

TEACHER AND STUDENT RESOURCE

Recursive Functions and Numerical Sequences



Precalculus • PC.F.6

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.F.6 | North Carolina Precalculus mathematics

This sample contains 21 complete selected pages from the 96-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62, 77-78

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62, 77-78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply mathematical reasoning to build recursive functions and solve problems.

T1 PC.F.6.1 Build recursive functions from algebraic representations.

T2 PC.F.6.2 Construct recursive functions from numerically represented sequences.

PC.F.6.1 Use algebraic representations to build recursive functions.

PC.F.6.2 Construct a recursive function for a sequence represented numerically.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M03, M05, M07, M09, M11, M13, M15, M17, M19, M21, M23, M25, M27, M29, O01, O03, O05, O07, O09, O11, O13, O15, O17, O19, O21, O23, O25, O27, O29

T2: M02, M04, M06, M08, M10, M12, M14, M16, M18, M20, M22, M24, M26, M28, M30, O02, O04, O06, O08, O10, O12, O14, O16, O18, O20, O22, O24, O26, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.F.6.1 Build recursive functions from algebraic representations.

Learning target: PC.F.6.2 Construct recursive functions from numerically represented sequences.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.F.6.2 Construct recursive functions from numerically represented sequences.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Function notation, integer indices, differences and ratios, exponent rules and algebraic substitution.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

A sequence is a function on the stated integer indices. Each recursive definition must include enough initial values, an update rule and the indices where the rule applies. a_n means the term at index n ; a_{n-1} means the preceding term. A finite list alone never determines a unique infinite continuation. When a task specifies a pattern family or continuation hypothesis, work within that assumption; otherwise state the assumption and acknowledge other possibilities. Sequence plots use separate points rather than continuous line segments. Exact fractions are preferred when given values are exact.

TEACHER REFERENCE

A recursive rule is more than a pattern

Three required ingredients

State the initial value or values, the recursive update and its starting index. A rule involving the previous two terms usually needs two initial values; a rule involving the term three places back needs three. If an algebraic expression is undefined during an update, the process cannot simply continue.

From algebra to recursion

For an explicit formula a_n , compare a_n with a_{n-1} . Differences help with polynomial expressions; ratios help with multiplicative ones. A vertical shift of an exponential usually produces an affine update, not a pure geometric ratio. Subtracting a fixed point can reveal the simpler geometric structure.

From numbers to a candidate rule

Inspect first differences, ratios, differences of differences, alternating operations, interleaved subsequences and dependence on two or more prior terms. Verify the candidate against every supplied transition. To fit $a_n = ra_{n-1} + b$, use two distinct transitions to determine r and b , then test the rest.

