

TEACHER AND STUDENT RESOURCE

Midpoints and missing endpoints



NC Math 1 • NC.M1.G-GPE.6

Coordinate geometry • Algebraic reasoning

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.G-GPE.6 | NC Math 1 mathematics

This sample contains 21 complete selected pages from the 108-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76-77

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76-77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use coordinates to find the midpoint or endpoint of a line segment.

T1 Find and justify the midpoint of a coordinate segment.

T2 Find and verify a missing endpoint from one endpoint and the midpoint.

T3 Use coordinate relationships to solve for unknown values.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M04, M06, M09, M11, M14, M16, M19, M21, M24, M26, M29, O03, O05, O08, O10, O13, O15, O18, O20, O23, O25, O28, O30

T2: M02, M05, M07, M10, M12, M15, M17, M20, M22, M25, M27, M30, O01, O04, O06, O09, O11, O14, O16, O19, O21, O24, O26, O29

T3: M03, M08, M13, M18, M23, M28, O02, O07, O12, O17, O22, O27

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Find and explain

Learning target: Find and justify the midpoint of a coordinate segment.

Select 2–4 items labeled T1, then use one fresh open task as independent evidence.

Session 2: Find and explain

Learning target: Find and verify a missing endpoint from one endpoint and the midpoint.

Select 2–4 items labeled T2, then use one fresh open task as independent evidence.

Session 3: Use and explain

Learning target: Use coordinate relationships to solve for unknown values.

Select 2–4 items labeled T3, then use one fresh open task as independent evidence.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Plot ordered pairs, subtract signed numbers, work with exact fractions, and interpret slope as vertical change divided by horizontal change. For polygons, review the Pythagorean theorem and area formulas before using them as evidence.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

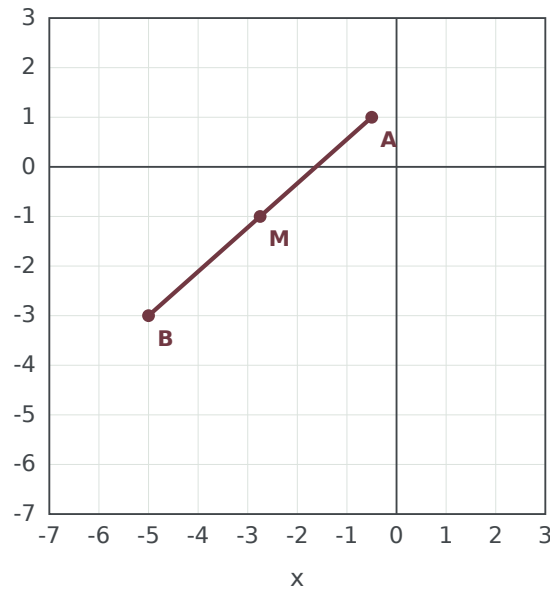
Coordinates are exact. Use exact fractions and radicals unless a task requests an approximation. Axes use equal units. A graph supports an algebraic argument; appearance alone is not proof. Distinct parallel lines never meet; coincident lines are identified separately.

TEACHER MODEL

Model the coordinate reasoning

A(-1/2, 1) and B(t, -3) have midpoint M(-11/4, -1). What is $t + -1/2$? Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed.

y



The x-coordinate equation is $(-1/2 + t)/2 = -11/4$. Hence $t = -5$ and $t + -1/2 = -11/2$. The y-coordinate average is -1, matching M. Both coordinates must satisfy the midpoint relationship. Explicit displacement check: $M - A = B - M = (-9/4, -2)$. These equal direction steps place M on AB and split it into two equal lengths.

COORDINATE ANCHOR

Build a verifiable argument

Distance and perimeter

For endpoints (x_1, y_1) , (x_2, y_2) , $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. Take the nonnegative square root for length. Perimeter adds the lengths of all boundary sides once; state linear units.

Area and classification

Rectangle area = base $\times$ perpendicular height; triangle area = half that product. Area uses square units. Equal squared lengths establish equal lengths. Parallel sides need equal slopes or two vertical directions; right angles need perpendicular directions. State sufficient properties for the claimed polygon.

Slope criteria and exceptions

For distinct nonvertical lines, equal slopes characterize parallel lines. If both slopes are finite, perpendicular lines have product -1 . A horizontal and a vertical line are perpendicular. Distinct vertical lines are parallel. Coincident equations describe the same line.

