

TEACHER AND STUDENT RESOURCE

Matrix Structure and Operations



Precalculus • PC.N.2

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.N.2 | North Carolina Precalculus mathematics

This sample contains 20 complete selected pages from the 95-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties and operations with matrices.

T1 PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.

T2 PC.N.2.2 Use associativity and distributivity while preserving factor order.

T3 PC.N.2.3 Justify and use commutativity of matrix addition.

T4 PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

T5 PC.N.2.5 Calculate and interpret row-by-column matrix products.

PC.N.2.1 Execute the sum and difference algorithms to combine matrices of appropriate dimensions.

PC.N.2.2 Execute associative and distributive properties to matrices.

PC.N.2.3 Execute commutative property to add matrices.

PC.N.2.4 Execute properties of matrices to multiply a matrix by a scalar.

PC.N.2.5 Execute the multiplication algorithm with matrices.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M06, M11, M16, M21, M26, O01, O06, O11, O16, O21, O26

T2: M02, M07, M12, M17, M22, M27, O02, O07, O12, O17, O22, O27

T3: M03, M08, M13, M18, M23, M28, O03, O08, O13, O18, O23, O28

T4: M04, M09, M14, M19, M24, M29, O04, O09, O14, O19, O24, O29

T5: M05, M10, M15, M20, M25, M30, O05, O10, O15, O20, O25, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.

Learning target: PC.N.2.2 Use associativity and distributivity while preserving factor order.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.N.2.2 Use associativity and distributivity while preserving factor order.

Learning target: PC.N.2.3 Justify and use commutativity of matrix addition.

Learning target: PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

Learning target: PC.N.2.5 Calculate and interpret row-by-column matrix products.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, straightedge, blank matrix organizers and graph paper. A basic calculator is optional after dimension and operation choices. No matrix software is needed; an optional calculator check must retain the written row-column calculation.

Prerequisites

Signed arithmetic, the distributive law for real numbers, indexed rectangular arrays, and linear equations.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Matrix notation $[a, b; c, d]$ separates rows with semicolons. An $m \times n$ matrix has m rows and n columns; entry (i, j) means row i , column j . Addition requires equal shapes and matching contextual labels. In AB , inner dimensions must match. A^2 means AA , not entrywise squaring. The symbol 0 denotes the zero matrix of the dimensions required by the expression.

VISUAL ANCHOR

A product entry is a route through matching labels

Read the two axes

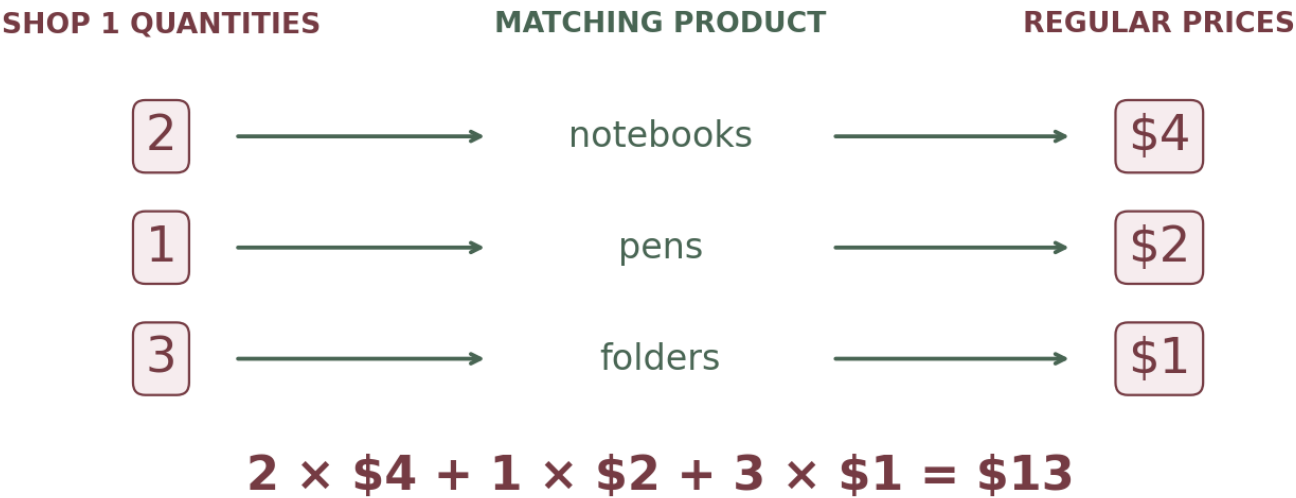
A records two shops by three products. B records those three products by two price scenarios. In AB, a shop row and a scenario column meet through all three products.

