

TEACHER AND STUDENT RESOURCE

Justifying Every Step of a Solution



NC Math 2 • NC.M2.A-REI.1

Reasoning with Equations and Inequalities

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-REI.1 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.

T1 Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

T2 Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

T3 Justify inverse-equation manipulations using nonzero denominators and valid reversible operations.

T4 Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M04, O04, M05, O05, M08, O08, M12, O12, M15, O15, M19, O19, M28, O28

T2: M02, O02, M06, O06, M10, O10, M16, O16, M20, O20, M23, O23, M25, O25, M29, O29

T3: M03, O03, M07, O07, M11, O11, M13, O13, M17, O17, M21, O21, M22, O22, M26, O26, M30, O30

T4: M09, O09, M14, O14, M18, O18, M24, O24, M27, O27

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Learning target: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Learning target: Justify inverse-equation manipulations using nonzero denominators and valid reversible operations.

Learning target: Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Learning target: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Learning target: Justify inverse-equation manipulations using nonzero denominators and valid reversible operations.

Learning target: Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Real square roots, factoring, completing the square, solving linear equations, nonzero denominators, and distinguishing an expression from an equation.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All solution sets in this packet are over the real numbers unless a prompt states a parameter case. Record the original domain. Equivalent steps preserve the exact solution set; one-way implications may generate candidates that require original checks. Multiplication or division by a variable expression needs a nonzero condition or a complete case split. Do not add $\pm$ when merely squaring a known number.

TEACHER MODEL

Clear the denominator on its actual domain

Solve $8/(x-1)=x+1$ over the reals. Explain why a negative denominator is allowed and why every candidate must still be checked.

First record $x \neq 1$. On that domain, multiplying both sides by nonzero $x-1$ is reversible and gives $8=(x-1)(x+1)=x^2-1$. Hence $x^2=9$, so $x=3$ or -3 . Both are allowed: at 3, $8/2=4=3+1$; at -3 , $8/(-4)=-2=-3+1$. A nonzero denominator need not be positive. The original domain belongs to every later step, and the checks confirm both candidates satisfy the original quotient equation.

VISUAL ANCHOR

What makes a step reversible?

A written reason should identify the condition that preserves all and only the original solutions.

Operation	Condition or safeguard	Reasoning evidence
Add the same expression	Added expression is defined on the original domain	The operation can be undone by subtraction
Divide or clear a factor	The factor is nonzero, or split cases	No solution is silently lost or added
Square both sides	Retain original signs and domain	Equal squares may hide opposite signs
Take square-root branches	A real square equals a nonnegative value	Keep both bases when the square is positive

A candidate is not yet a solution. Substitute in the original equation and its original domain, not merely in the final transformed equation.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask learners to name the operation and explain how it can be undone. Separate an original domain condition from the eventual solution set.

Core task

Solve $x^2=25$ over the real numbers. Explain why both signs must be retained and verify every solution in the original equation.

Teacher solution and discussion

Taking nonnegative square roots gives $|x|=5$, not merely $x=5$. Thus $x=5$ or $x=-5$. Checks: $5^2=25$ and $(-5)^2=25$. A real square loses the sign of its base, so both bases produce the same positive square.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Write an equality chain with a reason beside each arrow. State whether the arrow is reversible or only produces candidates. Keep branches and domain notes visible until the original checks are complete.

Core task

Solve $x^2 - 4x = 2$ by completing the square. Explain the compensating operation, retain all real square-root branches, and verify your solutions without decimal approximations.

Teacher solution and discussion

Add 4 to both sides because half of -4 is -2 and $(-2)^2 = 4$. Then $(x-2)^2 = 6$, so $x-2 = \pm\sqrt{6}$ and $x = 2 \pm \sqrt{6}$. For either sign, $x^2 - 4x = (x-2)^2 - 4 = 6 - 4 = 2$. Every step preserves the real solutions when both signs are included.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Pause at a variable denominator, square, or factor cancellation. Ask which inputs would make that move invalid and how to preserve a zero case rather than discard it.

Core task

A student solves $x^2 + 2x = 0$ by canceling x and obtains $x = -2$. Repair the argument without losing any solutions. Explain when the student's division would be valid and how to handle the missing case.

Teacher solution and discussion

Factor $x(x+2)=0$, so $x=0$ or -2 . Both check: $0+0=0$ and $4-4=0$. The student's division is valid only on the branch $x \neq 0$, where it correctly gives -2 . A complete case split first checks $x=0$ separately, then divides on the nonzero branch. Without that split, one solution is lost.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign solver, equivalence checker, domain checker and counterexample finder roles. Compare methods by their logic, not only speed. Require a concrete rejected input when an argument fails.

Core task

Solve $x^2 - 8x + 12 = 0$ in two ways: factoring and completing the square. Explain why their solution sets agree, then recommend one method for this equation without claiming the other is invalid.

Teacher solution and discussion

Factoring gives $(x-2)(x-6)=0$, so $x=2$ or 6 . Completing the square gives $x^2 - 8x = -12$, then $(x-4)^2 = 4$, so $x=4 \pm 2$, again 2 or 6 . Each rewrite is equivalent, and both signs or zero factors are retained. Factoring is shorter here because 2 and 6 are easy to identify; completing the square is also valid. Checks give $4-16+12=0$ and $36-48+12=0$.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect method choice, reasons, complete candidates and original checks as separate evidence. Do not supply the missing sign condition or branch while assessing the first response.