Midpoint and endpoint

$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$. If A and M are known, $B = 2M - A$ coordinate by coordinate. The equal displacements $M - A$ and $B - M$ show both collinearity and equal halves.

A graph is supporting evidence

Plot exact coordinates using the axis labels and equal scales. Use a straightedge. Label vertices, lines and any required midpoint. Join polygon vertices in the stated order. Teacher line diagrams show finite portions of infinite lines. Use algebra to establish conclusions, rather than measuring the printed drawing.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

A segment has endpoint A(-4, -4) and midpoint M(1/2, -9/2). Find its other endpoint B. Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed.

Teacher solution and discussion

From $(-4 + x)/2 = 1/2$ and $(-4 + y)/2 = -9/2$, $B = 2M - A = (5, -5)$. Check: averaging A and this B gives (1/2, -9/2). Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (9/2, -1/2)$. These equal direction steps place M on AB and split it into two equal lengths.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Find and justify the midpoint of a coordinate segment.

Optional bank: M05–M08; 005–008. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 005, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Endpoints $A(-5, 5)$ and $B(-1, 3/2)$ define a segment. Find its midpoint. Plot and label A , M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed.

Teacher solution and discussion

$M = ((-5 + -1)/2, (5 + 3/2)/2) = (-3, 13/4)$. The displacement A to M is $(2, -7/4)$ and equals M to B , so M lies halfway along AB . Explicit displacement check: $M - A = B - M = (2, -7/4)$. These equal direction steps place M on AB and split it into two equal lengths.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

A segment has endpoint A(-6, 3) and midpoint M(-13/2, -1/2). Find its other endpoint B. Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed. Explain why averaging A and M would not find B.

Teacher solution and discussion

From $(-6 + x)/2 = -13/2$ and $(3 + y)/2 = -1/2$, $B = 2M - A = (-7, -4)$. Check: averaging A and this B gives $(-13/2, -1/2)$. Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (-1/2, -7/2)$. These equal direction steps place M on AB and split it into two equal lengths. Averaging A and M gives a point halfway from A to M, only one-quarter of the way from A to B; it cannot be B for this nonzero segment.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

A segment has endpoint A(-2, -5) and midpoint M(-4, 1/2). Find its other endpoint B. Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed. Explain why averaging A and M would not find B.

Teacher solution and discussion

From $(-2 + x)/2 = -4$ and $(-5 + y)/2 = 1/2$, $B = 2M - A = (-6, 6)$. Check: averaging A and this B gives $(-4, 1/2)$. Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (-2, 11/2)$. These equal direction steps place M on AB and split it into two equal lengths. Averaging A and M gives a point halfway from A to M, only one-quarter of the way from A to B; it cannot be B for this nonzero segment.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

A segment has endpoint A(-2, -2) and midpoint M(-7/2, -1/2). Find its other endpoint B. Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed. Explain why averaging A and M would not find B.

Teacher solution and discussion

From $(-2 + x)/2 = -7/2$ and $(-2 + y)/2 = -1/2$, $B = 2M - A = (-5, 1)$. Check: averaging A and this B gives $(-7/2, -1/2)$. Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (-3/2, 3/2)$. These equal direction steps place M on AB and split it into two equal lengths. Averaging A and M gives a point halfway from A to M, only one-quarter of the way from A to B; it cannot be B for this nonzero segment.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

A segment has endpoint $A(0, 1)$ and midpoint $M(-2, -3/2)$. Find its other endpoint B . Plot and label A , M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed. Explain why averaging A and M would not find B .

Teacher solution and discussion

From $(0 + x)/2 = -2$ and $(1 + y)/2 = -3/2$, $B = 2M - A = (-4, -4)$. Check: averaging A and this B gives $(-2, -3/2)$. Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (-2, -5/2)$. These equal direction steps place M on AB and split it into two equal lengths. Averaging A and M gives a point halfway from A to M , only one-quarter of the way from A to B ; it cannot be B for this nonzero segment.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Find and justify the midpoint of a coordinate segment.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Endpoints $A(-1, -1)$ and $B(7, 4)$ define a segment. Find its midpoint. Plot and label A , M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed. Explain why averaging the coordinates puts the point on the segment, not merely at equal distance from its ends.

