

TEACHER AND STUDENT RESOURCE

Solving Inverse and Root Models in Context



NC Math 2 • NC.M2.A-REI.2

Reasoning with Equations and Inequalities

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-REI.2 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Solve and interpret one variable inverse variation and square root equations arising from a context, and explain how extraneous solutions may be produced.

T1 Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

T2 Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

T3 Explain how squaring or discarding an original denominator restriction can produce extraneous candidates, distinguishing these from physical exclusions.

T4 Evaluate model feasibility, compare conditions and construct contextual conclusions from inverse or square-root equations.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M03, O03, M05, O05, M07, O07, M11, O11, M13, O13, M19, M23, O23, M24, M26, O26, M30, O30

T2: M02, O02, M04, O04, M08, O08, M14, O14, M18, O18, O19, M20, O20, O22, O24, M25

T3: M06, O06, M10, O10, M12, O12, M16, O16, M21, O21, M22, O25, M27, O27, M29, O29

T4: M09, O09, M15, O15, M17, O17, M28, O28

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

Learning target: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Learning target: Explain how squaring or discarding an original denominator restriction can produce extraneous candidates, distinguishing these from physical exclusions.

Learning target: Evaluate model feasibility, compare conditions and construct contextual conclusions from inverse or square-root equations.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

Learning target: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Learning target: Explain how squaring or discarding an original denominator restriction can produce extraneous candidates, distinguishing these from physical exclusions.

Learning target: Evaluate model feasibility, compare conditions and construct contextual conclusions from inverse or square-root equations.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Solving linear equations, factoring quadratics, real principal square roots, inverse variation, nonzero denominators, unit conversion and meaningful contextual domains.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All contexts are invented instructional models, not empirical claims or safety guidance. Use the stated units and domain. A principal square root is nonnegative. Keep exact values until a prompt asks for approximation. A contextual exclusion may satisfy the original algebraic equation; an algebraically extraneous candidate does not. Always check the original equation before interpreting a result.

TEACHER MODEL

A candidate must match the original readings

Two dimensionless instrument readings agree when $\sqrt{s+6}=s-6$, where $s \geq 0$ is a setting. Find every matching setting and explain what squaring can hide.

The principal root is nonnegative, so $s-6 \geq 0$ and $s \geq 6$. Squaring gives $s+6=(s-6)^2=s^2-12s+36$, hence $s^2-13s+30=(s-3)(s-10)=0$. Candidate 10 gives $\sqrt{16}=4=10-6$, so it works. Candidate 3 gives $\sqrt{9}=3$ while $3-6=-3$, so it fails. The valid setting is $s=10$. Squaring made opposite values have the same square; the original sign condition and check reject the extraneous 3.

VISUAL ANCHOR

Checking a contextual solution

A computed value is only a candidate until it passes the original equation and the model’s conditions.

Stage	Question	Evidence
Original domain	Are denominators nonzero and roots real?	Write exclusions before transforming
Original equality	Do both original sides actually match?	Check signs, units and exact values
Contextual meaning	Is the value an allowed count, rate, length or setting?	Apply whole-number, positive and operating-range conditions
Interpretation	What does the accepted value answer?	Name the quantity and units, not just a number

A value may fail in different ways. A negative length can satisfy an abstract equation yet be physically excluded. A candidate whose original sides are 3 and -3 is algebraically extraneous. Explain the actual reason.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask learners to identify what the variable measures and which values are meaningful. Check units before choosing inverse or root operations.

Core task

A cyclist travels 90 km at constant speed v km/h in 1.5 hours. Write and solve an inverse equation for v , state the meaningful domain, and check the time with units.

Teacher solution and discussion

For $v > 0$, travel time is $90/v$ hours, so $90/v = 1.5$. Multiply by nonzero v : $90 = 1.5v$, giving $v = 60$ km/h. Check: $90 \text{ km} \div 60 \text{ km/h} = 1.5 \text{ h}$. Zero speed would make the travel-time expression undefined, and a negative speed is not meaningful for this model.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model three gates: original domain, original equality and contextual feasibility. When squaring, display a sign condition next to the equation and substitute every candidate before interpreting it.

