

TEACHER AND STUDENT RESOURCE

Volumes of Solids and Composite Models



NC Math 3 • NC.M3.G-GMD.3

Geometric Measurement & Dimension

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.G-GMD.3 | NC Math 3 mathematics

This sample contains 23 complete selected pages from the 121-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-18, 33-34, 66, 91-92

Representative activities

M01: sample PDF page(s) 19; original page(s) 18

M02: sample PDF page(s) 19; original page(s) 18

O01: sample PDF page(s) 20; original page(s) 33

O02: sample PDF page(s) 21; original page(s) 34

Selected teacher prompts and keys: original page(s) 66, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

T1 Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

T2 Recover positive dimensions, compare solid volumes and justify how independent or uniform dimension changes affect volume.

T3 Solve composite, hollow, capacity, displacement and recasting problems using consistent units and correct overlap or conservation reasoning.

T4 Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M04, M05, M06, M07, O01, O03, O05

T2: M08, M09, M10, M11, M12, M13, M17, M23, M25, M28, O07, O08, O11, O13, O19, O28

T3: M14, M15, M16, M18, M19, M20, M21, M22, M26, M29, O04, O09, O10, O15, O16, O17, O18, O20, O22, O27

T4: M24, M27, M30, O02, O06, O12, O14, O21, O23, O24, O25, O26, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Select formulas and meaningful dimensions

Learning target: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Learning target: Recover positive dimensions, compare solid volumes and justify how independent or uniform dimension changes affect volume.

Use Warm-up and I Do items to distinguish base area from base length, perpendicular height from slant and radius from diameter. Construct O01 and O03, then compare the equal-volume designs in O08.

Session 2 Compose conserve and recover

Learning target: Recover positive dimensions, compare solid volumes and justify how independent or uniform dimension changes affect volume.

Learning target: Solve composite, hollow, capacity, displacement and recasting problems using consistent units and correct overlap or conservation reasoning.

Use We Do and Collaborative items for inverse dimensions, nonuniform scaling, cavities and non-overlap decompositions. Give O14 time for the failed full-submersion assumption before developing the corrected equation.

Session 3 Audit designs and explain constraints

Learning target: Solve composite, hollow, capacity, displacement and recasting problems using consistent units and correct overlap or conservation reasoning.

Learning target: Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.

Use Independent practice to connect volume to feasible fits, material loss and overflow. Reteach the identified dimensional or domain error, then collect Exit responses requiring a proof, an exact feasible range and a multi-constraint decision.

PLANNING

Materials, access and pacing

Materials

Pencil, ruler, graph paper and optional physical solid models or transparent containers. Use a calculator only for requested estimates and checks; retain exact pi, square-root and cube-root forms. Provide separate blank construction grids when two solids are requested.

Prerequisites

Find areas of rectangles, triangles and circles; distinguish radius from diameter; use perpendicular height and the Pythagorean Theorem; solve equations with squares and cubes; apply similarity, fractions, percentages and length-to-volume unit conversions.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

