

TEACHER AND STUDENT RESOURCE

Model Together: Equations, Constraints and Feasible Choices



NC Math 3 • NC.M3.A-CED.3

Creating Equations

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-CED.3 | NC Math 3 mathematics

This sample contains 22 complete selected pages from the 112-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Create systems of equations and/or inequalities to model situations in context.

T1 Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

T2 Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

T3 Create systems of inequalities or mixed equations and inequalities for budgets, capacities, requirements and design choices, retaining boundary and discrete-domain meanings.

T4 Audit and construct contextual systems by checking units, domains, logical requirements, feasibility, independence and whether the available information determines a unique solution.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M05, M10, M17, M24, O01, O05, O11, O17, O24

T2: M03, M06, M07, M08, M11, M14, M18, M21, M25, M29, O02, O04, O06, O09, O13, O14, O19, O22, O25, O29

T3: M04, M09, M12, M15, M19, M22, M26, M28, O03, O07, O12, O15, O18, O21, O26, O28

T4: M13, M16, M20, M23, M27, M30, O08, O10, O16, O20, O23, O27, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Translate independent facts

Learning target: Create unit-consistent linear and nonlinear systems.

Learning target: Explain what each variable, coefficient and relation represents.

Use Warm-up and I Do. Collect a model before solving, then verify the original quantities rather than only a transformed equation.

Session 2: Describe and test feasible sets

Learning target: Create upper, lower and mixed constraints.

Learning target: Interpret intersections, regions, discrete counts and alternative regimes.

Use We Do and Collaborative tasks. Ask students to identify the exact constraint that rejects a proposed plan. Include a graph, a table and a parameterized description as different ways to make a feasible set explicit.

Session 3: Diagnose, design and justify

Learning target: Distinguish inconsistent, redundant and uniquely determining information.

Learning target: Construct systems that meet prescribed contextual or feasibility conditions.

Use Independent practice, Small-group reteach and Exit evidence. Require an argument for completeness or an attaining design when a task asks for an extremum.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Define variables and units, translate weighted totals and comparisons, solve basic linear and factored quadratic equations, interpret graph intersections and inequality boundaries, and recognize rates as quotients.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

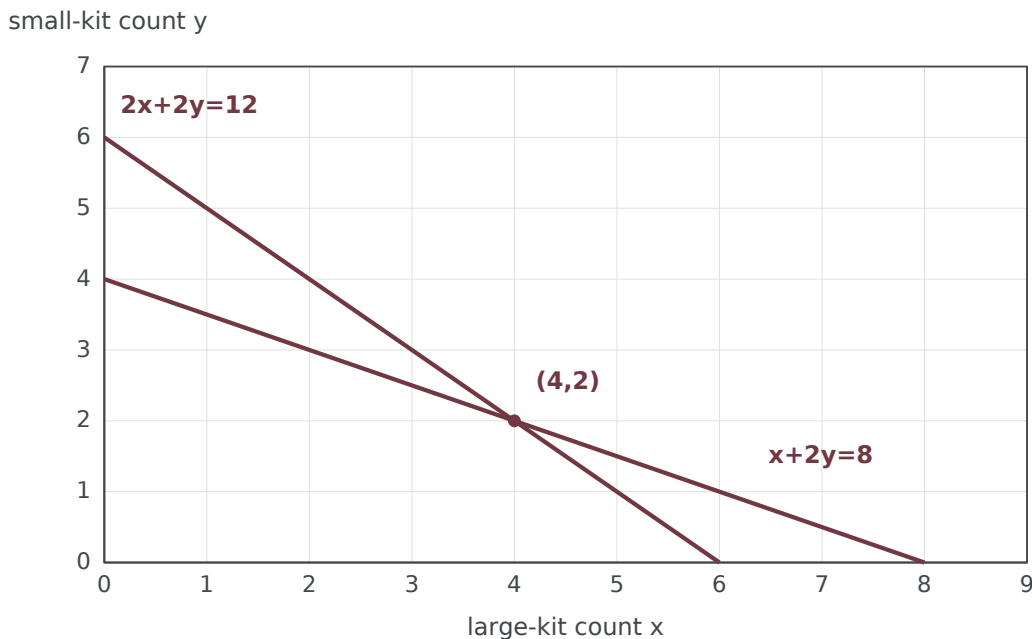
Review convention

All contexts and data are hypothetical instructional models, not operational recommendations. State units and domains explicitly. A system means all listed constraints hold simultaneously unless an alternative or is stated. A real algebraic intersection may be excluded by a positive-length or whole-number context. A verified feasible point does not by itself prove uniqueness or optimality. Keep exact values until a requested approximation.

