

TEACHER AND STUDENT RESOURCE

Rational Expressions in Quotient and Remainder Form



NC Math 3 • NC.M3.A-APR.6

Arithmetic with Polynomial and Rational Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-APR.6 | NC Math 3 mathematics

This sample contains 21 complete selected pages from the 100-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 80-81

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 80-81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.

T1 Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

T2 Preserve domains and equivalence when canceling factors, scaling denominators or repairing invalid rewrites.

T3 Use the polynomial and residual parts to interpret models, graph differences and prove exact value bounds.

T4 Construct division identities, determine missing information and justify the conditions for canonical uniqueness.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M05, M06, M07, M09, M22, M26, M27, M28, O01, O04, O05, O08, O21, O26, O28

T2: M04, M15, M16, M20, M23, M24, O09, O10, O14, O19, O24

T3: M10, M11, M17, M18, M19, M29, O02, O11, O12, O17, O22

T4: M08, M12, M13, M14, M21, M25, M30, O03, O06, O07, O13, O15, O16, O18, O20, O23, O25, O27, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Divide, reconstruct and restrict

Learning target: Use exact division including missing powers and nonmonic divisors.

Learning target: Check identity, degree and domain as separate requirements.

Use Warm-up and I Do tasks selectively. Ask learners to reconstruct the numerator before deciding whether a proposed answer is complete.

Session 2: Choose forms that reveal information

Learning target: Interpret residual parts in cost, graph and bound problems.

Learning target: Use parameter conditions to construct division records.

Use We Do and Collaborative tasks. Have each learner derive a first equation or counterexample before group comparison.

Session 3: Challenge equivalence claims

Learning target: Distinguish canonical uniqueness from arbitrary equivalent decompositions.

Learning target: Transfer division reasoning to exact proofs and fresh applications.

Use Independent work, targeted Small-group tasks and fresh Exit evidence. Keep original responses separate from later hints or completed graph models.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Expand and factor polynomials; align like powers; divide leading monomials; subtract signed expressions; solve linear equations and inequalities; read real coordinate graphs. Degree arguments use the fact that a nonzero polynomial has only finitely many zeros. Exact rational coefficients are allowed.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

The canonical form means $a(x)/b(x)=q(x)+r(x)/b(x)$, where b is a nonzero polynomial and r is zero or has degree strictly below degree b . The zero polynomial has no ordinary nonnegative degree. Every original real denominator restriction remains in force after rewriting, even if all denominator factors cancel. Proper describes degrees, not numerical magnitude. Context inputs may be further restricted to positive values or integers. All data and coefficients are exact unless stated otherwise.

TEACHER MODEL

One identity, three separate checks

Rewrite $(2x^2+7x+9)/(x+2)$, explain why the division is finished and state exactly where the rewrite is valid.

The leading quotient term is $2x$. Subtracting $2x(x+2)=2x^2+4x$ leaves $3x+9$. The next term is 3 ; subtracting $3(x+2)=3x+6$ leaves 3 . Thus $2x^2+7x+9=(x+2)(2x+3)+3$, and the rational form is $2x+3+3/(x+2)$. Check 1: multiplication reconstructs the numerator. Check 2: the constant remainder degree is below the linear divisor degree. Check 3: the expression is valid only for $x \neq -2$. A correct identity, a finished division and a preserved domain are related but different requirements.

VISUAL ANCHOR

From a division record to an equivalent form

Original fraction: $a(x)/b(x)$, with $b(x) \neq 0$

Division identity: $a(x) = b(x)q(x) + r(x)$

Rewritten form: $q(x) + r(x)/b(x)$

Finish condition: $r=0$ or degree $r < \text{degree } b$

Value difference: $a(x)/b(x) - q(x) = r(x)/b(x)$

Reconstruction test: multiply q by b , then add r

Domain test: keep every original zero of b excluded

A remainder numerator is not itself the residual contribution. The contribution is r/b . A small residual fraction need not be zero, and a degree-proper residual fraction need not be small.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O01 to connect integer and polynomial division, then test the analogy at $x=1$ so learners see its limit. O02 separates extra area 4 from extra height $4/(x+2)$; require both labels. On O03, ask first whether the equality is true and only then whether the form is finished. O04 keeps fractional coefficients exact rather than rounding.

Core task

Write $29/6$ as a whole number plus a proper numerical fraction. Then rewrite $(4x+5)/x$ as a polynomial plus a proper rational remainder. Explain the common identity structure and one important difference between the two remainder conditions.