PLANNING CONTINUED

Materials, access and pacing

Review convention

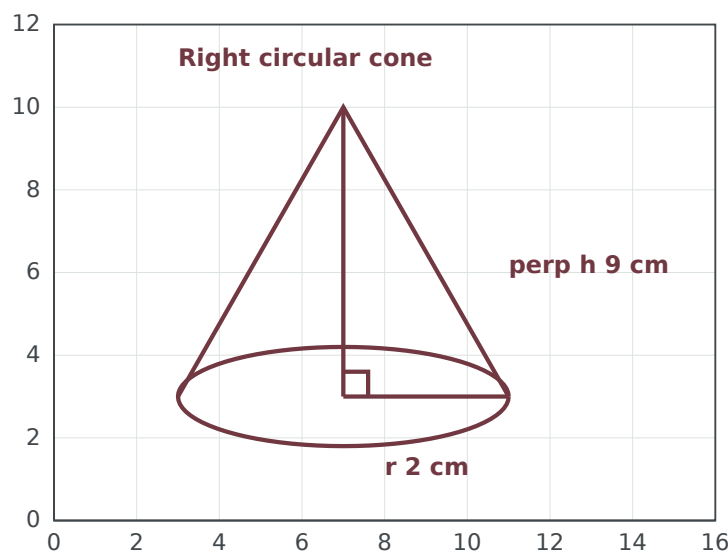
Dimensions are positive unless a task explicitly investigates a zero or impossible boundary. In $V=Bh$ or $V=Bh/3$, B is base area and h is perpendicular height, never a slanted edge. Cylinder and cone radii are not diameters. A sphere uses radius cubed. Retain exact π unless an approximation is requested; cube_root denotes the real cube root. Lengths must share a unit before substitution, and all volumes have cubic units. Internal dimensions determine capacity. Composite interiors are disjoint only when stated or established; shared faces have zero volume. Drawings are schematics: plotted coordinates position the illustration and do not supply physical lengths or angles. Use stated dimension labels, not measurements from the drawing. Completed construction diagrams appear only in the key. Accept equivalent exact answers and alternative valid models meeting every stated condition.

TEACHER MODEL

Separate the formula from the dimension and the requested region

A cone of radius 2 cm and perpendicular height 9 cm is contained in a cylinder with the same base and height. Find the cone volume, cylinder volume and volume inside the cylinder but outside the cone. Explain why the subtraction is legitimate.

Not to scale



Schematic; use dimension labels

First identify $B = \pi r^2 = 4\pi \text{ cm}^2$ and perpendicular $h = 9 \text{ cm}$. The cylinder volume is $Bh = 36\pi \text{ cm}^3$; the cone volume is $Bh/3 = 12\pi \text{ cm}^3$. Since the cone is entirely contained, the remaining region has volume $36\pi - 12\pi = 24\pi \text{ cm}^3$. The ratio is 1:3 for cone to cylinder, so the remaining fraction is $2/3$. The shared boundary surfaces have zero volume. If the cone were not contained, subtracting its whole volume would not describe this region; containment matters. The diagram shows the cone dimensions only, and the comparison cylinder has exactly the same base radius and height.

VISUAL ANCHOR

Choose the solid and track the occupied region

Prism: $V=Bh$. For a triangular base, $B=(1/2)ab$ with perpendicular a and b .

Cylinder: $V=\pi r^2h$, with $r=d/2$ and perpendicular h .

Pyramid: $V=Bh/3$. Cone: $V=\pi r^2h/3$. Slant height is not h .

Sphere: $V=(4/3)\pi r^3=\pi d^3/6$. Hemisphere: one half of the sphere.

Disjoint interiors: add volumes. A contained cavity: subtract its volume.

Overlapping regions: $V(\text{union})=V_1+V_2-V(\text{intersection})$.

Uniform linear factor k gives volume factor k^3 ; nonuniform changes need separate factors.

Capacity and displacement use internal dimensions and the actually submerged region.

VISUAL ANCHOR CONTINUED

Choose the solid and track the occupied region

Check units, positive dimensions, containment, overflow and every design restriction.

The shape determines the formula; the question determines whether to add, subtract, compare or solve backward. A correct number without a valid geometric model does not establish a feasible solution.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Before calculations, have learners point to a base region and name its area units. On O01 ask “Which height belongs to the triangle, and which belongs to the prism?” Require a completed prism rather than an unlabeled triangle. Use M04 to show why a matching numerical coefficient is insufficient when area and volume units differ. In O02 ask learners to predict whether the diameter error doubles or quadruples volume, then verify the factor. On O04 keep both unit-conversion routes visible and reconcile them.

Core task

Construct a labeled schematic of a right triangular prism whose triangular base has perpendicular legs 6 cm and 8 cm and whose prism length is 5 cm. Mark the right angle in the base. Find its volume and explain which two dimensions form the base area and which dimension measures the separation of the bases. Your drawing need not be to scale.