Core task

A packaging run takes $T=5+90/r$ minutes, where $r>0$ is the production rate in packages per minute. Find the rate for $T=20$ minutes. Then determine the total time if that rate is doubled and explain why the total time is not cut exactly in half.

Teacher solution and discussion

Set $5+90/r=20$, so $90/r=15$ and $r=6$ packages/min. Check $5+90/6=20$ minutes. Doubling to $r=12$ gives $5+90/12=12.5$ minutes, not 10. Only the variable production time falls from 15 to 7.5 minutes; the fixed 5-minute setup does not change.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Evaluate model feasibility, compare conditions and construct contextual conclusions from inverse or square-root equations.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Compare zero targets, shared charges, physical lengths and sign mismatches. Ask whether a rejected candidate fails the equation itself or only the context; demand a concrete original check.

Core task

A delivery route is 90 km long. A vehicle can travel only at constant speeds from 20 through 40 km/h, inclusive. Solve $90/v=1$ for a 1-hour trip, then decide whether that result is feasible. Is the rejected value extraneous because of clearing the denominator, or excluded by the context?

Teacher solution and discussion

The equation gives $v=90$ km/h, and $90/90=1$, so the result truly satisfies the original algebraic equation. It is outside the allowed interval $[20, 40]$, so no permitted speed achieves the target. It is a contextual exclusion, not a new false solution created by denominator clearing. At the fastest permitted speed, time is $90/40=2.25$ hours.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign model interpreter, solver, domain checker and verifier roles. Compare methods, calibrate a model, inspect exact versus rounded rates, and connect graph intersections with agreement times.

Core task

A classroom distance model uses $v=3\sqrt{d-1}$, with $d \geq 1$ meters and v a speed index. Find d when $v=12$. Show two valid routes: divide before squaring, and square the whole product first. Explain why both routes agree.

Teacher solution and discussion

Division first: $\sqrt{d-1}=4$, so $d-1=16$ and $d=17$ m. Whole-product squaring: $[3\sqrt{d-1}]^2=12^2$ gives $9(d-1)=144$, hence $d-1=16$ and $d=17$. Both sides were nonnegative, and each route preserves the original equality. Check $3\sqrt{17-1}=3 \times 4=12$. The outside 3 must be squared to 9 if squaring is done first.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require the first complete contextual argument without answer-supplying cues. Look for bounds, identities, total-distance structure and fixed-product reasoning rather than one memorized sequence.

Core task

A simplified travel model is $d = av/t$, where $t \geq 0$ is time in minutes and d is distance in meters. A device records $d = 6$ m at $t = 4$ min. Find the positive constant a , then solve for the time when $d = 12$ m. Explain why doubling this modeled distance does not merely double time.

Teacher solution and discussion

From $6 = a\sqrt{4} = 2a$, obtain $a = 3$. Then $12 = 3\sqrt{t}$ gives $\sqrt{t} = 4$ and $t = 16$ min. Check $3\sqrt{16} = 12$. The original reading was at 4 min; doubling distance multiplies the root by 2 and therefore time by 4, from 4 to 16, not from 4 to 8.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use the learner’s observed error: decimal-time conversion, outside coefficient squaring, omitted sign condition or shifted denominator. Repair that step, then check a fresh original model.

Core task

A simplified sensor model reports rate $R=12/\sqrt{t}$, where $t>0$ is elapsed time in minutes. Find the time when $R=3$. Explain why $t=0$ is excluded and verify the original quotient rather than only the squared equation.

Teacher solution and discussion

Because $\sqrt{t}$ is a denominator, $t>0$ is required. Solve $12/\sqrt{t}=3$: multiply by positive $\sqrt{t}$ to get $12=3\sqrt{t}$, hence $\sqrt{t}=4$ and $t=16$ minutes. Check $12/\sqrt{16}=12/4=3$. At $t=0$ the root is zero and the quotient is undefined.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Evaluate model feasibility, compare conditions and construct contextual conclusions from inverse or square-root equations.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Collect exact algebra, domain decisions, original checks and a sentence answering the context. Ask learners to distinguish unavailable group counts from false algebraic solutions.