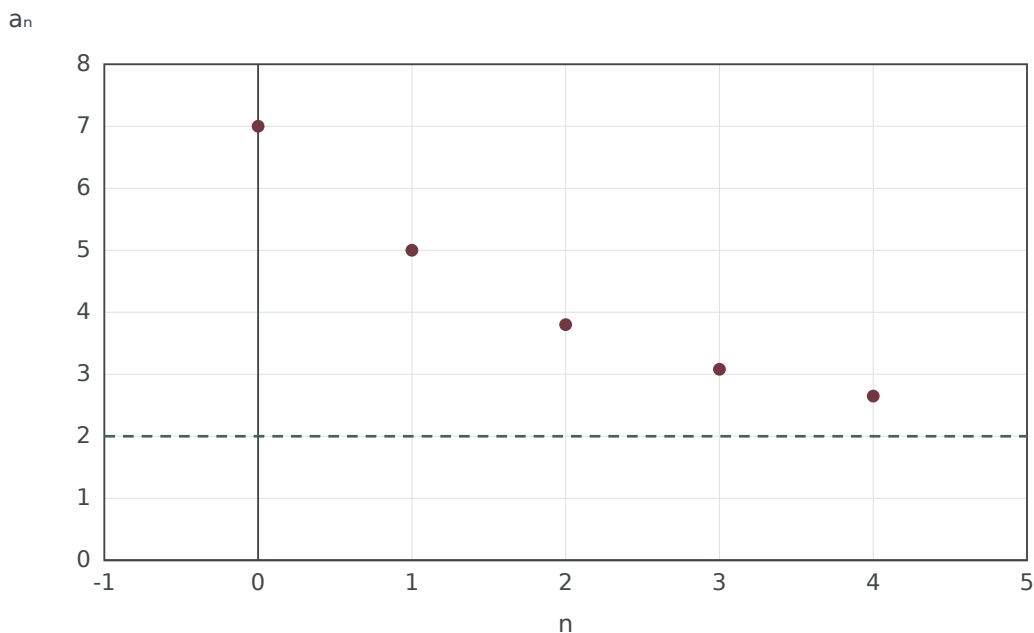
Evidence and ambiguity

A recurrence that fits four terms is a model under a stated hypothesis, not a proof of the intended next term. Ask for a different valid continuation when the data alone do not determine one. Do not mark an alternative mathematically justified recurrence wrong merely because it differs from the teacher's simplest pattern.

TEACHER MODEL

An offset changes the update

For $a_n = 2 + 5(0.6)^n$, derive a recurrence rather than guessing a constant ratio from the first two values.



The seed is $a_0 = 7$. Subtract the baseline 2: $a_n - 2 = 0.6(a_{n-1} - 2)$. Rearranging gives $a_n = 0.6a_{n-1} + 0.8$ for $n \geq 1$. The first terms 7, 5, 3.8, 3.08 agree with the explicit formula. The deviation from 2 has a constant ratio; the unshifted values do not.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

For $a_n = 5 - 2n$, $n = 0, 1, 2, \dots$, derive a recursive definition. List the first five terms and plot them as sequence points. Explain why connecting the points would add inputs not in the stated domain.

Teacher solution and discussion

$a_0 = 5$ and $a_n - a_{n-1} = (5 - 2n) - [5 - 2(n-1)] = -2$, so $a_n = a_{n-1} - 2$ for integers $n \geq 1$. The first five terms are 5, 3, 1, -1 , -3 at indices 0, 1, 2, 3, 4. Plot those five separate points. Joining them would visually include noninteger inputs, whereas this sequence is defined only on nonnegative integers.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

For $a_n = n^3$, $n \geq 0$, derive an additive recursive rule by comparing consecutive cubes. Generate a_0 through a_4 and explain why adding a fixed constant cannot work.

Teacher solution and discussion

$a_0 = 0$. Expand $n^3 - (n-1)^3 = 3n^2 - 3n + 1$, so $a_n = a_{n-1} + 3n^2 - 3n + 1$ for $n \geq 1$. The first five terms are 0, 1, 8, 27, 64, with increments 1, 7, 19, 37. These increments vary with n , so a constant-addition update cannot reproduce all terms.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Let $a_n = (-1)^n / (n+1)$, $n \geq 0$. Derive a recurrence with an index-dependent multiplier and list the first four terms. Explain why a single constant ratio would be incorrect.

Teacher solution and discussion

$a_0 = 1$. For $n \geq 1$, $a_n / a_{n-1} = [(-1)^n / (n+1)] / [(-1)^{n-1} / n] = -n / (n+1)$. Therefore $a_n = -[n / (n+1)] a_{n-1}$. The first four terms are 1, $-1/2$, $1/3$, $-1/4$. The ratios $-1/2$, $-2/3$, $-3/4$ vary, even though the signs alternate every step.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

The values at $n=0, 1, 2, 3, 4$ are 0, 1, 4, 9, 16. Assume the second differences stay equal to 2. Construct a second-order recurrence, state its seeds, verify it and find a_5 .