Teacher solution and discussion

$M = ((-1 + 7)/2, (-1 + 4)/2) = (3, 3/2)$. The displacement A to M is $(4, 5/2)$ and equals M to B , so M lies halfway along AB . Explicit displacement check: $M - A = B - M = (4, 5/2)$. These equal direction steps place M on AB and split it into two equal lengths.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Endpoints A(-3, -5) and B(-8, -7/2) define a segment. Find its midpoint.

- A. $(-11/2, -17/4)$
- B. $(-17/2, -9/4)$
- C. $(-9/2, -33/4)$
- D. $(-21/2, -9/4)$

Reasoning: _____

M02 | Warm-up | T2

A segment has endpoint A(-3, 0) and midpoint M(-2, -5/2). Find its other endpoint B.

- A. (3, -1)
- B. (-1, -5)
- C. (1, -3)
- D. (-4, -3)

Reasoning: _____

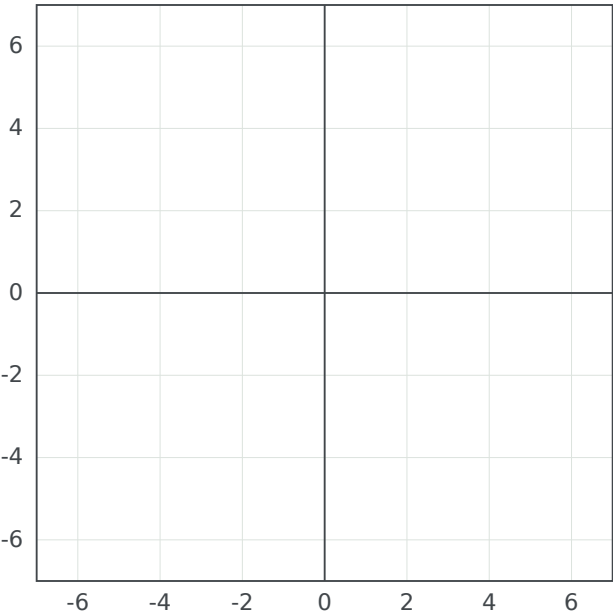
WARM-UP

001 • Show your reasoning

T2: Find and verify a missing endpoint from one endpoint and the midpoint.

A segment has endpoint $A(-4, -4)$ and midpoint $M(1/2, -9/2)$. Find its other endpoint B . Plot and label A , M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed.

vertical axis: _____



horizontal axis: _____

Explanation and check

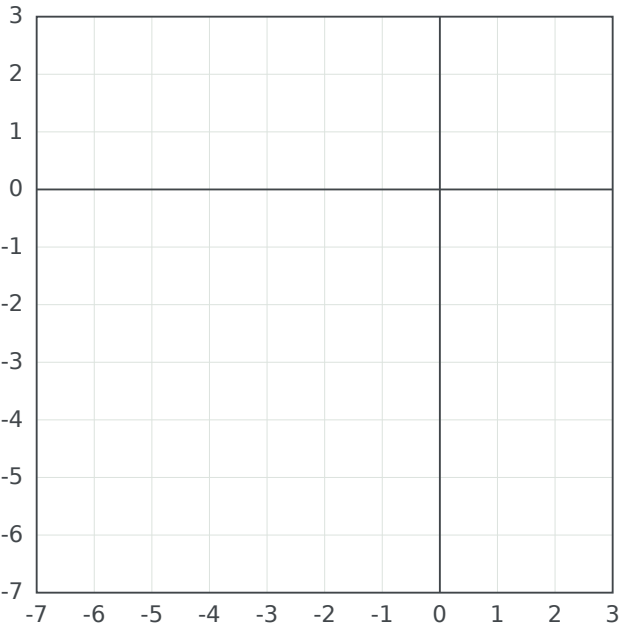
WARM-UP

002 • Show your reasoning

T3: Use coordinate relationships to solve for unknown values.

A(-1/2, 1) and B(t, -3) have midpoint M(-11/4, -1). What is t + -1/2? Plot and label A, M and B on the grid, join the segment, and justify the answer using equal coordinate displacements. Give exact fractions when needed.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Find and justify the midpoint of a coordinate segment.****Answer: A. $(-11/2, -17/4)$**

$M = ((-3 + -8)/2, (-5 + -7/2)/2) = (-11/2, -17/4)$. The displacement A to M is $(-5/2, 3/4)$ and equals M to B, so M lies halfway along AB.

