

TEACHER AND STUDENT RESOURCE

Matrix and Vector Operations



DCS.N.1

Number and Quantity

Seven instructional sections. Thirty multiple-choice and thirty open or performance tasks, original mathematical visuals, generous workspaces, teaching scripts, and worked keys.

Free sample | Contents

DCS.N.1 | NC Discrete Mathematics for Computer Science mathematics

This sample contains 19 complete selected pages from the 92-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply operations with matrices and vectors.

DCS.N.1.1 Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.

DCS.N.1.2 Implement procedures of addition, subtraction, and scalar multiplication on vectors.

DCS.N.1.3 Implement procedures to find the inverse of a matrix.

The decimal codes are the official objectives. All are represented in the item bank. These materials support several sessions, not one 60-task lesson.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

DCS.N.1.1: M01, M02, M03, M04, M05, M06, M14, M15, M16, M17, M18, M19, M20, M21, M30, O01, O02, O03, O04, O05, O06, O07, O08, O09, O10, O11, O12

DCS.N.1.2: M07, M08, M09, M10, M22, M23, M24, M25, O13, O14, O15, O16, O17, O18, O19, O20

DCS.N.1.3: M11, M12, M13, M26, M27, M28, M29, O21, O22, O23, O24, O25, O26, O27, O28, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Build and interpret representations

Learning target: Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices. Implement procedures of addition, subtraction, and scalar multiplication on vectors.

Use selected Warm-up and I Do tasks; compare an initial prediction with the worked model.

Session 2 Apply and explain methods

Learning target: Implement procedures to find the inverse of a matrix.

Use We Do and collaborative tasks across the objectives. Require each learner to explain one mathematical choice.

Session 3 Transfer and collect independent evidence

Learning target: Apply the listed objectives with accurate representations, justified methods, and contextual checks.

Choose independent tasks covering each objective. Reteach an evidenced gap, then use a fresh exit task without answer-supplying cues.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, printed tasks, and optional scratch paper. A calculator may support arithmetic after the method is stated. Use graph paper or movable labels when useful without supplying the requested construction.

Prerequisites

Integers, ordered pairs and triples, and arithmetic with fractions.

Pacing

Select two to four tasks for a session. Preserve first independent evidence, discuss a model, and revisit with a fresh task. The full banks are reusable choices, not a timed requirement.

Neutral access

Read directions aloud without identifying a method or answer. Enlarge diagrams, allow dictated or typed reasoning, and provide accessible tables. Preserve all mathematical conditions.

Keep evidence interpretable

Record neutral supports separately from answer-supplying cues. Retain first responses. A selected distractor is not by itself a diagnosis.

Conventions

Matrix $[a,b; c,d]$ lists rows separated by semicolons. Vectors may be written as ordered tuples or explicit columns. Matrix products use row-by-column dot products; a matrix inverse is not entrywise reciprocation.

TEACHER MODEL

Make row-column structure visible

Multiply $A=[1,2;0,-1]$ by $B=[3,0;2,1]$. Explain the four dot products.

Entry	Row from A	Column from B
(1,1)	(1,2)	(3,2)
(1,2)	(1,2)	(0,1)
(2,1)	(0,-1)	(3,2)
(2,2)	(0,-1)	(0,1)

Entry $(1,1)=1\cdot3+2\cdot2=7$; $(1,2)=1\cdot0+2\cdot1=2$; $(2,1)=0\cdot3+(-1)\cdot2=-2$; $(2,2)=0\cdot0+(-1)\cdot1=-1$. Thus $AB=[7,2;-2,-1]$. Each output combines a row of the first factor with a column of the second.

Think aloud: I check the inner dimensions, then name the row and column before calculating. A result with the right shape can still be wrong if I multiply entries in place.

REUSABLE REASONING GUIDE

Represent justify interpret

Givens and conditions

Model and calculation

Justification and independent check

Conclusion in context

WARM-UP

Teaching script

Core 001 | DCS.N.1.1

Collect a short first response before discussion. Ask learners what is given and which condition controls the method.

Display and ask

Two data arrays are $A = [4, 1; 2, 3]$ and $B = [-1, 5; 0, 2]$. Compute $A+B$ and $A-B$. Then recover A by averaging those two results. Explain why the recovery works.

Work and discuss

$A+B = [3, 6; 2, 5]$; $A-B = [5, -4; 2, 1]$. Their sum is $[8, 2; 4, 6] = 2A$, so half is A . In general $(A+B) + (A-B) = 2A$ because B and $-B$ cancel entrywise.