Teacher solution and discussion

The second-difference equation $a_n - 2a_{n-1} + a_{n-2} = 2$ gives $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n \geq 2$, with $a_0 = 0$, $a_1 = 1$. It produces 4, 9, 16 from the seeds. Next $a_5 = 2 \cdot 16 - 9 + 2 = 25$. This definition depends on two previous terms even though the same values also have the explicit square formula under this continuation.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

For $a_n = n \cdot 2^n$, $n \geq 0$, derive the first-order rule $a_n = 2a_{n-1} + 2^n$. Then derive a constant-coefficient second-order rule by eliminating the added exponential. State all required seeds and verify a_0 through a_4 .

Teacher solution and discussion

$a_0 = 0$ and $a_n - 2a_{n-1} = n2^n - (n-1)2^n = 2^n$. At the previous index, $a_{n-1} - 2a_{n-2} = 2^{(n-1)}$; multiplying by 2 and subtracting gives $a_n - 4a_{n-1} + 4a_{n-2} = 0$. Thus $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$ with $a_0 = 0$, $a_1 = 2$. Both definitions produce 0, 2, 8, 24, 64.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

The values 2, -4 , 8, -16 , 32, ... start at $n=1$. Assume a constant ratio. Construct the recurrence, give the next term, plot the first six points and describe both sign behavior and magnitude growth without claiming a real limit.

Teacher solution and discussion

The ratio is -2 , so $a_1=2$ and $a_n=-2a_{n-1}$ for $n \geq 2$. The next term is -64 . Signs alternate and magnitudes double: 2, 4, 8, 16, 32, 64. The sequence has no real limit; it does not tend to $+\infty$ or $-\infty$ because the signs keep alternating while magnitudes grow. Plot separate points at indices 1 through 6.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

The values 1, 10, 2, 20, 3, 30, ... begin at $n=1$. Assume the odd-index terms increase by 1 and the even-index terms increase by 10. Write a complete recurrence using the term two places back and find the next two terms.

Teacher solution and discussion

Use $a_1=1$, $a_2=10$. For odd $n \geq 3$, set $a_n = a_{n-2} + 1$; for even $n \geq 4$, set $a_n = a_{n-2} + 10$. This keeps the two interleaved subsequences separate and produces $a_7=4$, $a_8=40$. A single seed would leave one parity class unspecified. The increments apply within each parity class, not between every consecutive pair.

Evidence and response

Assess the seed, index convention, update structure and verification separately. Ask learners to generate several terms and compare them with the original representation. For numerical data, distinguish a demonstrated fit from a uniquely determined rule. A missing seed or an update that fails at a later supplied term is not a complete recursive model.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

For $a_n=3n-2$, $n \geq 1$, which recursive definition is equivalent?

- A. $a_1=3$; $a_n=a_{n-1}+3$ for $n \geq 2$
- B. $a_1=1$; $a_n=3a_{n-1}$ for $n \geq 2$
- C. $a_1=1$; $a_n=a_{n-1}-2$ for $n \geq 2$
- D. $a_1=1$; $a_n=a_{n-1}+3$ for $n \geq 2$

Reasoning: _____

M02 | Warm-up | T2

Assume 5, 10, 20, 40, ... is geometric, indexed from $n=1$. Which is its recursive definition?

- A. $a_1=5$; $a_n=a_{n-1}+5$ for $n \geq 2$
- B. $a_0=5$; $a_n=2a_{n-1}$ for $n \geq 1$
- C. $a_1=5$; $a_n=2a_{n-1}$ for $n \geq 2$
- D. $a_1=5$; $a_n=5a_{n-1}$ for $n \geq 2$