Core task

A \$420 event fee is shared equally among n participants, who must register in complete teams of 5. Solve for n when each share is exactly \$28 and decide whether it is allowed. Repeat for exactly \$30. Distinguish a contextually excluded count from an algebraically extraneous solution.

Teacher solution and discussion

For \$28, solve $420/n=28$ to get $n=15$, which is a positive multiple of 5. Check $420/15=28$. For \$30, solve $420/n=30$ to get $n=14$, which satisfies the original equation but is not a multiple of 5. Thus the \$30 equality is not feasible under the team rule. The excluded 14 was not created falsely by algebra; it is a valid algebraic solution rejected by the contextual registration constraint.

Evidence and response

Record equation, original restrictions, operation choices, exact candidates, original checks and contextual interpretation separately. Look for explanations of why a candidate fails, not a blanket rule that all negatives or all zero inputs are forbidden. Whole-number constraints and allowed operating ranges must be applied after algebra without silently rounding an exact target.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A bus travels 240 km at constant speed v km/h. Its travel time is $240/v$ hours. What speed gives a 4-hour trip?

- A. 60 km/h
- B. 960 km/h
- C. 236 km/h
- D. $4/240$ km/h

Reasoning: _____

M02 | Warm-up | T2

A square panel has area A square meters and side length $\sqrt{A}$ meters. If its side is 9 m, what area satisfies the model?

- A. -81 square meters
- B. 81 square meters
- C. 3 square meters
- D. 18 square meters

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

A cyclist travels 90 km at constant speed v km/h in 1.5 hours. Write and solve an inverse equation for v , state the meaningful domain, and check the time with units.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

A square garden has perimeter 52 m. Let A be its area in square meters, so its perimeter is $4\sqrt{A}$ meters. Solve for A using this equation and explain why a negative side length is not another physical answer.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

Answer: A. 60 km/h

For $v > 0$, solve $240/v = 4$: $240 = 4v$, so $v = 60$. The check $240/60 = 4$ confirms a 4-hour trip.

Check each alternative

- A. For $v > 0$, solve $240/v = 4$: $240 = 4v$, so $v = 60$. The check $240/60 = 4$ confirms a 4-hour trip.
- B. This multiplies distance by time rather than dividing.
- C. Subtracting hours from kilometers does not give a speed.
- D. This inverts the speed ratio; speed is distance divided by time.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02

M02 • T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Answer: B. 81 square meters

The equation $\sqrt{A}=9$ has nonnegative sides. Squaring gives $A=81$, and $\sqrt{81}=9$. Area is nonnegative.

Check each alternative

- A. A negative area is not in the real square-root model domain.
- B. The equation $\sqrt{A}=9$ has nonnegative sides. Squaring gives $A=81$, and $\sqrt{81}=9$. Area is nonnegative.
- C. This takes a root of the given side rather than squaring it to find area.
- D. Doubling a side does not produce a square area.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T1: Solve one-variable inverse equations and interpret rates, counts, times and units in their original contexts.

For $v > 0$, travel time is $90/v$ hours, so $90/v = 1.5$. Multiply by nonzero v : $90 = 1.5v$, giving $v = 60$ km/h. Check: $90 \text{ km} \div 60 \text{ km/h} = 1.5 \text{ h}$. Zero speed would make the travel-time expression undefined, and a negative speed is not meaningful for this model.

Evidence to assess

Require a one-variable inverse equation, positive-speed domain, correct solution, units and an original time check.

WORKED TEACHER KEYS

002

002 • T2: Solve one-variable square-root equations in context using principal-root signs, meaningful domains and original checks.

Solve $4\sqrt{A}=52$ with $A \geq 0$. Divide by 4: $\sqrt{A}=13$. Squaring gives $A=169$ square meters. Check $4\sqrt{169}=52$ m. The principal root represents the nonnegative side length 13 m; -13 m is not a second side length or a second area solution.

Evidence to assess

Require the root equation, isolation before squaring, area with square units, perimeter check and a correct principal-root interpretation.