Trace one entry

For row [2,1,3] and column [4;2;1], multiply matching product quantities and prices, then add: $2 \cdot 4 + 1 \cdot 2 + 3 \cdot 1 = 13$. The result is one shop value, not three separate entries.

Contrast the operations

Addition combines the same position in two equally shaped arrays. Scalar multiplication changes every entry. Matrix multiplication sums paired products across an entire row and column.



Teacher reference only. Lines connect the three matching product categories. Diagram lengths do not represent values.

TEACHER MODEL

Keep the shapes, labels and order visible

$Q = [2, 1, 3; 0, 4, 2]$ lists two shops by three products; $P = [4, 3; 2, 1; 1, 1]$ lists products by regular and sale prices. Find QP and explain what can and cannot be reordered.

The shared product axis has length 3, so $(2 \times 3)(3 \times 2)$ gives a 2×2 shop-by-scenario matrix. Row-column sums give $QP = [13, 10; 10, 6]$ dollars. Within each scalar sum the real-number terms may be regrouped or reordered, but the factors Q and P cannot simply be switched: PQ has different dimensions and a different interpretation. Matrix addition is commutative when shapes and labels match; matrix multiplication is not generally commutative.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

For $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 & 1 \\ 0 & -2 & 6 \end{bmatrix}$, calculate $A+B$ and $A-B$. Check the difference by adding B back.

Teacher solution and discussion

$A+B = \begin{bmatrix} -1 & 4 & 1 \\ 3 & 2 & 4 \end{bmatrix}$. $A-B = \begin{bmatrix} 5 & -6 & -1 \\ 3 & 6 & -8 \end{bmatrix}$. Adding B to the latter gives $\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \end{bmatrix} = A$. Both operations pair entries in the same 2×3 positions.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T5: PC.N.2.5 Calculate and interpret row-by-column matrix products.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Compute every entry of AB for $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -1 & 3 & 5 & 0 \end{bmatrix}$. Write the dimensions first, then show all four row-column sums.

Teacher solution and discussion

$(2 \times 3)(3 \times 2)$ gives 2×2 . $AB = \begin{bmatrix} 1 \cdot 2 + (-2)(-1) + 3 \cdot 5 & 1 \cdot 1 + (-2) \cdot 3 + 3 \cdot 0 \\ 0 \cdot 2 + 4(-1) + (-1) \cdot 5, \\ 0 \cdot 1 + 4 \cdot 3 + (-1) \cdot 0 \end{bmatrix} = \begin{bmatrix} 19 & -5 \\ -9 & 12 \end{bmatrix}$.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

$A = [125, 200; 75, 40]$ records lengths in centimeters, with rows for two designs and columns for width and height. Convert the whole array to meters with one scalar operation. Explain why adding 0.01 is not a unit conversion.

Teacher solution and discussion

One centimeter is 0.01 meter, so $0.01A = [1.25, 2; 0.75, 0.40]$ meters. Every entry is multiplied by the same unit ratio. Adding 0.01 would shift lengths by a fixed numerical amount rather than change the scale of the unit.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T4: PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Prove $(r+s)A=rA+sA$ using an arbitrary entry of A . Then check your identity for $r=1/2$, $s=3/2$ and $A=[2, -4; 0, 1]$.

Teacher solution and discussion

At each position, $((r+s)A)_{ij}=(r+s)a_{ij}=ra_{ij}+sa_{ij}=(rA+sA)_{ij}$ by real distributivity. In the example, $(r+s)A=2A=[4, -8; 0, 2]$. Also $rA=[1, -2; 0, 1/2]$ and $sA=[3, -6; 0, 3/2]$, summing to the same matrix.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

A learner cancels c in $cA=cB$ without any restriction. Explain the missing condition. Give a counterexample with $c=0$ and distinct 1×2 matrices, then justify the cancellation when $c \neq 0$.

Teacher solution and discussion

The condition is $c \neq 0$. With $c=0$, $A=[1, 0]$ and $B=[2, 0]$, both scalar multiples are $[0, 0]$ although $A \neq B$. When $c \neq 0$, each equality $ca_{ij}=cb_{ij}$ can be divided by c , giving equality of every corresponding entry.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: PC.N.2.4 Apply scalar multiplication and its properties to every matrix entry.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Temperature arrays from two trials are $A = [10, 14; 18, 22]$ and $B = [12, 10; 20, 18]$, measured in degrees Celsius. Find the entrywise mean M . A learner instead uses $(1/2)A + B$. Diagnose the error with a complete comparison.