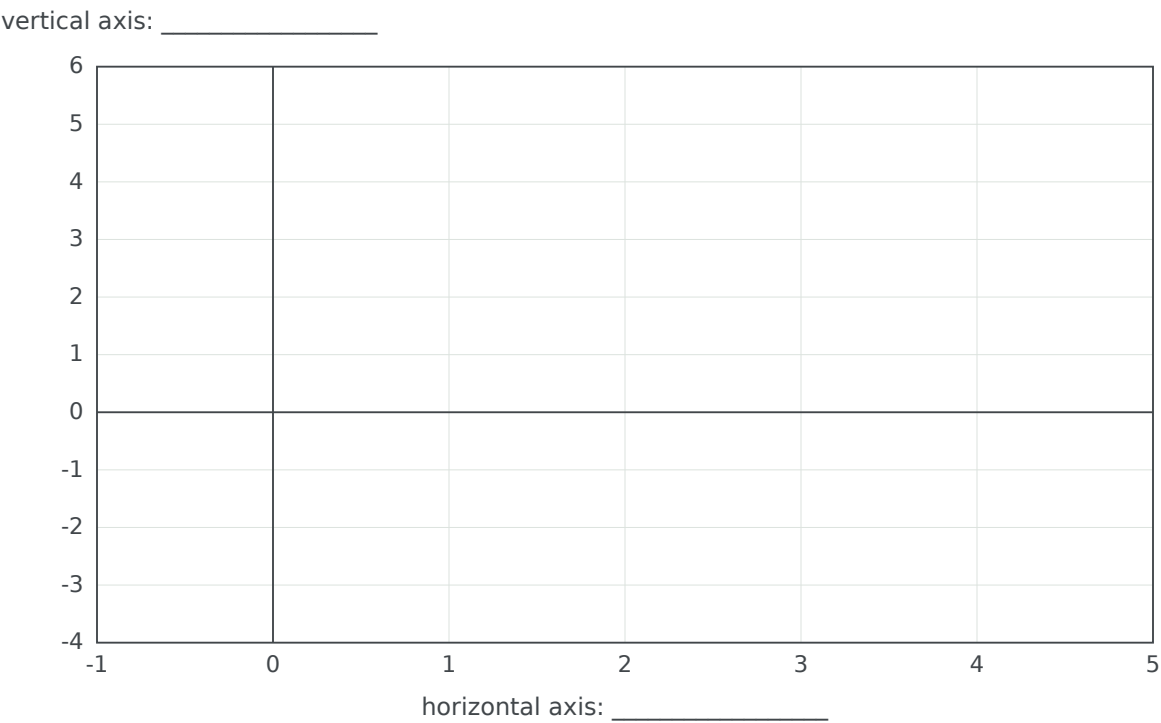
Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.F.6.1 Build recursive functions from algebraic representations.

For $a_n=5-2n$, $n=0, 1, 2, \dots$, derive a recursive definition. List the first five terms and plot them as sequence points. Explain why connecting the points would add inputs not in the stated domain.



Explanation and check

WARM-UP

002 • Show your reasoning

T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.

The sequence starts 7, 11, 15, 19, ... at $n=1$. Assuming a constant consecutive difference, construct a complete recurrence, verify all displayed transitions and find a_{20} . Explain what the assumption contributes.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: PC.F.6.1 Build recursive functions from algebraic representations.****Answer: D. $a_1=1$; $a_n=a_{n-1}+3$ for $n \geq 2$** The first term is 1 and $a_n - a_{n-1} = 3$.**Check each alternative**

- A. The first value is $3 \cdot 1 - 2 = 1$, not 3.
- B. A constant slope gives repeated addition, not multiplication.
- C. The constant -2 is an offset in the explicit formula, not the consecutive difference.
- D. Correct. The first term is 1 and $a_n - a_{n-1} = 3$.

M02 • T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.**Answer: C. $a_1=5$; $a_n=2a_{n-1}$ for $n \geq 2$**

Each term divided by its predecessor equals 2; include the specified first index.

