

TEACHER AND STUDENT RESOURCE

Function Features, Optimization and Conic Sections



Precalculus • PC.F.4

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.F.4 | North Carolina Precalculus mathematics

This sample contains 24 complete selected pages from the 107-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-19, 34-35, 64, 80-81

Representative activities

M01: sample PDF page(s) 20; original page(s) 19

M02: sample PDF page(s) 20; original page(s) 19

O01: sample PDF page(s) 21; original page(s) 34

O02: sample PDF page(s) 22; original page(s) 35

Selected teacher prompts and keys: original page(s) 64, 80-81

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.
Its required companion lab is included with the complete paid download.
The sample does not include that complete companion archive.

ALIGNMENT

Standard and learning targets

Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.

T1 PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

T2 PC.F.4.2 Build exponential growth and decay models from contextual information.

T3 PC.F.4.3 Interpret logarithmic features, including continuity and end behavior.

T4 PC.F.4.4 Solve contextual exponential and logarithmic equations with algebra and technology.

T5 PC.F.4.5 Interpret rational features, including holes, concavity, limits and asymptotes.

T6 PC.F.4.6 Solve constrained polynomial and rational optimization problems with technology.

T7 PC.F.4.7 Construct transformed power, exponential and logarithmic graphs.

T8 PC.F.4.8 Identify ellipses, hyperbolas and parabolas from algebraic forms.

T9 PC.F.4.9 Interpret ellipse axes, hyperbola vertices/transverse axes and parabola vertex/symmetry.

PC.F.4.1 Interpret algebraic and graphical representations to determine key features of exponential functions. Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, limits, and asymptotes.

PC.F.4.2 Integrate information to build exponential functions to model phenomena involving growth or decay.

PC.F.4.3 Interpret algebraic and graphical representations to determine key features of logarithmic functions. Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.

OFFICIAL OBJECTIVES

Scope and coverage

PC.F.4.4 Implement graphical and algebraic methods to solve exponential and logarithmic equations in context with support from technology.

PC.F.4.5 Interpret algebraic and graphical representations to determine key features of rational functions. Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.

PC.F.4.6 Implement graphical and algebraic methods to solve optimization problems given rational and polynomial functions in context with support from technology.

PC.F.4.7 Construct graphs of transformations of power, exponential, and logarithmic functions showing key features.

PC.F.4.8 Identify the conic section (ellipse, hyperbola, parabola) from its algebraic representation in standard form.

PC.F.4.9 Interpret algebraic and graphical representations to determine key features of conic sections (ellipse: center, length of the major and minor axes; hyperbola: vertices, transverse axis; parabola: vertex, axis of symmetry)

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M10, M19, O01, O10, O19

T2: M02, M11, M20, O02, O11, O20

T3: M03, M12, M21, O03, O12, O21

T4: M04, M13, M22, O04, O13, O22

T5: M05, M14, M23, M28, O05, O14, O23, O28

T6: M06, M15, M24, M29, O06, O15, O24, O29

T7: M07, M16, M25, M30, O07, O16, O25, O30

T8: M08, M17, M26, O08, O17, O26

T9: M09, M18, M27, O09, O18, O27

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

Learning target: PC.F.4.2 Build exponential growth and decay models from contextual information.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.F.4.2 Build exponential growth and decay models from contextual information.

Learning target: PC.F.4.3 Interpret logarithmic features, including continuity and end behavior.

Learning target: PC.F.4.4 Solve contextual exponential and logarithmic equations with algebra and technology.

Learning target: PC.F.4.5 Interpret rational features, including holes, concavity, limits and asymptotes.

Learning target: PC.F.4.6 Solve constrained polynomial and rational optimization problems with technology.

Learning target: PC.F.4.7 Construct transformed power, exponential and logarithmic graphs.

Learning target: PC.F.4.8 Identify ellipses, hyperbolas and parabolas from algebraic forms.

Learning target: PC.F.4.9 Interpret ellipse axes, hyperbola vertices/transverse axes and parabola vertex/symmetry.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Graphing calculator or the supplied free Python lab (numpy and scipy), pencils and straightedge. No calculus is required for the student methods: use transformations, algebra, graphs and numerical optimization. Optional exact teacher certificates establish selected global optima without relying only on a screen.

Prerequisites

Function transformations, exponent/logarithm rules, factoring, interval notation and coordinate geometry.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All variables are real unless a context requires whole numbers. $\ln$ denotes logarithm base e ; $\log_2$ denotes base 2. Limits describe approaching behavior and do not assert that a value is attained. A hole remains excluded after cancellation. Concavity describes how slopes change: increasing slopes mean concave up; decreasing slopes mean concave down. Monotonicity is stated on connected intervals of the domain. Optimization must respect the contextual domain and compare feasible endpoints when included. Conic diagrams use equal coordinate scales; labels and equations, not ruler measurements, are authoritative.

TEACHER REFERENCE

Read a complete function, not just a curve

A feature checklist

Record domain and range; all intercepts; positive/negative intervals; increasing/decreasing intervals; concavity; continuity; one-sided and end limits; and each genuine asymptote. A horizontal asymptote can be crossed. A canceled factor can remove a vertical asymptote while leaving a hole and a missing range value.

Concavity without differentiation

Use graph behavior and how secant slopes change, supported by known parent-function shapes. Reflections reverse concavity. For the rational function $x/(x^2+1)$, the specified possible inflection inputs in O23 support the graph investigation; calculus is not a prerequisite for that task.

