

TEACHER AND STUDENT RESOURCE

Develop conditional probability and independence



NC Math 2 • NC.M2.S-CP.3

Conditional Probability and the Rules for Probability

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.S-CP.3 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 125-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 33-34, 63-64, 93-94

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 33

O02: sample PDF page(s) 19; original page(s) 34

Selected teacher prompts and keys: original page(s) 63-64, 93-94

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Develop and understand independence and conditional probability.

- a. Use a 2-way table to develop understanding of the conditional probability of A given B (written $P(A|B)$) as the likelihood that A will occur given that B has occurred. That is, $P(A|B)$ is the fraction of event B's outcomes that also belong to event A.
- b. Understand that event A is independent from event B if the probability of event A does not change in response to the occurrence of event B. That is $P(A|B)=P(A)$.

T1 Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

T2 Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M04, M07, M08, M10, M20, M24, M25, M26, M28, O01, O02, O03, O04, O07, O08, O10, O20, O24, O25, O26, O28

T2: M05, M06, M09, M11, M12, M13, M14, M15, M16, M17, M18, M19, M21, M22, M23, M27, M29, M30, O05, O06, O09, O11, O12, O13, O14, O15, O16, O17, O18, O19, O21, O22, O23, O27, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Learning target: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Learning target: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Use fractions, percentages, row and column totals, and the meaning of intersection. Understand that an event is a subset of a specified sample space.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Choose a member uniformly from the stated finite population unless a different model is named. A table proportion is exact for that empirical model, not proof about a broader population. Conditional probabilities require a positive conditioning total. In $P(A|B)$, B is the condition. Independence does not mean disjointness or prove causation.

TEACHER MODEL

Restrict the group before forming the fraction

For a random member of the table, find $P(A|B)$, compare it to $P(A)$, and decide whether knowing B changes the likelihood of A.

	A	Not A	Total
B	6	4	10
Not B	18	12	30
Total	24	16	40

Counts describe the stated population; each member appears in one interior cell.

Say: Given B means I temporarily cover the not B row. Six of the ten B outcomes also belong to A, so $P(A|B)=6/10=3/5$. Uncover the whole table: 24 of 40 outcomes belong to A, so $P(A)=24/40=3/5$. The equality means A is independent of B. Different row totals do not matter if their proportions agree.

VISUAL ANCHOR

The denominator tells you the question

The same overlap count answers different questions when the reference group changes.

	A	Not A	Total
B	8	12	20
Not B	2	18	20
Total	10	30	40

Counts describe the stated population; each member appears in one interior cell.

$P(A|B)=8/20$: among B, how many are A? $P(B|A)=8/10$: among A, how many are B?
 $P(A \cap B)=8/40$: among everyone, how many are both? $P(A)=10/40$: among everyone, how many are A? Read the words before selecting a denominator.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Sample space, events and known information**Representation and calculation****Justification or check****Conclusion in context**

WARM-UP

Teaching script

Core: 001 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask learners to circle the conditioning group in the story. Ask, “Out of which people?” before asking for a fraction.

Core task

In a group of 40 people, 16 wear glasses. Of those 16, 6 also wear hats. Explain $P(\text{hat} | \text{glasses})$ using a restricted reference group. Calculate it and contrast it with the joint probability.

Teacher solution and discussion

Restrict to the 16 glasses-wearers; 6 also wear hats. $P(\text{hat} | \text{glasses}) = 6/16 = 3/8$. The joint probability is $6/40 = 3/20$, the fraction of the whole group wearing both. The numerator stays 6 while the denominator changes.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T2: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Cover the irrelevant row, model the restricted fraction, then uncover the full table for the marginal probability. State explicitly why unchanged proportions establish independence.

Core task

Use the table to decide whether A is independent of B for a random member of this population. Compare conditional and marginal probabilities and explain what remains unchanged.

Teacher solution and discussion

$P(A|B)=6/10=3/5$. $P(A)=(6+18)/40=24/40=3/5$. Thus A and B are independent in this finite-population model: knowing B does not change the chance of A. Row counts need not match.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Have one learner locate the overlap and another choose the denominator. Compare reverse conditionals and ask whether the actual groups or only the fractions coincide.

Core task

Construct a 2-by-2 table with 40 people, 20 in A and 10 in B, such that A and B are independent. Include all margins and verify the conditional criterion.

Teacher solution and discussion

The overlap must satisfy $x/10=20/40=1/2$, so $x=5$. B row=[5, 5]; not B row=[15, 15]; column totals=[20, 20]; grand total 40. $P(A|B)=5/10=1/2=P(A)$.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign table builder, fraction checker, independence critic and context explainer. Compare counterexamples to claims that equal margins or overlap alone establish independence.

Core task

Create two tables on 20 members that both have 10 in A and 10 in B. Make one independent and one dependent. Explain why the same margins cannot determine independence.