Teacher solution and discussion

Integer division gives $29=6\cdot4+5$, so $29/6=4+5/6$. Polynomial division gives $4x+5=x\cdot4+5$, so $(4x+5)/x=4+5/x$ for $x\neq0$. Both use $\text{dividend}=\text{divisor}\cdot\text{quotient}+\text{remainder}$. For the positive integer divisor 6, the remainder satisfies $0\leq5<6$. For polynomial divisor x , the condition is $\text{degree}(5)=0<\text{degree}(x)=1$, not $5<x$ at every input. For example $x=1$ is allowed even though $5>1$.

Evidence and response

Produce both identities, retain $x\neq0$ and explicitly distinguish numerical size from polynomial degree. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

For O05, write every missing-power placeholder and subtract whole rows with parentheses. In O06, reconstruct the claimed numerator before editing the record; the quotient is correct, so do not discard all of the learner’s work. O07 reverses division through $a=bq+r$. O08 distinguishes a zero function on a restricted domain from a function defined everywhere.

Core task

Use polynomial division to rewrite $(x^4-3x^2+2x+5)/(x^2-1)$. Show the missing-power placeholders, each subtraction, the completed quotient and remainder, and all excluded real inputs.

Teacher solution and discussion

Write $x^4+0x^3-3x^2+2x+5$. The first quotient term is x^2 ; subtracting $x^2(x^2-1)=x^4-x^2$ leaves $-2x^2+2x+5$. The next quotient term is -2 ; subtracting $-2(x^2-1)=-2x^2+2$ leaves $2x+3$. Therefore the form is $x^2-2+(2x+3)/(x^2-1)$, with $x \neq -1, 1$. The remainder degree 1 is below 2. Checking gives $(x^2-1)(x^2-2)+(2x+3)=x^4-3x^2+2x+5$.

Evidence and response

Maintain coefficient positions, subtract signed polynomials and retain both quadratic-denominator exclusions. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: O09 • T2: Preserve domains and equivalence when canceling factors, scaling denominators or repairing invalid rewrites.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

On O09, insist that the $x=2$ table entry remain undefined for f even though the polynomial gives 4. O10 compares records with different scaled divisors, so ask which numerator and denominator were scaled together. O11 sketches only the difference d , not f ; use its sign to compare f and q . O12 needs an integer minimum and a check of the preceding integer, not just an inequality answer.

Core task

Let $f(x)=(x^2-4)/(x-2)$ on its original real domain, and let $g(x)=x+2$ on all real numbers. Rewrite f by division. Make a table of f and g at $x=1,2,3$, and explain precisely in what sense the formulas agree and in what sense the functions differ.

Teacher solution and discussion

The identity $x^2-4=(x-2)(x+2)$ gives $f(x)=x+2$ for $x \neq 2$, with zero remainder. At $x=1$, $f=g=3$; at $x=2$, f is undefined and $g=4$; at $x=3$, $f=g=5$. Thus the formulas give the same outputs wherever f is defined, but f and g are not the same function because their domains differ. Restricting g to $x \neq 2$ would give the same function as f ; defining $f(2)=4$ would instead be an extension.

Evidence and response

Include the undefined table entry and distinguish a restricted identity from an added function value. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Preserve domains and equivalence when canceling factors, scaling denominators or repairing invalid rewrites.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

For O14, one learner writes the original identity and another the scaled identity, then explain the changed remainder numerators. O15 and O16 contrast uniqueness under a degree condition with infinitely many unrestricted decompositions. In O17, a claimed range needs both bounds and attainable values. In O18, compare the two exceptional parameter values before specializing.

Core task

Rewrite $(3x^2+x+2)/(3x-2)$. Then normalize the denominator to $x-2/3$ and compare the residual numerators. Explain using an identity why changing both numerator and denominator by the same nonzero scalar preserves the function but changes the division record.

Teacher solution and discussion

The product $(3x-2)(x+1)=3x^2+x-2$ leaves remainder 4, so the form is $x+1+4/(3x-2)$, $x \neq 2/3$. Dividing both numerator and denominator of the original fraction by 3 gives $(x^2+x/3+2/3)/(x-2/3)=x+1+(4/3)/(x-2/3)$. The identities are $a=b(x+1)+4$ and $a/3=(b/3)(x+1)+4/3$. Thus the quotient stays the same while the remainder scales with the dividend and divisor. Uniqueness applies to a fixed division problem, not to two scaled records.