Check each alternative

- A. Matches the verified calculation and all conditions.
- B. Does not match the verified result $(-11/2, -17/4)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- C. Does not match the verified result $(-11/2, -17/4)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- D. Does not match the verified result $(-11/2, -17/4)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.

M02 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.**Answer: B. $(-1, -5)$**

From $(-3 + x)/2 = -2$ and $(0 + y)/2 = -5/2$, $B = 2M - A = (-1, -5)$. Check: averaging A and this B gives $(-2, -5/2)$. Equal displacement on each side of M proves the point is on the segment.

Check each alternative

- A. Does not match the verified result $(-1, -5)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- B. Matches the verified calculation and all conditions.
- C. Does not match the verified result $(-1, -5)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- D. Does not match the verified result $(-1, -5)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.

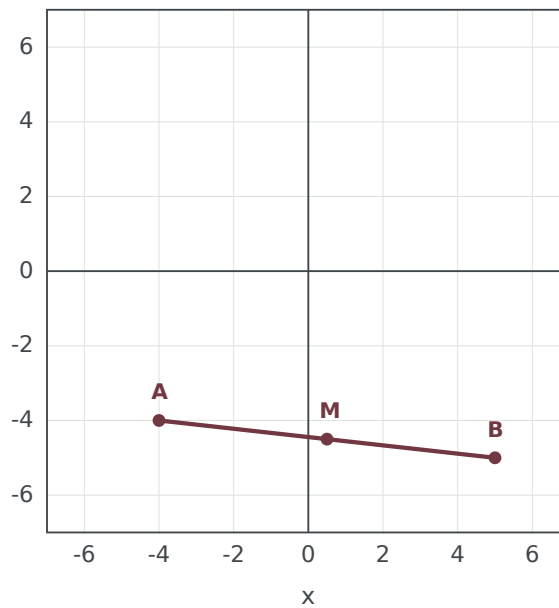
WORKED TEACHER KEYS

001

001 • T2: Find and verify a missing endpoint from one endpoint and the midpoint.

From $(-4 + x)/2 = 1/2$ and $(-4 + y)/2 = -9/2$, $B = 2M - A = (5, -5)$. Check: averaging A and this B gives $(1/2, -9/2)$. Equal displacement on each side of M proves the point is on the segment. Explicit displacement check: $M - A = B - M = (9/2, -1/2)$. These equal direction steps place M on AB and split it into two equal lengths.

y

**Evidence to assess**

Assess the plotted coordinates, algebraic relationships, logical justification and final conclusion. A correct value without the requested argument is incomplete. Accept equivalent exact forms and other valid proofs.

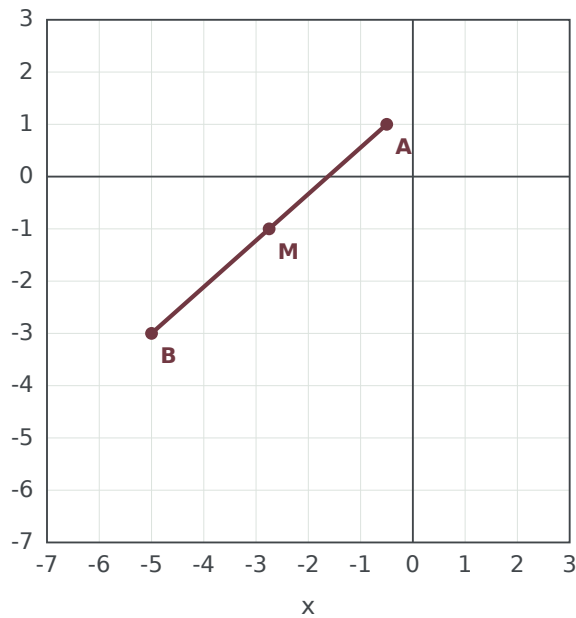
WORKED TEACHER KEYS

002

002 • T3: Use coordinate relationships to solve for unknown values.

The x-coordinate equation is $(-1/2 + t)/2 = -11/4$. Hence $t = -5$ and $t + -1/2 = -11/2$. The y-coordinate average is -1 , matching M. Both coordinates must satisfy the midpoint relationship. Explicit displacement check: $M - A = B - M = (-9/4, -2)$. These equal direction steps place M on AB and split it into two equal lengths.

y

**Evidence to assess**

Assess the plotted coordinates, algebraic relationships, logical justification and final conclusion. A correct value without the requested argument is incomplete. Accept equivalent exact forms and other valid proofs.