Core task

Classify the real solution set of $a(x-1)^2=4$ for each case $a>0$, $a=0$ and $a<0$. Justify when division and square roots are allowed, rather than applying one formula to every case.

Teacher solution and discussion

If $a>0$, divide by nonzero a : $(x-1)^2=4/a>0$. Thus $x=1\pm2/\sqrt{a}$, two real solutions; substitution gives $a(4/a)=4$. If $a=0$, the equation is $0=4$, so none exist and division by a is forbidden. If $a<0$, the left side is nonpositive, so again none exist over the reals. Equivalently $4/a$ would be negative and cannot be a real square.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Critique or construct solution arguments by distinguishing equivalence, implication, omitted cases and extraneous candidates.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use the observed issue: omitted branch, unnecessary $\pm$, mixed squaring rules, zero multiplier or wrong denominator exclusion. Repair that step and test it in a fresh argument.

Core task

Solve $\sqrt{(x-2)^2}=5$ over the reals. Explain why replacing the left side by $x-2$ without an absolute value can lose a solution, and verify every answer.

Teacher solution and discussion

For real x , $\sqrt{(x-2)^2}=|x-2|$. Thus $x-2=5$ or $x-2=-5$, giving $x=7$ or -3 . Checks: at either input $(x-2)^2=25$ and its principal root is 5. Replacing the root by $x-2$ silently assumes $x \geq 2$ and loses $x=-3$, where $x-2=-5$ but its absolute value is 5.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Require a complete exact solution argument for a new equation. Learners must justify each conditional operation, keep every valid branch and explain any rejected candidate.

Core task

Solve $2x^2+4x-3=0$ using a method you choose. Explain why that method is suitable, justify every equivalence step, and verify all exact roots. Do not rely only on decimal approximations.

Teacher solution and discussion

Completing the square is suitable because the linear coefficient becomes 2 after dividing by 2. Divide: $x^2+2x=3/2$. Add 1: $(x+1)^2=5/2$. Take both real branches: $x=-1\pm\sqrt{10}/2$. For either root, $(x+1)^2=5/2$, so $2x^2+4x-3=2(x+1)^2-5=0$. The quadratic formula is also valid if the learner justifies its use and obtains the same complete exact set.

Evidence and response

Assess the reason for the operation, not only the resulting answer. Look for preserved domains, correct square-root branches, scaled compensation, and explicit original checks. An answer-supplying cue should be recorded separately from a learner's independent argument.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A solver changes $3x^2=12$ to $x^2=4$. Which reason justifies an equivalent equation?

- A. Remove the exponent because 3 is positive.
- B. Divide only the left side by 3.
- C. Divide both sides by the same nonzero constant 3.
- D. Take the square root of both sides.

Reasoning: _____

M02 | Warm-up | T2

Before solving $\sqrt{x+4}=3$ over the reals, which domain condition is necessary?

- A. x can be any real number
- B. $x \leq -4$
- C. $x \geq 4$
- D. $x \geq -4$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Solve $x^2=25$ over the real numbers. Explain why both signs must be retained and verify every solution in the original equation.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Solve $2\sqrt{x-1}=6$ over the reals. State the domain and justify each operation, including why the final squared step does not introduce an unchecked sign branch.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Answer: C. Divide both sides by the same nonzero constant 3.

Division by the nonzero number 3 is reversible by multiplication by 3, so it preserves exactly the same solution set.

Check each alternative

- A. A positive coefficient does not cancel an exponent.
- B. Changing only one side does not preserve equality.
- C. Division by the nonzero number 3 is reversible by multiplication by 3, so it preserves exactly the same solution set.
- D. Square roots would change the powers and produce $\sqrt{(3x^2)}=\sqrt{12}$, not the stated step.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02

M02 • T2: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Answer: D. $x \geq -4$

The radicand must satisfy $x+4 \geq 0$, which gives $x \geq -4$. This is a domain condition, not yet the solution set.

Check each alternative

- A. Some real inputs make the radicand negative.
- B. This reverses the radicand inequality.
- C. This subtracts the shift in the wrong direction.
- D. The radicand must satisfy $x+4 \geq 0$, which gives $x \geq -4$. This is a domain condition, not yet the solution set.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T1: Justify quadratic solution methods and equality-preserving steps while retaining every required branch.

Taking nonnegative square roots gives $|x|=5$, not merely $x=5$. Thus $x=5$ or $x=-5$. Checks: $5^2=25$ and $(-5)^2=25$. A real square loses the sign of its base, so both bases produce the same positive square.

Evidence to assess

Require both solutions, an absolute-value or equivalent sign argument, and substitution of both roots rather than an unexplained $\pm$ symbol.

WORKED TEACHER KEYS

002

002 • T2: Justify square-root equation steps using real domains, principal-root signs, logical direction and original checks.

Need $x \geq 1$. Divide by the nonzero constant 2 to get $\sqrt{x-1}=3$. Both sides are nonnegative, so squaring is reversible here and gives $x-1=9$. Add 1: $x=10$. It meets the domain and checks because $2\sqrt{9}=6$. There is no negative root value to retain for a principal square root already isolated as 3.

Evidence to assess

Require the domain, reversible constant division, nonnegative squaring justification, correct solution and original-equation check.