TEACHER MODEL

Two resources, two quantities, one simultaneous plan

A project makes x large kits and y small kits. Each kit uses 2 fabric panels, and the large/small kits use 1 and 2 fasteners respectively. Exactly 12 panels and 8 fasteners must be used. Create a complete count model, solve it and interpret the graph.



Panel conservation gives $2x+2y=12$, while fastener conservation gives $x+2y=8$. Require nonnegative integer counts. Subtract the second equation from the first to get $x=4$; then $4+2y=8$ gives $y=2$. Check panels: $2 \cdot 4 + 2 \cdot 2 = 12$. Check fasteners: $4 + 2 \cdot 2 = 8$. The two lines meet at $(4,2)$, so the shared point simultaneously describes both exact resource totals. If the instructions instead said at most, the equalities would need to become inequalities and the intersection alone would no longer describe the whole feasible set.

VISUAL ANCHOR

A system is a promise to satisfy every condition

Define a variable for each unknown quantity and record its unit.

Translate each separate fact into its own equation or inequality.

Use rates for simultaneous work; use weighted totals for prices, solute and resources.

Keep contextual domains: positive dimensions, nonnegative amounts, whole counts or continuous times.

Check and versus or, exact versus bounded, and strict versus included boundaries.

Solve or test the entire system, then verify in the original statements.

Ask separately: is a point feasible, is it unique, and is it optimal?

Do not let algebra silently change what the model promises. The same equations can have different feasible answers when units, domains or logical requirements change.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

In O01 name kilograms before writing dollar coefficients. O02 contrasts a linear perimeter equation with a product area equation and gives two ordered intersections for one shape. O03 asks for each failing resource rather than a vague wrong label. O04 uses a finite table to make the domain of the comparison explicit.

Core task

A shop blends two hypothetical dry ingredients priced at \$4 and \$7 per kilogram. It needs 12 kg costing exactly \$60 before any fees. Define both variables, create a system, solve it and check the result using mass and cost separately. Explain why adding $4+7$ cannot replace the weighted-cost equation.

Teacher solution and discussion

Let x and y be kilograms of the \$4/kg and \$7/kg ingredients. The system is $x+y=12$ and $4x+7y=60$, with $x,y \geq 0$. Subtract $4(x+y)=48$ from the cost equation to obtain $3y=12$, so $y=4$ and $x=8$. The mass check is $8+4=12$ kg; the cost check is $4 \cdot 8 + 7 \cdot 4 = 32 + 28 = \$60$. The sum $4+7$ adds unit prices, not total cost; each price must be multiplied by the amount bought.

Evidence and response

Define variables with units, create both conservation equations, solve without losing the quantity labels and verify the two different totals. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 with three separate quantities rather than hiding one in an incomplete total. O06 considers both absolute-value regions to justify uniqueness. In O07 distinguish the closed triangular region from its three vertices. O08 shows why a real intersection and a valid count plan are different conclusions.

Core task

Three hypothetical meal portions contain x grams of grain, y grams of beans and z grams of vegetables. Total mass is 240 g, bean mass is half grain mass, and vegetable mass is 30 g greater than bean mass. Create a three-variable system, solve it and explain why the total equation must include all three unknowns.

Teacher solution and discussion

The system is $x+y+z=240$, $y=x/2$ and $z=y+30$, with nonnegative real masses. Write $x=2y$ and $z=y+30$; substitution gives $2y+y+y+30=240$, so $4y=210$ and $y=52.5$. Then $x=105$ and $z=82.5$. The total is $105+52.5+82.5=240$ g, beans are half of grain, and vegetables exceed beans by 30 g. Omitting a component from the total would describe only a partial mass, not the full portion. Fractional grams are allowed by the continuous mass model.