Evidence and response

Connect fraction normalization to scaled polynomial identities and keep the excluded input unchanged. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Preserve domains and equivalence when canceling factors, scaling denominators or repairing invalid rewrites.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 requires both restrictions after complete cancellation. O20 and O23 use different types of information to determine remainder coefficients; record the equation each condition supplies. O21 recognizes structure without losing the original variable's degree condition. O22 should be an interval proof from the residual fraction, not a set of calculator trials.

Core task

Rewrite $(x^3 - x^2 - 4x + 4)/(x^2 - 4)$ by two routes: factor first, and divide first. State the domain each route must preserve. Give the values suggested by the polynomial quotient at the two excluded inputs and explain why these are not original function values.

Teacher solution and discussion

Grouping the numerator gives $x^2(x-1) - 4(x-1) = (x-1)(x^2-4)$. Canceling yields $x-1$ only when $x^2-4 \neq 0$, so $x \neq -2, 2$. Division gives the same identity directly: $(x^2-4)(x-1) + 0 = x^3 - x^2 - 4x + 4$, with $q = x-1$ and $r = 0$. The quotient polynomial gives -3 at -2 and 1 at 2 , but the original fraction is undefined at both inputs. Those numbers would be the values of a continuous extension, not values supplied by the original function.

Evidence and response

Compare two valid rewrite methods while tracking both exclusions through complete cancellation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Preserve domains and equivalence when canceling factors, scaling denominators or repairing invalid rewrites.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for canceled-term errors, O25 for confusing degree with magnitude, O26 for divisor hypotheses, or O27 for confusing sample agreement with an identity. Ask for an explicit counterexample or reconstruction tied to that misconception. After discussion, give a fresh task without pointing out the needed denominator or subtraction.

Core task

A learner claims $(x^2+2x+3)/(x+1)=x+2+3/(x+1)$. Disprove the claim at an allowed input, identify the algebraic error, and supply a fully checked canonical rewrite.

Teacher solution and discussion

At $x=1$ the original expression is $(1+2+3)/2=3$, while the proposed expression is $1+2+3/2=9/2$, so the identity fails. The claimed polynomial part $x+2$ would contribute $(x+1)(x+2)=x^2+3x+2$, not x^2+2x . Correct division gives $(x+1)(x+1)+2=x^2+2x+3$. Thus the canonical form is $x+1+2/(x+1)$, with $x \neq -1$. At 1 it gives $1+1+1=3$, and reconstruction proves equality for every allowed input.

Evidence and response

Give a valid numerical disproof, identify the product mismatch and replace it with an exact identity rather than only one matching value. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 checks exact division and reconstruction together. O29 asks which information fixes the remainder beyond an already known quotient. O30 probes why a nonzero proper rational expression cannot become a polynomial on its whole domain. Require the essential reason, not just a selected option or memorized rule, and record neutral access supports separately from answer-supplying help.

Core task

Rewrite $(2x^3 - 3x^2 + 4x - 5)/(2x - 1)$ in canonical form using exact coefficients. Show the successive remainders, state the domain, and verify the final identity by multiplication.

Teacher solution and discussion

The first quotient term x^2 leaves $-2x^2 + 4x - 5$ after subtracting $2x^3 - x^2$. The next term $-x$ leaves $3x - 5$ after subtracting $-2x^2 + x$. The final term $3/2$ leaves $-7/2$ after subtracting $3x - 3/2$. Hence the form is $x^2 - x + 3/2 + (-7/2)/(2x - 1)$, $x \neq 1/2$. Multiplying $(2x - 1)(x^2 - x + 3/2)$ gives $2x^3 - 3x^2 + 4x - 3/2$; adding $-7/2$ gives the original constant -5 . The constant remainder is proper.

Evidence and response

Complete a three-stage nonmonic division, preserve exact fractional coefficients and independently reconstruct the full numerator. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****For $x \neq 0$, which expression equals $(2x+7)/x$?**

- A. $7/x$.
- B. $2+7$.
- C. $2+7/x$.
- D. $2x+7/x$.

Reasoning: _____

M02 | Warm-up | T1**A nonzero divisor $b(x)$ has degree 3. Which nonzero remainder is allowed in the required quotient-plus-remainder form?**

- A. x^4 .
- B. $1/x$.
- C. $4x^2-1$.
- D. x^3+2 .

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Write $29/6$ as a whole number plus a proper numerical fraction. Then rewrite $(4x+5)/x$ as a polynomial plus a proper rational remainder. Explain the common identity structure and one important difference between the two remainder conditions.

