3 workshops, 4 start times for each, and 2 room choices for every workshop-time pair. Count the possible bookings using a decision tree structure. Explain what independence means here.

Representation and calculation

Explanation and check

WARM-UP

002 Show your reasoning

DCS.SP.1.1 | Implement the Fundamental Counting Principle to solve problems.

A code is one letter A-E followed by 3 digits 0-9, with repetition allowed. Count all codes and those whose final digit is odd. Explain why a leading 0 in the digit portion is permitted.

Representation and calculation

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 | DCS.SP.1.1****Answer C: 24**

Each of 3 mains pairs with 4 sides, and each of those 12 pairs has 2 drinks: 24 outcomes.

- A. This adds stage choices instead of counting combined outcomes.
- B. This omits the drink choice.
- C. Correct. Multiply $3 \cdot 4 \cdot 2$.
- D. This omits the main choice.

Evidence: Independent staged choices and a product interpretation.

M02 | DCS.SP.1.1**Answer B: 81**

The ordered code is determined by four independent choices, each with 3 options.

- A. This uses $4!$ as though four distinct symbols were arranged.
- B. Correct. Each position has 3 choices, so $3^4=81$.
- C. This adds 3 choices across 4 positions instead of multiplying.
- D. This counts permutations of all 3 symbols, not 4 positions with repetition.

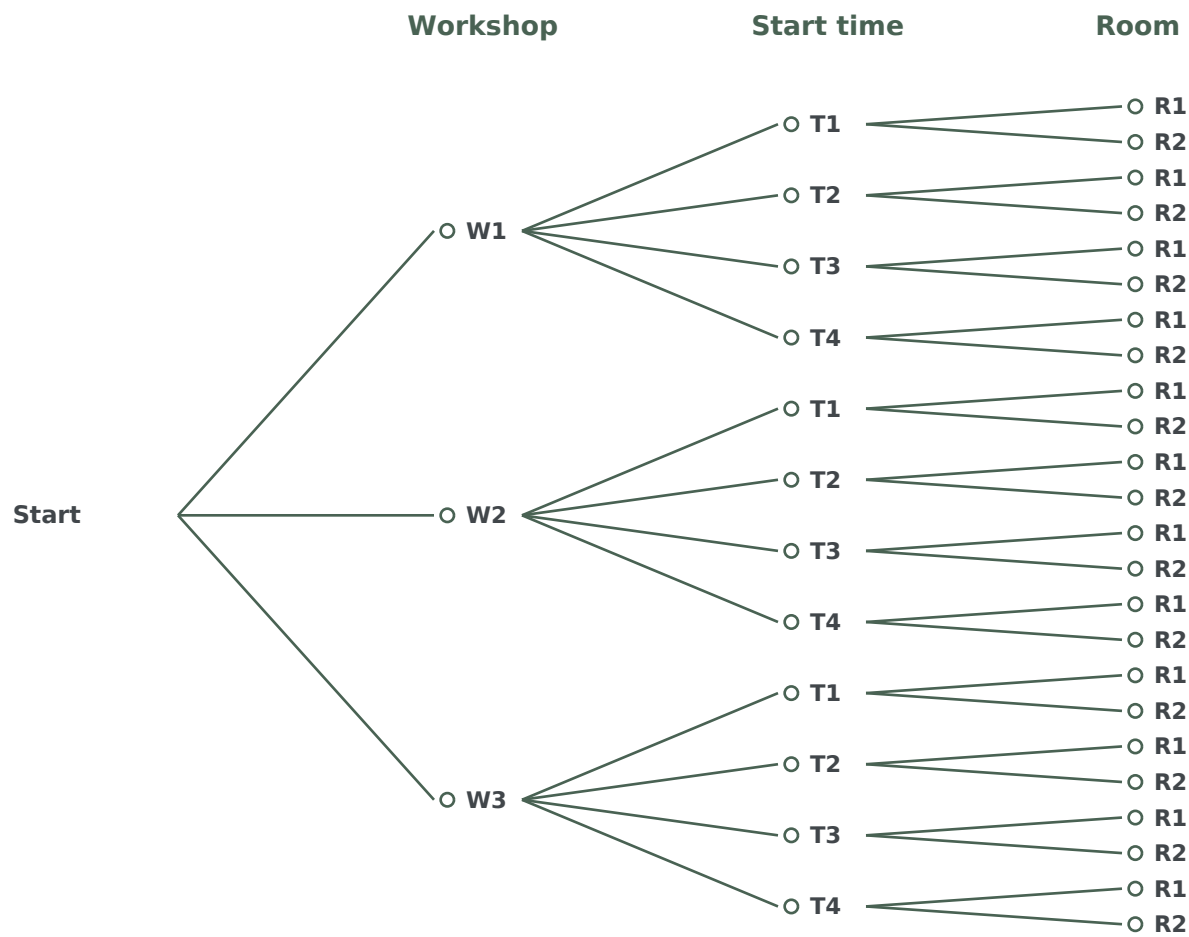
Evidence: Position order and allowed repetition.

WORKED TEACHER KEYS

001

001 | DCS.SP.1.1

The tree has 3 first branches, 4 branches from each, and 2 from each of those, giving $3 \cdot 4 \cdot 2 = 24$ leaves. Independence here means the stated number of choices remains available after every earlier choice; it is not a claim about random events.



Evidence: A labeled stage structure and interpretation of multiplication.

WORKED TEACHER KEYS

002 / 003**002 | DCS.SP.1.1**

All codes: $5 \cdot 10^3 = 5000$. Odd final digit has 5 choices, so $5 \cdot 10 \cdot 10 \cdot 5 = 2500$. The digit portion is part of a code, not a three-digit number, so the prompt allows 0 in its first digit position.

Evidence: Position-specific choices and distinction between codes and numerical representations.

003 | DCS.SP.1.1

First digit has 9 choices. Second has 9 choices from the remaining digits, including 0. Third has 8 and fourth 7. Product $9 \cdot 9 \cdot 8 \cdot 7 = 4536$. Do not exclude 0 after the first position unless it has already been used.

Evidence: A correct conditional choice count and explanation of zero placement.