Teacher solution and discussion

Draw two congruent right triangles and connect their corresponding vertices. Label the perpendicular legs of one triangle 6 cm and 8 cm, mark their right angle, and label a joining prism edge 5 cm. The triangular base area is $B = (1/2)(6)(8) = 24 \text{ cm}^2$, while the perpendicular distance between the triangular planes is 5 cm. Therefore $V = 24 \cdot 5 = 120 \text{ cm}^3$. The 8 cm base altitude is not the prism length; those two different height roles belong to different steps.

Evidence and response

Require a completed prism schematic with two triangular bases, the marked base right angle, all three lengths, a 24 cm^2 base-area calculation and a 120 cm^3 volume with the two height roles distinguished. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Think aloud through O05: name B first, choose the solid family second and use the perpendicular height third. Model the axial right triangle in O06 before substituting into a volume formula; ask which segment is the hypotenuse. In O07 derive the diameter formula by substituting $d/2$ into the cube rather than memorizing another rule. For O08 solve both equations before drawing and explicitly label each schematic as not to scale. The different heights are mathematical evidence, not a visual-size comparison.

Core task

A prism and a pyramid each have base area 35 cm^2 and perpendicular height 12 cm. Calculate both volumes. Explain why knowing the base area is enough even if the actual base polygon is not specified. Then find the pyramid height that would match the original prism volume while retaining base area 35 cm^2 .

Teacher solution and discussion

The prism volume is $Bh=35 \cdot 12=420 \text{ cm}^3$. The pyramid volume is $Bh/3=35 \cdot 12/3=140 \text{ cm}^3$. Both formulas depend on the given base area, not the individual side lengths or the base perimeter, so the exact polygon is unnecessary. To make a pyramid of base area 35 have volume 420, solve $35h/3=420$, giving $h=36 \text{ cm}$. Check: $35 \cdot 36/3=420$. The height must triple to compensate for the pyramid factor.

Evidence and response

Compute both solid-family volumes, explain why B summarizes all needed base geometry, solve the equal-volume height and verify it in the pyramid formula. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Solve composite, hollow, capacity, displacement and recasting problems using consistent units and correct overlap or conservation reasoning.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Have one learner identify the disjoint components in O09 and another explain why the roof cannot be counted as a full-height prism. In O10 trace the cavity and then rebuild the material as a bottom plus a shorter sidewall, checking that no bottom material is counted twice. On O11 retain the negative algebraic root until the length domain is applied. For O12 compare width before volume, then explain why the larger case volume does not rescue the fit. Use O13 to connect square-base scaling to an inverse-square height change.

Core task

A model building consists of a rectangular prism with an 8 m by 6 m floor and 5 m high walls. A rectangular pyramid roof sits directly on the top face and has perpendicular height 3 m. Construct a dimension-labeled schematic, calculate the enclosed volume, and give the total building height. Explain why using the total height in the prism formula is wrong.

Teacher solution and discussion

Draw the 8 m by 6 m rectangular base, 5 m vertical walls and a pyramid over the matching top rectangle with apex 3 m above that plane. The shared base area is 48 m^2 . The prism contributes $48 \cdot 5 = 240 \text{ m}^3$; the pyramid contributes $48 \cdot 3/3 = 48 \text{ m}^3$. Their interiors are disjoint and their common face has zero volume, so the total is 288 m^3 . Total height is $5 + 3 = 8 \text{ m}$. Using $48 \cdot 8 = 384 \text{ m}^3$ would replace the tapering roof by a full rectangular prism and overcount by 96 m^3 .

Evidence and response

Require the completed composite schematic, separate correct component volumes, total height, a justified non-overlap addition and a quantitative diagnosis of the full-height prism error. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Assign modeler, calculator and constraint checker roles after independent predictions. O14 deliberately challenges an unverified full-submersion assumption: test it, observe its contradiction and then count only the submerged part of the block. In O15 have the checker distinguish middle length from total capsule length. In O16 use similarity before subtracting cones. O17 should end with a half-height volume check, not merely a cube-root answer. On O18 require an explicit leftover and ask whether an empty cavity supplies any material.