Listen for evidence

Both operations, recovery calculation, and a general entrywise explanation.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M01 for contrast after discussion. Select another task from M01-M04 or O01-O04 to check transfer. Record support and the new evidence separately.

I DO

Teaching script

Core O05 | DCS.N.1.1

Make a prediction before modeling. Think aloud through the core task, making each calculation and check visible.

Display and ask

Can one scalar k make $k[2, -1; 0, 3]=[6, -3; 0, 8]$? Solve the entry conditions and decide whether such k exists.

Work and discuss

The top-left gives $2k=6$, so $k=3$. The top-right also gives $k=3$. But the bottom-right requires $3k=8$, or $k=8/3$. These conditions conflict, so no single scalar works. The zero entry imposes no extra restriction.

Listen for evidence

Use at least two conflicting nonzero entry conditions and address the zero entry.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M05 for contrast after discussion. Select another task from M05-M08 or O05-O08 to check transfer. Record support and the new evidence separately.

WE DO

Teaching script

Core 009 | DCS.N.1.1

Ask learners to propose the next step and have a partner verify it against the original conditions.

Display and ask

$A = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 & 3 \end{bmatrix}$, and column $v = \begin{bmatrix} 1 & 2 \end{bmatrix}$. Calculate $(AB)v$ and $A(Bv)$. Explain why this does not show that $AB=BA$.

Work and discuss

$AB = \begin{bmatrix} 2 & 3 & 0 & 3 \end{bmatrix}$, so $(AB)v = \begin{bmatrix} 2+6 & 0+6 \end{bmatrix} = \begin{bmatrix} 8 & 6 \end{bmatrix}$. $Bv = \begin{bmatrix} 2 & 6 \end{bmatrix}$, then $A(Bv) = \begin{bmatrix} 2+6 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 6 \end{bmatrix}$. This checks associativity for the example. It changes grouping, not factor order; $BA = \begin{bmatrix} 2 & 2 & 0 & 3 \end{bmatrix}$ differs from AB .

Listen for evidence

Two routes to the same product and a clear distinction between associativity and commutativity.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M09 for contrast after discussion. Select another task from M09-M13 or O09-O13 to check transfer. Record support and the new evidence separately.

COLLABORATIVE PRACTICE

Teaching script

Core O14 | DCS.N.1.2

Assign reader, modeler, checker, and explainer roles. Require a private first attempt before comparing approaches.

Display and ask

Points $P=(-3, 2)$, $Q=(1, 5)$, and $R=(4, -1)$ are positions. Compute vectors PQ , QR , and PR . Verify $PQ+QR=PR$ and explain the cancellation of Q .

Work and discuss

$PQ=Q-P=(4,3)$; $QR=R-Q=(3,-6)$; $PR=R-P=(7,-3)$. Adding $(4,3)+(3,-6)=(7,-3)$. Algebraically $(Q-P)+(R-Q)=R-P$ because Q cancels.

Listen for evidence

Correct subtraction order, all three vectors, and the vector-identity explanation.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M14 for contrast after discussion. Select another task from M14-M18 or O14-O18 to check transfer. Record support and the new evidence separately.

INDEPENDENT PRACTICE

Teaching script

Core O19 | DCS.N.1.2

Provide only neutral access supports. Keep the first response and ask for justification without selecting a method.

Display and ask

Determine whether $v=(6, -9, 3)$ and $w=(6, -8, 3)$ are scalar multiples of $u=(2, -3, 1)$. Justify each decision using the same scalar across all components.

Work and discuss

For v , factor 3 gives $3u=(6, -9, 3)$, so yes. For w , the first and third components force factor 3, but then the second must be -9 rather than -8 . No single scalar produces w .

Listen for evidence

One common scalar for the valid case and a contradictory component for the invalid case.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M19 for contrast after discussion. Select another task from M19-M23 or O19-O23 to check transfer. Record support and the new evidence separately.

SMALL-GROUP RETEACH

Teaching script

Core O24 | DCS.N.1.3

Use the earlier work to identify a specific gap. Model that connection, then remove the cue on a fresh item.

Display and ask

Explain why $A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ has no inverse in two ways: use its determinant and find a nonzero column v with $Av = 0$.

Work and discuss

$\det(A) = 12 - 12 = 0$. Also $v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is nonzero and $Av = \begin{bmatrix} -4 + 4 \\ -6 + 6 \end{bmatrix} = 0$. If an inverse existed, multiplying $Av = 0$ by it would force $v = 0$, a contradiction.