Teacher solution and discussion

Independent table: B row [5, 5], not B [5, 5], giving $5/10 = 10/20$. Dependent example: B row [8, 2], not B [2, 8], giving $8/10 \neq 10/20$. Both retain the same margins; the overlaps differ, changing the conditional proportions.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Determine independence by comparing $P(A|B)$ with $P(A)$, and explain unchanged probability.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Keep the first table and calculation. Provide neutral reading supports but do not point to a denominator or supply the independence comparison.

Core task

The B row is [4 in A, 6 not A]. The not B row must total 25. Find the counts in that row that make A independent of B; justify the choice.

Teacher solution and discussion

The B-row A proportion is $4/10=2/5$. Thus the not B row needs $(2/5)25=10$ in A and 15 not in A. Overall $P(A)=14/35=2/5$, equal to $P(A|B)$.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use a two-row table with unequal group sizes. Ask the learner to physically trace the restricted group, then transfer to a reversed or complemented condition.

Core task

Use the table to critique the claim $P(A|B)=14/50$. Calculate the requested value and explain what $14/50$ represents.

Teacher solution and discussion

The B total is 20, so $P(A|B)=14/20=7/10$. The fraction $14/50=7/25$ uses the whole population and is $P(A \cap B)$, the chance of both events before B is known.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Collect a conditional fraction, its group-specific interpretation, and an independence comparison. Revisit students who compute correctly but reverse the context.

Core task

A museum table shows 120 visitors, 45 members, and 27 members who used an audio guide. Give $P(\text{audio guide} | \text{member})$ as a fraction and percentage. Write one correct sentence and one plausible but incorrect reversed interpretation, labeling the latter.

Teacher solution and discussion

The conditional is $27/45 = 3/5 = 60\%$. Correct: 60% of museum members in the table used a guide. Incorrect reversal: 60% of guide users were members. The guide-user total is not given, so that reversed percentage is not established.

Evidence and response

Require learners to point to the numerator cell and trace the conditioning row or column before calculating. Record reference-group understanding separately from arithmetic. For independence, require the two compared proportions and an interpretation, not a yes/no label alone.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

To find $P(A|B)$ from a two-way frequency table, which group is the new reference group?

- A. Only outcomes in B
- B. Only outcomes outside B
- C. Only outcomes in A
- D. All outcomes in the table

Reasoning: _____

M02 | Warm-up | T1

There are 60 students. Twenty ride a bus, and 8 of those bus riders bring lunch. What is $P(\text{brings lunch} | \text{rides bus})$?

- A. $8/20$
- B. $8/60$
- C. $20/8$
- D. $20/60$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

In a group of 40 people, 16 wear glasses. Of those 16, 6 also wear hats. Explain $P(\text{hat} | \text{glasses})$ using a restricted reference group. Calculate it and contrast it with the joint probability.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Use the table to calculate $P(A|B)$ and $P(B|A)$. Explain the difference using the actual reference groups rather than just notation.

	A	Not A	Total
B	8	12	20
Not B	2	18	20
Total	10	30	40

Counts describe the stated population; each member appears in one interior cell.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

Answer: A. Only outcomes in B

Given B restricts attention to B. Count outcomes also in A within that B group.

Check each alternative

A. Given B restricts attention to B. Count outcomes also in A within that B group.

B. This conditions on B^c instead of B.

C. This reverses the condition and would support $P(B|A)$.

D. This denominator gives an unconditional or joint probability, not the requested conditional probability.

Evidence to assess

Require naming the conditioning group and explaining the restriction.

WORKED TEACHER KEYS

M02**M02 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.****Answer: A. $8/20$**

The known group is the 20 bus riders. Eight also bring lunch, so the probability is $8/20=2/5$.

Check each alternative

- A. The known group is the 20 bus riders. Eight also bring lunch, so the probability is $8/20=2/5$.
- B. This is the joint fraction of all students in both categories.
- C. This inverts the favorable and reference counts and exceeds 1.
- D. This is the marginal probability of riding a bus.

Evidence to assess

Require numerator 8 and denominator 20 with their meanings.

WORKED TEACHER KEYS

001**001 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.**

Restrict to the 16 glasses-wearers; 6 also wear hats. $P(\text{hat}|\text{glasses})=6/16=3/8$. The joint probability is $6/40=3/20$, the fraction of the whole group wearing both. The numerator stays 6 while the denominator changes.

Evidence to assess

Item evidence: restricted group of 16; $3/8$ conditional; $3/20$ joint; denominator comparison. Award one point per named requirement, up to 4. Accept equivalent fractions and valid alternative reasoning.

WORKED TEACHER KEYS

002

002 • T1: Use a two-way table to interpret $P(A|B)$ as the fraction of B outcomes also in A.

$P(A|B)=8/20=2/5$; $P(B|A)=8/10=4/5$. The first refers to 20 B members, the second to 10 A members. Both numerators are the same eight members in the overlap.

Evidence to assess

Item evidence: $2/5$; $4/5$; two correct reference groups; common-overlap explanation. Award one point per named requirement, up to 4. Accept equivalent fractions and valid alternative reasoning.