Core task

A vertical cylinder of internal radius 5 cm and height 10 cm contains water 4 cm deep. A 3 cm by 4 cm rectangular block, 5 cm tall, is placed upright on the bottom; its footprint fits inside the tank. No water is lost. First test the assumption that the block becomes fully submerged. Then find the actual final water depth and decide whether the block is fully submerged or any water spills.

Teacher solution and discussion

Initial water volume is $25\pi \cdot 4 = 100\pi$ cm³. If all 60 cm³ of the block were submerged, the predicted depth would be $(100\pi + 60)/(25\pi) = 4 + 12/(5\pi) \approx 4.764$ cm, below the block top at 5 cm. That contradicts full submersion. For a depth $y < 5$, the submerged block volume is its footprint 12 times y , so water occupies $(25\pi - 12)y$. Solve $(25\pi - 12)y = 100\pi$ to obtain $y = 100\pi/(25\pi - 12) \approx 4.721$ cm. This is between 4 and 5 and below the 10 cm rim, so the block is partly submerged and there is no spill. The dry top portion must not be counted as displaced water.

Evidence and response

Require the failed full-submersion test, a piecewise-appropriate volume equation using only submerged block volume, the exact final depth and numerical/domain checks for partial submersion and no overflow. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Recover positive dimensions, compare solid volumes and justify how independent or uniform dimension changes affect volume.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Collect O19–O23 without choosing formulas for the learner. O19 must exclude all other integer triples, not stop at a successful guess. In O20 distinguish a 12.5 volume ratio from the number of complete cylinders. O21 requires the whole feasible radius interval, including endpoint checks. In O22 determine overflow from the initial headspace, then verify the final submersion assumption. O23 needs two finished schematics with different unordered edge triples; equal volume is not evidence of congruence.

Core task

A rectangular prism has positive integer edge lengths that are three consecutive integers. Its volume is 120 cm^3 . Determine the dimensions and justify that there is no other possible triple. Your argument must do more than exhibit one successful guess.

Teacher solution and discussion

Let the smallest edge be $n \text{ cm}$, with n a positive integer. Then $V = n(n+1)(n+2) = 120$. At $n=4$, $4 \cdot 5 \cdot 6 = 120$, giving edges 4 cm, 5 cm and 6 cm. For positive integers, each of the three positive factors strictly increases as n increases, so their product strictly increases. Values $n \leq 3$ give at most $3 \cdot 4 \cdot 5 = 60 < 120$; values $n \geq 5$ give at least $5 \cdot 6 \cdot 7 = 210 > 120$. Hence $n=4$ is the only possibility, up to reordering the same three edges.

Evidence and response

Translate consecutive lengths into a product equation, verify the triple and supply a complete monotonic or bounding argument excluding every other positive integer. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Select O24 for a difference-of-squares misconception, O25 for impossible slant and zero-height cases, O26 for ignoring a physical parameter domain, or O27 for diameter and cubic-unit conversions. Preserve the learner’s first response. Rebuild the missing disk-area subtraction, axial right triangle, domain inequality or internal-dimension model with one visible example. Remove that support before a fresh explanation. Require a geometric reason for each correction rather than accepting a replaced numerical answer alone.

Core task

A cylindrical pipe has outer radius 7 cm, inner radius 5 cm and length 4 cm. A student calculates material volume as $\pi(7-5)^2(4)=16\pi \text{ cm}^3$. Find the correct material volume and compare it to the student’s value. Use algebra to explain why subtracting radii before squaring does not measure the annular base area.