Evidence and response

Represent three distinct quantities, combine proportional and additive constraints and verify both comparisons as well as the overall total. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

O09 combines rates, not times; use job-per-hour units to check the equation. O10 asks what extra information is actually new. O11 permits either speed equations or distance equations if units match. O12 keeps ingredient ranges linked by the exact total. O13 requires the diagonal, not area, formula and checks the negative root.

Core task

A machine can finish a job in x hours and a second in y hours, with $x, y > 0$. Together they need 3 hours, and the second takes 8 hours longer alone. Create and solve a system. Explain why adding completion times instead of rates would model the wrong situation.

Teacher solution and discussion

Use $1/x + 1/y = 1/3$ and $y = x + 8$. Substituting gives $1/x + 1/(x+8) = 1/3$. Multiply by $3x(x+8)$, valid for $x > 0$: $3(x+8) + 3x = x(x+8)$, so $x^2 + 2x - 24 = (x+6)(x-4) = 0$. Only $x = 4$ is positive, so $y = 12$. Check rates: $1/4 + 1/12 = 1/3$ job per hour. Adding times would not represent simultaneous work; the quantities that add are the fractions of a job completed per hour.

Evidence and response

Create a rate equation and time-difference equation, solve with original restrictions and verify the combined rate. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O14 makes growth and decay comparable at one continuous time. O15 derives an attainable upper bound by combining two inequalities. O16 uses counterexamples to separate strictness errors from a forced-total error. O17 shows how adding invoice equations loses their separate information. O18 combines a continuous resource bound with an integer conclusion and verifies an attaining plan.

Core task

A quantity A starts at 18 and grows by 50% per period; a quantity B starts at 108 and decays by a factor of $\frac{1}{2}$ per period. Use continuous time $t \geq 0$. Create a system for their common amount y , solve exactly for the equality time using logarithms if needed, and give a numerical check.

Teacher solution and discussion

The system is $y = 18(3/2)^t$ and $y = 108(1/2)^t$. Equality gives $(3/2)^t / (1/2)^t = 6$, hence $3^t = 6$ and $t = \ln(6)/\ln(3) \approx 1.63093$. Substitution into either model gives $y \approx 34.8710$. For instance evaluating the two expressions at the unrounded logarithmic time produces the same value because their ratio is $3^t/6 = 1$. The equation has one time since 3^t is strictly increasing. Time need not be a whole number in the specified continuous model.

Evidence and response

Create growth and decay equations, reduce their equality using the ratio of bases and verify the common output without prematurely imposing an integer time. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 retains a positive domain after clearing a denominator. O20 normalizes half-hour and hourly rates before equating costs. O21 contrasts equal-area designs with different perimeters. O22 separates the full algebraic solution set from a later contextual restriction. O23 proves an entire parameter interval rather than checking a few favorable cases.

Core task

A positive input x produces response $y=20/x$. Another device requires $y=x-1$ for the same input and response. Create the system, solve it and graph the two relationships in the positive-response window. Explain why the negative root of the derived quadratic is not a valid device setting.

Teacher solution and discussion

The system is $y=20/x$ and $y=x-1$ with $x>0$. Equating gives $20/x=x-1$, so $x^2-x-20=(x-5)(x+4)=0$ after multiplying by nonzero x . The candidate $x=-4$ violates the positive-input condition. Thus $x=5$ and $y=4$, which checks $20/5=5-1=4$. Draw the positive reciprocal branch and the visible increasing line, marking their sole feasible intersection. The excluded negative root is not an extra allowed device setting even though it solves the cleared quadratic.

Evidence and response

Create a rational-linear comparison, retain its original domain through elimination and connect the valid ordered pair to both graphs. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O24 for swapped totals and unit sense, O25 for students who stop after solving a subsystem, O26 for and-versus-or confusion, and O27 for dependent equations mistaken as independent data. Ask for a fresh counterexample or exact contradiction after discussing the initial error.

Core task

A total of 26 vehicles are bicycles or tricycles, with 62 wheels altogether. A learner writes $2b+3t=26$ and $b+t=62$. Repair the system, solve it and explain a quick upper or lower bound that would have exposed the swapped totals.

Teacher solution and discussion

Let b be bicycles and t tricycles. The correct system is $b+t=26$ and $2b+3t=62$, with nonnegative integer counts. Subtract twice the count equation from the wheels equation: $t=62-52=10$. Then $b=16$. Check: $16+10=26$ and $32+30=62$ wheels. With 26 vehicles each having at least two wheels, the wheel total must be at least 52, so treating 26 as the wheel total is impossible. Labels and units reveal the swap before elimination.