Representation / calculation

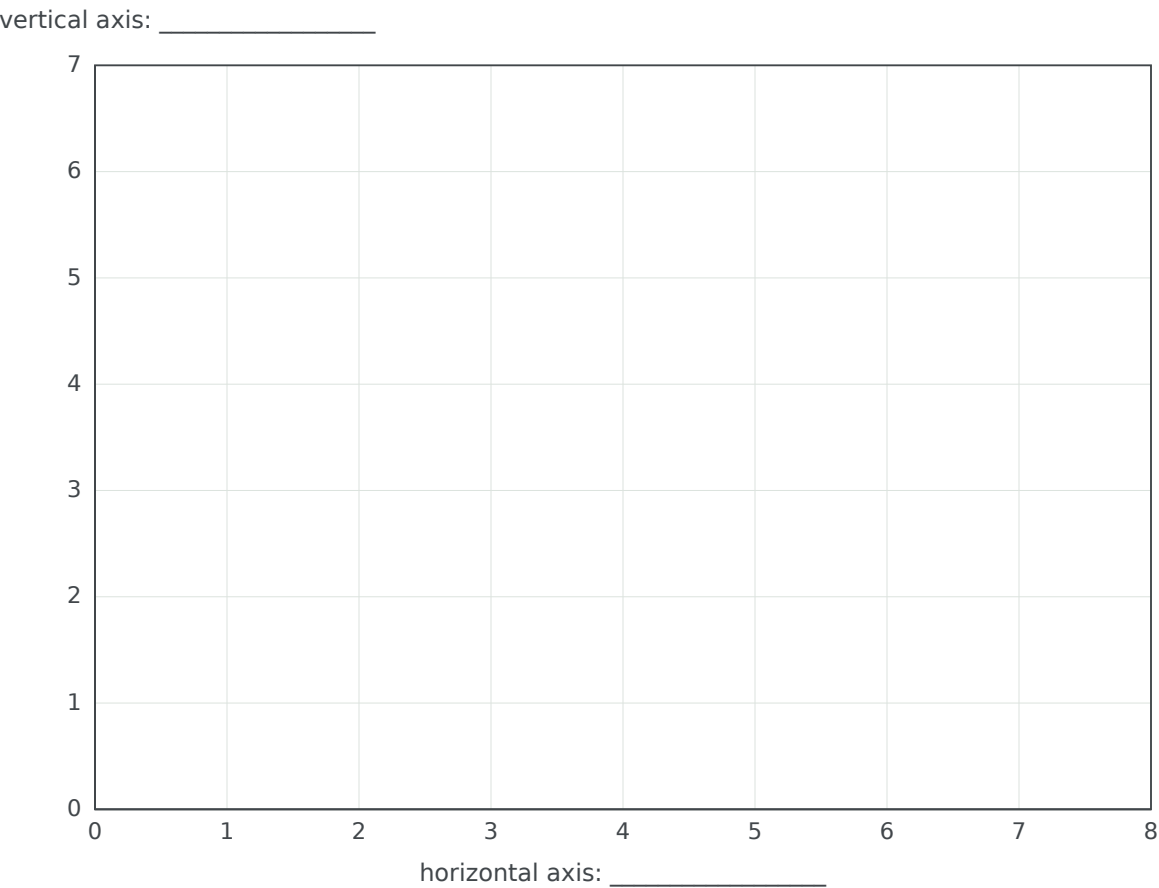
Explanation / check

WARM-UP

002 • Show your reasoning

T3: Use the polynomial and residual parts to interpret models, graph differences and prove exact value bounds.

A rectangular panel has positive width $x+2$ and total area $x^2+5x+10$, where $x>0$. Rewrite its height as a polynomial plus a residual fraction. Draw an area model separating a rectangle of polynomial height from an extra strip of area 4, label all relevant lengths, and verify the total area.



Explanation and check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Answer: C. $2+7/x$.

Divide both numerator terms by the common denominator: $(2x)/x+7/x=2+7/x$. Equivalently, the division identity is $2x+7=x\cdot 2+7$, with constant remainder 7 and linear divisor x .

Check each alternative

- A. This drops the contribution $(2x)/x=2$ from the numerator.
- B. Only the term $2x$ contains a factor x to cancel; $7/x$ does not equal 7 for all allowed x .
- C. Correct. Divide both numerator terms by the common denominator: $(2x)/x+7/x=2+7/x$. Equivalently, the division identity is $2x+7=x\cdot 2+7$, with constant remainder 7 and linear divisor x .
- D. Dividing the first numerator term by x gives 2, not $2x$.

Evidence to assess

Distribute division over numerator addition and identify the polynomial and residual parts.

M02 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.

Answer: C. $4x^2-1$.