Teacher solution and discussion

The correct annular base area is $\pi(7^2-5^2)=24\pi \text{ cm}^2$, so material volume is $24\pi \cdot 4=96\pi \text{ cm}^3$. This is six times the student’s $16\pi \text{ cm}^3$ and exceeds it by $80\pi \text{ cm}^3$. In general $R^2-r^2=(R-r)(R+r)$, whereas $(R-r)^2=(R-r)(R-r)$. For $R=7$ and $r=5$, the factors are $2 \cdot 12$ versus $2 \cdot 2$. The circular cavity removes a disk from the larger disk; the wall is not a new disk whose radius equals the thickness.

Evidence and response

Subtract actual cylinder volumes or disk areas, report both multiplicative and additive comparisons, and connect the algebraic factorization to the annular geometry. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Recover positive dimensions, compare solid volumes and justify how independent or uniform dimension changes affect volume.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

For O28 look for a^2b , the positive square-root radius factor and a verified counterexample, not a claim about all larger-radius cylinders. On O29 collect the composite construction and the strict volume interval; ask why the endpoints are excluded when both components must have positive height. On O30 every design needs both a capacity and an overall-height check. Use the stated π bounds to distinguish exact equality from strict comparison. Decide the next lesson from the missing model, unit or feasibility evidence, not just the final design choice.

Core task

For a cylinder, prove the new-to-old volume factor when the radius is multiplied by $a > 0$ and the height by $b > 0$. Use the result to determine the radius factor that doubles volume while height stays fixed. Finally give a numerical counterexample showing that a larger radius alone does not guarantee a larger volume.

Teacher solution and discussion

With original positive dimensions r, h , $V_{\text{new}}/V_{\text{old}} = [\pi(ar)^2(bh)]/[\pi r^2h] = a^2b$. If height is fixed, $b=1$, and doubling volume requires $a^2=2$; the positive radius factor is $a=\sqrt{2}$, not 2. For a counterexample, a cylinder with radius 2 cm and height 10 cm has volume $40\pi \text{ cm}^3$, while one with radius 3 cm and height 4 cm has volume $36\pi \text{ cm}^3$. The radius increased, but the height decrease more than offset the base-area increase. All dimensions are positive, and $36\pi < 40\pi$.

Evidence and response

Prove the general factor with valid cancellation, select the positive square root for a fixed-height doubling, and verify a genuine strict-decrease counterexample with all dimensions stated. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A right triangular prism has a triangular base with perpendicular legs 4 cm and 9 cm. The distance between its triangular bases is 7 cm. What is its volume?

- A. 42 cm^3 .
- B. 252 cm^3 .
- C. 18 cm^3 .
- D. 126 cm^3 .

Reasoning: _____

M02 | Warm-up | T1

A right circular cylinder has diameter 10 m and perpendicular height 4 m. Which exact value is its volume?

- A. $20\pi \text{ m}^3$.
- B. $400\pi \text{ m}^3$.
- C. $100\pi \text{ m}^3$.
- D. $100\pi/3 \text{ m}^3$.