Evidence and response

Repair mismatched count and weighted-count equations, solve and use a contextual bound as an independent sanity check. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Create systems of inequalities or mixed equations and inequalities for budgets, capacities, requirements and design choices, retaining boundary and discrete-domain meanings.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 requires all integer plans on a specified slice and one example showing the slice is not the full set. O29 reverses modeling direction by asking for a context and meaningful ordering condition. O30 designs constraints from prescribed outcomes. Assess equation creation, domain, verification and interpretation separately, including justified alternatives where construction is open-ended.

Core task

A class orders x reusable boxes costing \$7 each and y refill packs costing \$3 each. It needs at least 12 total items, at least 4 boxes, and may spend at most \$60. Create the system with the correct domain, find every feasible plan using exactly 12 items, and explain why this is only a slice of the full feasible set.

Teacher solution and discussion

The full model is x, y nonnegative integers, $x + y \geq 12$, $x \geq 4$ and $7x + 3y \leq 60$. On the requested slice $x + y = 12$, write $y = 12 - x$. The budget becomes $7x + 3(12 - x) = 36 + 4x \leq 60$, so $x \leq 6$. Together with $x \geq 4$ and integrality, x is 4, 5 or 6, yielding plans (4, 8), (5, 7), (6, 6). Their costs are 52, 56 and 60. The full system also permits more than 12 items, such as (4, 9) with total 13 and cost 55; the slice is not the entire feasible set.

Evidence and response

Create a complete integer system, exhaust a specified equality slice and explicitly avoid mistaking that slice for all allowed designs. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A booth sells 18 tickets: adult tickets cost \$8 and student tickets cost \$5. Receipts total \$120. If a and s are the two ticket counts, which system models both totals?

- A. $a+s=18$ and $5a+8s=120$.
- B. $a+s=18$ and $8a+5s=120$; a, s are nonnegative integers.
- C. $8a+5s=120$ and $a-s=18$.
- D. $a+s=120$ and $8a+5s=18$.

Reasoning: _____

M02 | Warm-up | T1

Mix x liters of 10% solution and y liters of 30% solution to make 20 L of 25% solution, with additive volumes. Which system is correct?

- A. $x+y=20$ and $0.10x+0.30y=0.25$.
- B. $x+y=20$ and $0.10x+0.30y=25$.
- C. $x+y=20$ and $0.10x+0.30y=5$, with $x, y \geq 0$.
- D. $x+y=5$ and $0.10x+0.30y=20$.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

A shop blends two hypothetical dry ingredients priced at \$4 and \$7 per kilogram. It needs 12 kg costing exactly \$60 before any fees. Define both variables, create a system, solve it and check the result using mass and cost separately. Explain why adding $4+7$ cannot replace the weighted-cost equation.