The remainder must be a polynomial of degree less than 3. The polynomial $4x^2-1$ has degree 2 and qualifies. The other polynomial candidates have degrees 3 and 4; $1/x$ is not a polynomial.

Check each alternative

- A. Its degree exceeds the divisor degree.
- B. A remainder in polynomial division must itself be a polynomial.
- C. Correct. The remainder must be a polynomial of degree less than 3. The polynomial $4x^2-1$ has degree 2 and qualifies. The other polynomial candidates have degrees 3 and 4; $1/x$ is not a polynomial.
- D. Its degree equals the divisor degree; the division is not finished.

Evidence to assess

Check both the polynomial requirement and the strict degree inequality.

WORKED TEACHER KEYS

001**001 • T1: Produce and verify quotient-plus-proper-remainder forms using polynomial degree and exact division.**

Integer division gives $29 = 6 \cdot 4 + 5$, so $29/6 = 4 + 5/6$. Polynomial division gives $4x + 5 = x \cdot 4 + 5$, so $(4x + 5)/x = 4 + 5/x$ for $x \neq 0$. Both use $\text{dividend} = \text{divisor} \cdot \text{quotient} + \text{remainder}$. For the positive integer divisor 6, the remainder satisfies $0 \leq 5 < 6$. For polynomial divisor x , the condition is $\text{degree}(5) = 0 < \text{degree}(x) = 1$, not $5 < x$ at every input. For example $x = 1$ is allowed even though $5 > 1$.

Evidence to assess

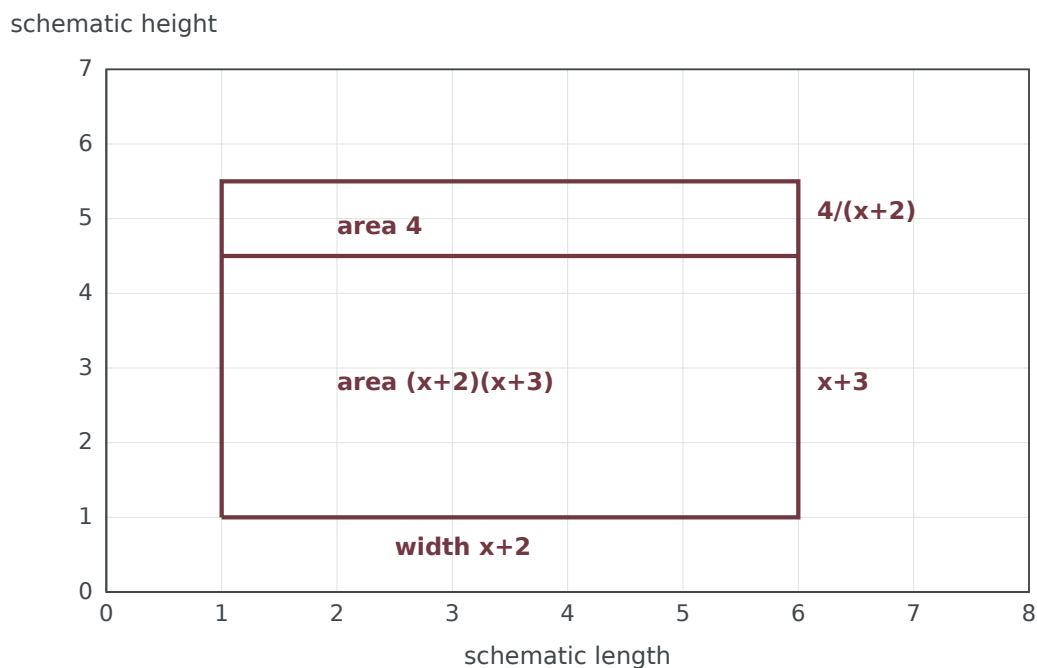
Produce both identities, retain $x \neq 0$ and explicitly distinguish numerical size from polynomial degree.

WORKED TEACHER KEYS

002

002 • T3: Use the polynomial and residual parts to interpret models, graph differences and prove exact value bounds.

Since $(x+2)(x+3)=x^2+5x+6$, the full area is $(x+2)(x+3)+4$. Dividing by the positive width gives height $x+3+4/(x+2)$. The model has a main rectangle of width $x+2$ and height $x+3$, plus a strip of the same width and height $4/(x+2)$, whose area is 4. Adding areas recovers $x^2+5x+10$. All lengths are positive for $x>0$. The schematic below represents labeled areas, not a fixed numerical scale for every x .



Evidence to assess

Connect division to total height, distinguish area from strip height and verify every labeled quantity against the original area.