Listen for evidence

Zero determinant, valid nonzero null vector, and the inverse contradiction.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M24 for contrast after discussion. Select another task from M24-M27 or O24-O27 to check transfer. Record support and the new evidence separately.

EXIT TICKET

Teaching script

Core O28 | DCS.N.1.3

Collect a concise claim and supporting computation before showing the key. Use the evidence to select a next task.

Display and ask

Compare inverse behavior for $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $2A$. Find both inverses and explain why $(2A)^{-1}$ is not $2A^{-1}$.

Work and discuss

$A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. Since $2A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$, its inverse is $\begin{bmatrix} 1/2 & -1/2 \\ 0 & 1/2 \end{bmatrix} = (1/2)A^{-1}$. Multiplying $2A$ by $2A^{-1}$ would give $4I$, not I . A nonzero scalar must be inverted too.

Listen for evidence

Both inverse calculations and a product-based explanation of the reciprocal scalar.

Probe and respond

Check row and column order before computing. If a learner multiplies entries in place, compare one dot product with that result, then ask for a fresh product.

Next: use M28 for contrast after discussion. Select another task from M28-M30 or O28-O30 to check transfer. Record support and the new evidence separately.

MULTIPLE-CHOICE BANK

M01 / M02 Select and justify**M01 | Warm-up | DCS.N.1.1****A=[1, 2; -1, 2] and B=[3, -1; 2, 1]. Find A + B.**

- A. [4, 1; 1, 3]
- B. [5, 1; 1, 4]
- C. [-4, -1; -1, -3]
- D. [5, 2; 2, 4]

Reasoning: _____

M02 | Warm-up | DCS.N.1.1**A=[1, 2; -1, 2] and B=[3, -1; 2, 1]. Find A – B.**

- A. [-2, 3; -3, 1]
- B. [-1, 3; -3, 2]
- C. [2, -3; 3, -1]
- D. [-2, -3; 3, 1]

Reasoning: _____

WARM-UP

001 Show your reasoning

DCS.N.1.1 | Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.

Two data arrays are $A=[4, 1; 2, 3]$ and $B=[-1, 5; 0, 2]$. Compute $A+B$ and $A-B$. Then recover A by averaging those two results. Explain why the recovery works.

Representation and calculation

Explanation and check

WARM-UP

002 Show your reasoning

DCS.N.1.1 | Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.

Construct a nonzero 2 by 2 matrix A with both positive and negative entries. Find B so that $A+B$ is the zero matrix. Verify all four entries.

Representation and calculation

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 | DCS.N.1.1****Answer A: [4, 1; 1, 3]**

Operate in matching row-column positions: [4, 1; 1, 3]. The result remains a 2 by 2 matrix.

A. Correct. Operate in matching row-column positions: [4, 1; 1, 3]. The result remains a 2 by 2 matrix.

B. Entry in row 1, column 1 is 5, but the required operation gives 4. This is not the correct matrix.

C. Entry in row 1, column 1 is -4, but the required operation gives 4. This is not the correct matrix.

D. Entry in row 1, column 1 is 5, but the required operation gives 4. This is not the correct matrix.

Evidence: Correct entries in the correct positions; an entrywise calculation, not just a copied result.

M02 | DCS.N.1.1**Answer A: [-2, 3; -3, 1]**

Operate in matching row-column positions: [-2, 3; -3, 1]. The result remains a 2 by 2 matrix.

A. Correct. Operate in matching row-column positions: [-2, 3; -3, 1]. The result remains a 2 by 2 matrix.

B. Entry in row 1, column 1 is -1, but the required operation gives -2. This is not the correct matrix.

C. Entry in row 1, column 1 is 2, but the required operation gives -2. This is not the correct matrix.

D. Entry in row 1, column 2 is -3, but the required operation gives 3. This is not the correct matrix.

Evidence: Correct entries in the correct positions; an entrywise calculation, not just a copied result.

WORKED TEACHER KEYS

001 / 002**001 | DCS.N.1.1**

$A+B=[3,6;2,5]$; $A-B=[5,-4;2,1]$. Their sum is $[8,2;4,6]=2A$, so half is A . In general $(A+B)+(A-B)=2A$ because B and $-B$ cancel entrywise.

Evidence: Both operations, recovery calculation, and a general entrywise explanation.

002 | DCS.N.1.1

One valid choice is $A=[1,-2;3,-4]$ and $B=[-1,2;-3,4]$. Their sum is $[0,0;0,0]$. Generally $B=-A$, with every entry negated. Other nonzero mixed-sign constructions with $B=-A$ are valid.

Evidence: A construction satisfying the sign requirement and four verified zero sums.