Representation / calculation

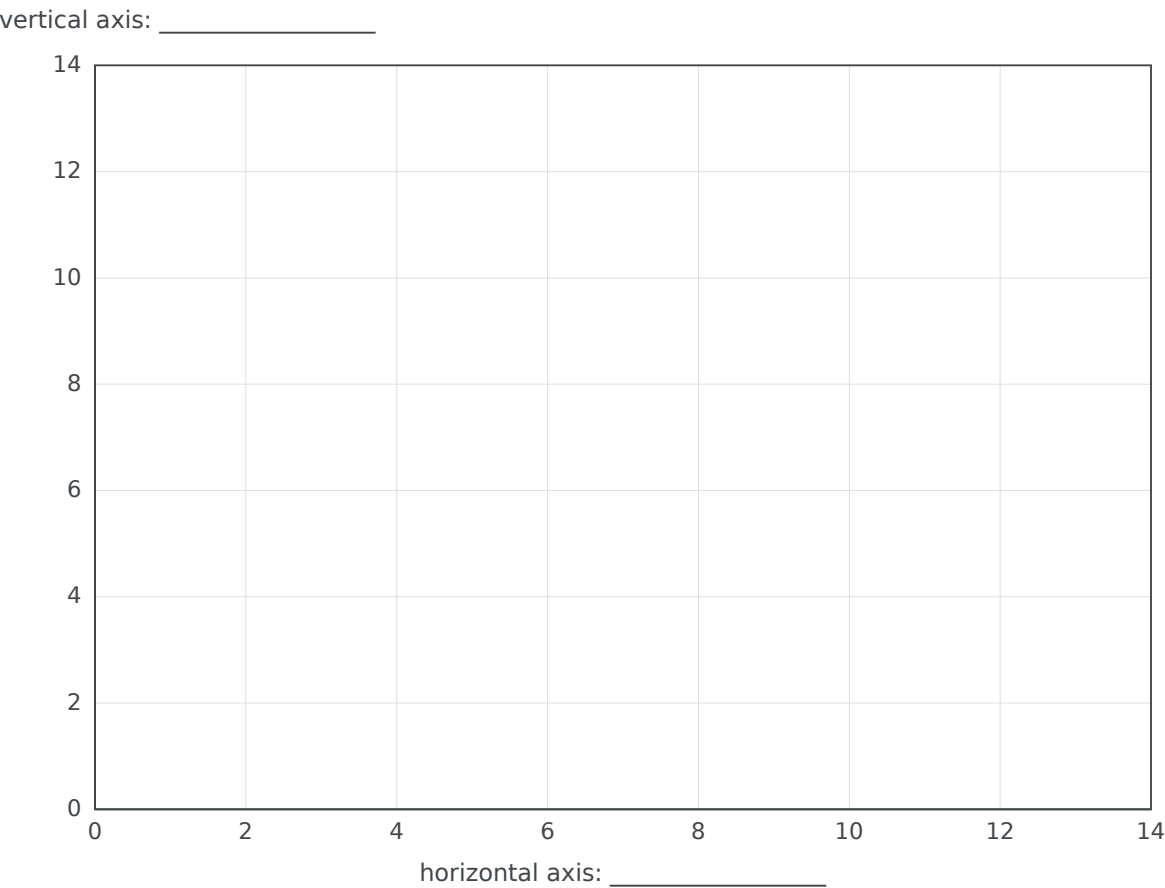
Explanation / check

WARM-UP

002 • Show your reasoning

T2: Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

A rectangle has positive width w and length L , perimeter 28 and area 48. Create a system, solve all positive ordered solutions and graph both relationships for positive dimensions in the provided window. Explain whether the two solutions necessarily represent two different shapes.



Explanation and check

WORKED TEACHER KEYS

M01

M01 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Answer: B. $a+s=18$ and $8a+5s=120$; a,s are nonnegative integers.

Each ticket contributes one to the count, so $a+s=18$. Adult receipts are $8a$ and student receipts $5s$, so $8a+5s=120$. The counts must be nonnegative integers. The pair $(a,s)=(10,8)$ checks both totals: 18 tickets and $80+40=\$120$.

Check each alternative

- A. This assigns the student price to adults and the adult price to students.
- B. Correct. Each ticket contributes one to the count, so $a+s=18$. Adult receipts are $8a$ and student receipts $5s$, so $8a+5s=120$. The counts must be nonnegative integers. The pair $(a,s)=(10,8)$ checks both totals: 18 tickets and $80+40=\$120$.
- C. The total number sold is a sum, not a difference.
- D. This interchanges a count total and a dollar total.

Evidence to assess

Assign each coefficient to the correct quantity and retain whole-number restrictions when creating two independent conservation equations.

WORKED TEACHER KEYS

M02

M02 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Answer: C. $x+y=20$ and $0.10x+0.30y=5$, with $x,y\geq 0$.

Total volume gives $x+y=20$. The mixture must contain $0.25\cdot 20=5$ L of solute, so solute conservation is $0.10x+0.30y=5$. The pair (5,15) supplies $0.5+4.5=5$ L of solute in 20 L total. Nonnegative real amounts are appropriate here; they need not be whole liters.

Check each alternative

- A. The right side must be solute amount, 25% of 20 L, not the concentration alone.
- B. A percent must be converted to a decimal before multiplying the total volume.
- C. Correct. Total volume gives $x+y=20$. The mixture must contain $0.25\cdot 20=5$ L of solute, so solute conservation is $0.10x+0.30y=5$. The pair (5,15) supplies $0.5+4.5=5$ L of solute in 20 L total. Nonnegative real amounts are appropriate here; they need not be whole liters.
- D. This exchanges total volume with solute volume.