Teacher solution and discussion

$M = (1/2)(A+B) = (1/2)[22, 24; 38, 40] = [11, 12; 19, 20]$ degrees Celsius. The learner obtains $[5, 7; 9, 11] + [12, 10; 20, 18] = [17, 17; 29, 29]$. Averaging requires halving the contribution of both trials: $(1/2)A + (1/2)B$.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: PC.N.2.3 Justify and use commutativity of matrix addition.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

For $A = \begin{bmatrix} 4 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & -3 \end{bmatrix}$, solve $A+X=B$ and $X+A=B$. Explain why these equations have the same solution set, using addition properties rather than assuming $A=X$.

Teacher solution and discussion

In either equation, $X=B-A=\begin{bmatrix} -3 & 7 & -3 \end{bmatrix}$. $A+X=\begin{bmatrix} 1 & 5 & -3 \end{bmatrix}$, and $X+A$ produces the same column. For any compatible X , $A+X=X+A$ by commutativity; hence the two equations are equivalent. Adding the additive inverse of A isolates X without requiring $A=X$.

Evidence and response

Ask the learner to name the operation, check dimensions, and identify one corresponding-entry calculation or one row-column sum. Distinguish regrouping from reordering. A numerical example verifies an instance; a general entry argument establishes an identity. Preserve a valid representation even when one arithmetic error remains.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A is 2×2 and B is 1×4 . Each contains four entries. Which statement about $A - B$ is correct?

- A. It is 2×2 because A has two rows.
- B. It is undefined because the shapes differ.
- C. It is 1×4 because B has four columns.
- D. It is a scalar because four entries cancel.

Reasoning: _____

M02 | Warm-up | T2

For matrices with compatible dimensions, which identity preserves the order in right distributivity?

- A. $(A+B)C=CA+CB$
- B. $(A+B)C=AC+B$
- C. $(A+B)C=AC+BC$
- D. $(A+B)C=AB+BC$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.

For $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 & 1 \\ 0 & -2 & 6 \end{bmatrix}$, calculate $A+B$ and $A-B$. Check the difference by adding B back.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: PC.N.2.2 Use associativity and distributivity while preserving factor order.

Use $A=[1, -2; 3, 0]$, $B=[4, 2; -1, 5]$, and $C=[-3, 1; 2, -4]$ to calculate $(A+B)+C$ and $A+(B+C)$. State what this example does and does not establish.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.

Answer: B. It is undefined because the shapes differ.

Both row counts and column counts must agree; 2×2 and 1×4 do not.

Check each alternative

- A. Subtraction needs corresponding entries in matching row and column positions.
- B. Correct. Both row counts and column counts must agree; 2×2 and 1×4 do not.
- C. The shape of one operand cannot override the other.
- D. Equal entry counts do not define entrywise subtraction.

M02 • T2: PC.N.2.2 Use associativity and distributivity while preserving factor order.

Answer: C. $(A+B)C=AC+BC$

Every entry in the product of the sum with C splits into an entry of AC plus the matching entry of BC.

Check each alternative

- A. This moves C to the left; matrix multiplication need not commute.
- B. The right factor must multiply both summands.
- C. Correct. Every entry in the product of the sum with C splits into an entry of AC plus the matching entry of BC.
- D. The first product should be AC, not AB.

WORKED TEACHER KEYS

001 / 002**001 • T1: PC.N.2.1 Add and subtract matrices with compatible dimensions and aligned entries.**

$A+B=[-1, 4, 1; 3, 2, 4]$. $A-B=[5, -6, -1; 3, 6, -8]$. Adding B to the latter gives $[2, -1, 0; 3, 4, -2]=A$. Both operations pair entries in the same 2×3 positions.

Full evidence: Two correctly arranged 2×3 results and a six-entry inverse-operation check. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

002 • T2: PC.N.2.2 Use associativity and distributivity while preserving factor order.

$A+B=[5, 0; 2, 5]$, so $(A+B)+C=[2, 1; 4, 1]$. $B+C=[1, 3; 1, 1]$, so $A+(B+C)=[2, 1; 4, 1]$. This verifies associativity for these three arrays; the universal property requires the real-number associative law at every entry.

Full evidence: Both intermediate sums, equal final matrices, and the distinction between checking an example and proving an identity. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.