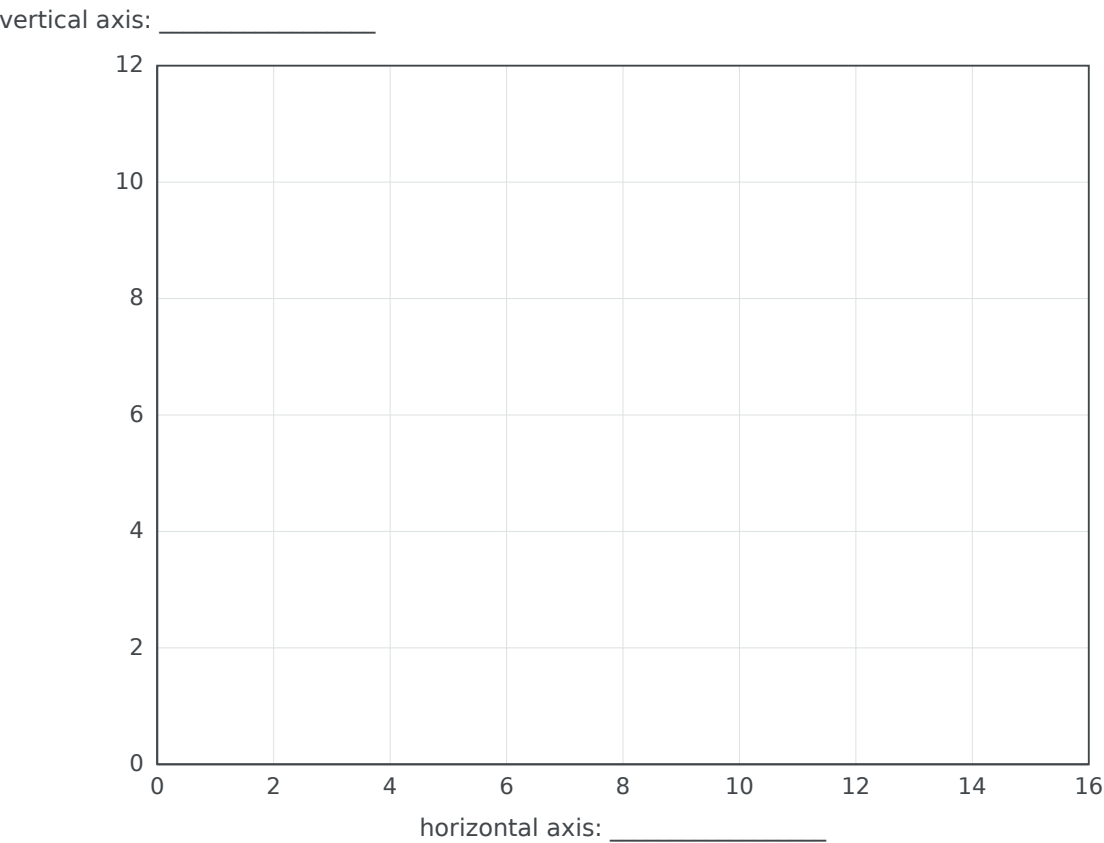
Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Construct a labeled schematic of a right triangular prism whose triangular base has perpendicular legs 6 cm and 8 cm and whose prism length is 5 cm. Mark the right angle in the base. Find its volume and explain which two dimensions form the base area and which dimension measures the separation of the bases. Your drawing need not be to scale.



Explanation and check

WARM-UP

002 • Show your reasoning

T4: Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.

A cylindrical package has diameter 14 cm and height 3 cm. A student reports its volume as $588\pi \text{ cm}^3$ by using $V=\pi(14)^2(3)$. Correct the calculation. Determine the factor by which the reported volume is too large, and explain why halving a diameter does not merely halve the computed volume.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Answer: D. 126 cm³.

The base area is $B = (1/2)(4)(9) = 18 \text{ cm}^2$. A prism has $V = Bh$, where h is the perpendicular distance between bases, so $V = 18 \cdot 7 = 126 \text{ cm}^3$. The triangular shape changes B , not the prism formula.

Check each alternative

- A. Dividing the prism volume by 3 incorrectly treats this prism as a pyramid.
- B. This uses $4 \cdot 9$ as the triangular base area and omits the factor $1/2$.
- C. Eighteen is the triangular base area in cm^2 ; it has not been multiplied by the prism length.
- D. Correct. The base area is $B = (1/2)(4)(9) = 18 \text{ cm}^2$. A prism has $V = Bh$, where h is the perpendicular distance between bases, so $V = 18 \cdot 7 = 126 \text{ cm}^3$. The triangular shape changes B , not the prism formula.

Evidence to assess

Identify the triangular base area and the separate distance between bases; preserve the factor $1/2$ and cubic units.

M02 • T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Answer: C. 100pi m³.

The radius is half the diameter, $r = 5 \text{ m}$. The circular base area is $\pi(5)^2 = 25\pi \text{ m}^2$, and $V = Bh = 25\pi \cdot 4 = 100\pi \text{ m}^3$. Keep π exact rather than rounding it before multiplication.