Evidence to assess

Keep concentration separate from solute amount and model both conserved quantities with compatible units.

WORKED TEACHER KEYS

001

001 • T1: Create linear systems from multiple contextual measurements or comparisons, define variables and units, and verify every original equation.

Let x and y be kilograms of the \$4/kg and \$7/kg ingredients. The system is $x+y=12$ and $4x+7y=60$, with $x,y \geq 0$. Subtract $4(x+y)=48$ from the cost equation to obtain $3y=12$, so $y=4$ and $x=8$. The mass check is $8+4=12$ kg; the cost check is $4 \cdot 8 + 7 \cdot 4 = 32 + 28 = \$60$. The sum $4+7$ adds unit prices, not total cost; each price must be multiplied by the amount bought.

Evidence to assess

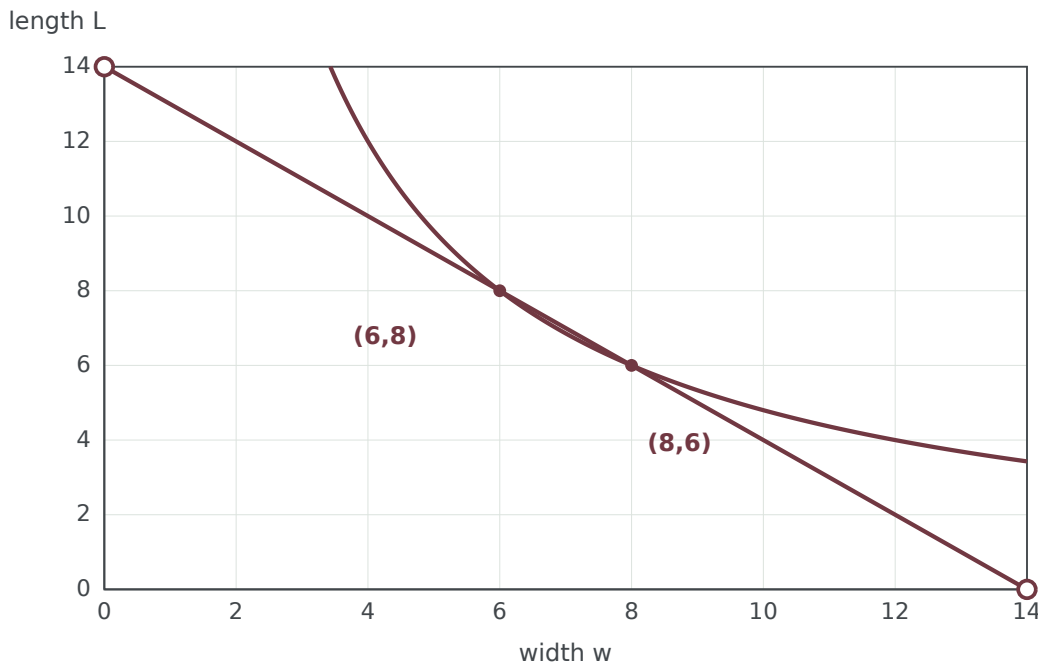
Define variables with units, create both conservation equations, solve without losing the quantity labels and verify the two different totals.

WORKED TEACHER KEYS

002

002 • T2: Create systems involving polynomial, absolute-value, exponential or rational relationships and interpret their simultaneous solutions within the contextual domain.

The system is $2w+2L=28$ and $wL=48$, with $w,L>0$. Rewrite $L=14-w$ and $L=48/w$. Substitution gives $w(14-w)=48$, or $w^2-14w+48=(w-6)(w-8)=0$. Thus the intersections are $(w,L)=(6,8),(8,6)$. Draw the decreasing line for $0<w<14$ with open zero-dimension endpoints, and the positive reciprocal branch through those two shared points. The reciprocal curve continues beyond the top and right edges of the displayed window. Both pairs describe a 6 by 8 rectangle with the names of the sides exchanged, so they do not necessarily mean different shapes. Each has perimeter 28 and area 48.

**Evidence to assess**

Create and graph a mixed linear-rational representation of the constraints, find all physical intersections and interpret ordered-pair symmetry.