Check each alternative

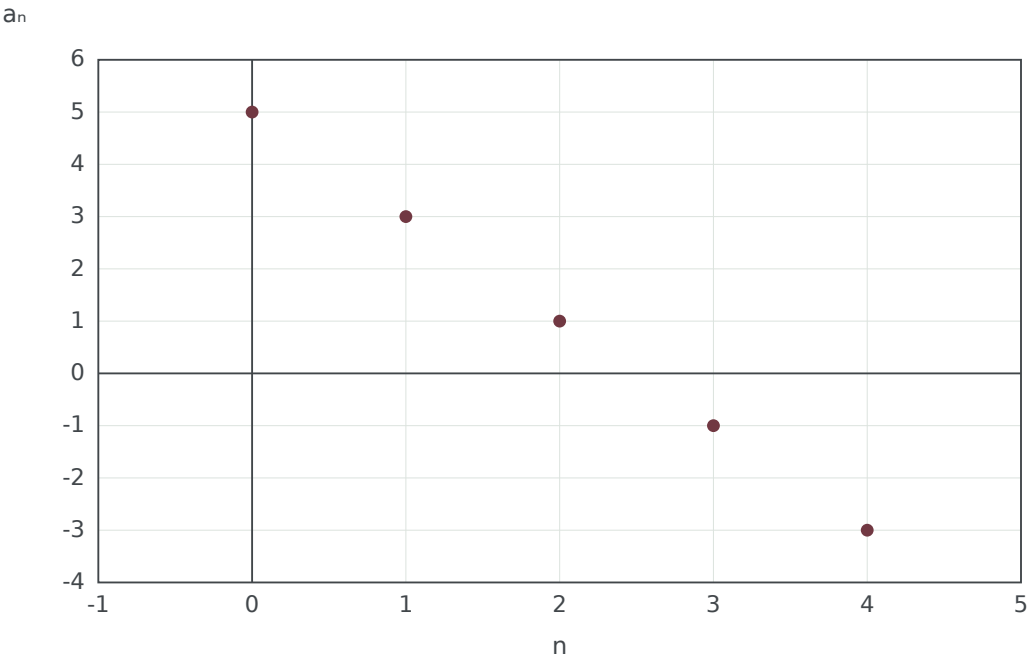
- A. The differences are 5, 10, 20, not a constant 5.
- B. This assigns the first listed value to index 0 rather than the specified index 1.
- C. Correct. Each term divided by its predecessor equals 2; include the specified first index.
- D. Five is the initial value; the common ratio is 2.

WORKED TEACHER KEYS

001

001 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

$a_0=5$ and $a_n-a_{n-1}=(5-2n)-[5-2(n-1)]=-2$, so $a_n=a_{n-1}-2$ for integers $n\geq 1$. The first five terms are 5, 3, 1, -1 , -3 at indices 0, 1, 2, 3, 4. Plot those five separate points. Joining them would visually include noninteger inputs, whereas this sequence is defined only on nonnegative integers.



Evidence to assess

Full evidence: Correct seed, difference derivation, starting index, five values and a discrete graph with the domain explanation. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002 / 003**002 • T2: PC.F.6.2 Construct recursive functions from numerically represented sequences.**

Each displayed difference is 4, so $a_1=7$ and $a_n=a_{n-1}+4$ for $n \geq 2$. Repeating 19 transitions from a_1 to a_{20} gives $a_{20}=7+19 \cdot 4=83$. The constant-difference hypothesis selects this arithmetic continuation; the finite prefix alone would not uniquely determine a_{20} .

Full evidence: Seed and valid index, all differences, correct count of 19 updates and a clear assumption-versus-data distinction. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

003 • T1: PC.F.6.1 Build recursive functions from algebraic representations.

$a_0=-1$. Since $a_n+4=2(a_{n-1}+4)$, $a_n=2a_{n-1}+4$ for $n \geq 1$. The terms are $-1, 2, 8, 20$. With $b_n=a_n+4$, $b_0=3$ and $b_n=2b_{n-1}$, giving $3, 6, 12, 24$. The shift -4 changes the recurrence from pure multiplication to an affine update.

Full evidence: Algebraic derivation, exact seed and four terms, shifted geometric recurrence and the role of the offset. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.