Check each alternative

- A. This uses πrh instead of πr^2h and therefore fails to compute the circular base area.
- B. This substitutes the diameter 10 for the radius 5, making the circular base area four times too large.
- C. Correct. The radius is half the diameter, $r = 5 \text{ m}$. The circular base area is $\pi(5)^2 = 25\pi \text{ m}^2$, and $V = Bh = 25\pi \cdot 4 = 100\pi \text{ m}^3$. Keep π exact rather than rounding it before multiplication.
- D. The factor $1/3$ belongs to a cone with the same base and height, not a cylinder.

Evidence to assess

Halve the stated diameter, square the radius and multiply by the perpendicular height with a cubic-unit conclusion.

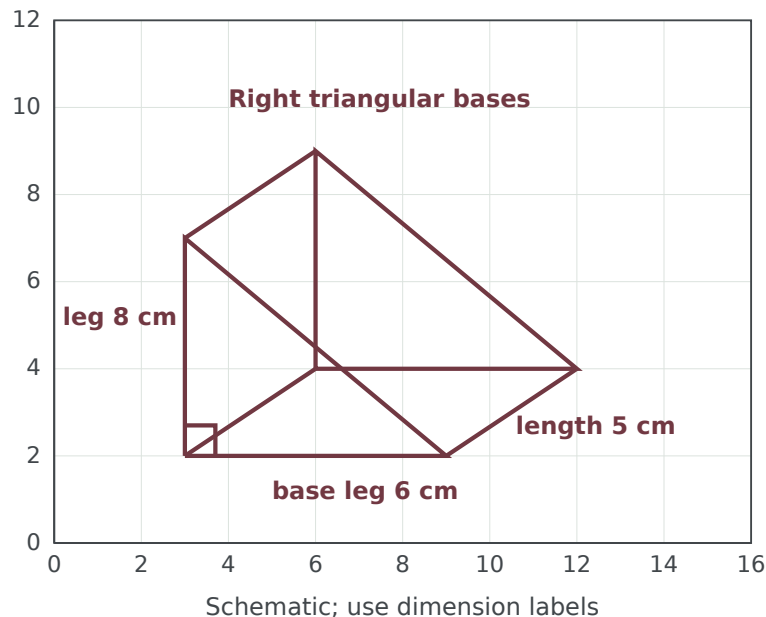
WORKED TEACHER KEYS

001

001 • T1: Select and explain the prism, cylinder, pyramid, cone or sphere volume formula using a valid base area, radius and perpendicular height.

Draw two congruent right triangles and connect their corresponding vertices. Label the perpendicular legs of one triangle 6 cm and 8 cm, mark their right angle, and label a joining prism edge 5 cm. The triangular base area is $B = (1/2)(6)(8) = 24 \text{ cm}^2$, while the perpendicular distance between the triangular planes is 5 cm. Therefore $V = 24 \cdot 5 = 120 \text{ cm}^3$. The 8 cm base altitude is not the prism length; those two different height roles belong to different steps.

Not to scale



Evidence to assess

Require a completed prism schematic with two triangular bases, the marked base right angle, all three lengths, a 24 cm^2 base-area calculation and a 120 cm^3 volume with the two height roles distinguished.

WORKED TEACHER KEYS

002**002 • T4: Construct and audit volume models, diagnose errors, test all constraints and distinguish valid solids from degenerate or impossible specifications.**

The radius is $14/2=7$ cm, so the correct volume is $\pi(7^2)(3)=147\pi$ cm³. The reported-to-correct ratio is $588\pi/(147\pi)=4$. Radius is squared in the base area: replacing 14 by 7 multiplies r^2 by $(1/2)^2=1/4$, while the height stays fixed. Thus the original value is four times the true volume; it is 441π cm³ too large, not just twice the true value.

Evidence to assess

Correct diameter to radius, compute the exact volume, compare the two volumes as a ratio and explicitly connect the factor of four to the squared-radius dependence.