Optimize on the actual domain

A calculator's local extremum command is a candidate finder, not a proof that a value is globally best. Set a meaningful window, examine the whole feasible domain, compare endpoints and use algebraic bounds when available. A continuous optimum must also be checked against integer restrictions.

Conic structure

A sum of two positive squared terms equal to 1 gives an ellipse; a difference gives a hyperbola. An equation with one squared coordinate and the other linear gives a parabola. Degenerate or empty equations must not be forced into one of these nondegenerate categories.

TECHNOLOGY LABORATORY

Three contextual equation investigations

Files and protocol

The accompanying technology_lab folder contains seven original instructional model CSV files, each with 41 samples, plus teacher_verified_results.json. source/function_lab.py reproduces them. Use a graphing calculator or the script; record the exact formula, domain, units, window and all crossings. Sample rows alone do not locate continuous roots.

A: cooling

$T(t) = 20 + 80e^{(-0.3t)}$ degrees Celsius for $0 \leq t \leq 15$ minutes. Compare with $T = 30$. Suggested window: $0 \leq t \leq 15$, $15 \leq T \leq 105$. The 20-degree baseline must be subtracted before using logarithms.

B: logarithmic calibration

$D(t) = 5 + 2\ln(t+1)$ sensor units for $0 \leq t \leq 25$ minutes. Compare with $D = 11$. Suggested window: $0 \leq t \leq 25$, $4 \leq D \leq 13$. Verify the logarithm domain before exponentiating.

G: growth

$P(t) = 250(1.12)^t$ population units for $0 \leq t \leq 10$ hours. Compare with $P = 600$. Suggested window: $0 \leq t \leq 10$, $200 \leq P \leq 850$. Distinguish the continuous crossing time from the first qualifying whole-hour observation.

TECHNOLOGY LABORATORY

Four constrained optimization investigations

C: open box

$V(x) = x(20 - 2x)(14 - 2x)$ cubic centimeters, where x is the cutout side length in centimeters. A usable box requires $0 < x < 7$; include $x = 0$ and $x = 7$ only as zero-volume boundary comparisons. Suggested window: $0 \leq x \leq 7$, $0 \leq V \leq 360$.

D: average cost

$C(x) = x + 400/x$ dollars per kilogram for $5 \leq x \leq 50$ kilograms. Suggested window: $5 \leq x \leq 50$, $30 \leq C \leq 90$. Search for the minimum and compare both feasible endpoints.

E: output rate

$R(x) = 120x/(x^2 + 9)$ units per hour for control setting $0 \leq x \leq 10$. Suggested window: $0 \leq x \leq 10$, $0 \leq R \leq 22$. A larger input does not necessarily produce a larger output.

F: whole-item profit

$P(q) = -2q^2 + 60q - 200$ dollars for whole-number quantities q from 0 to 25 items. Suggested continuous graph window: $0 \leq q \leq 25$, $-220 \leq P \leq 270$. Find the continuous peak, then check feasible integers and endpoints.

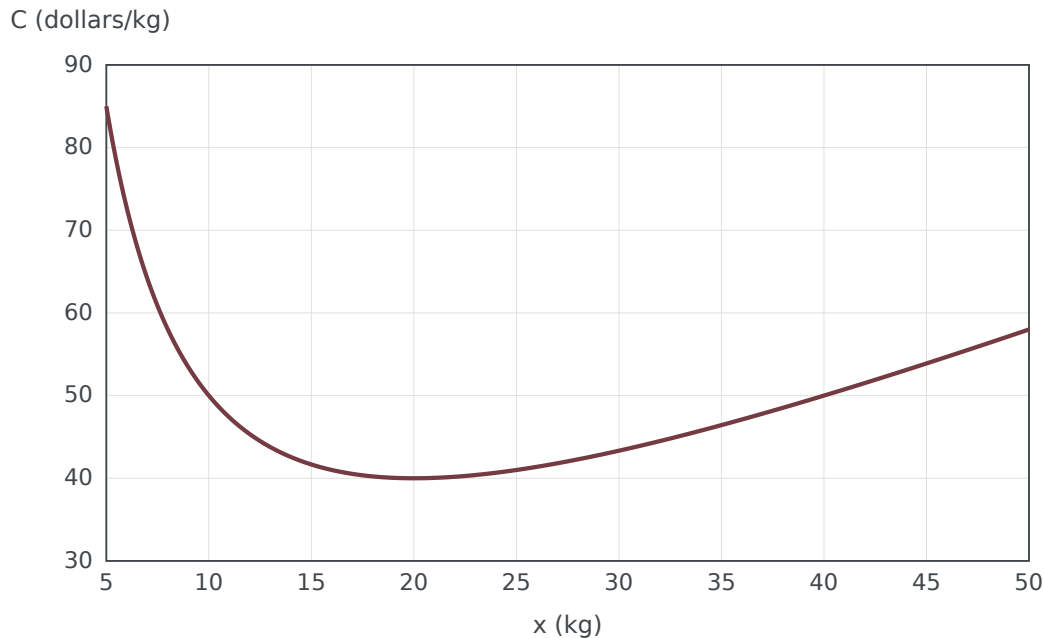
Verification and limitations

The Python script uses bracketing for the three monotone root equations and bounded optimization for these four unimodal examples, with endpoint checks. It additionally enumerates feasible integers for the profit model. This algorithm is not a guarantee for arbitrary multimodal functions. Keep the teacher verification file separate until students have produced their own evidence.

TEACHER MODEL

A numerical optimum needs an algebraic check

For $C(x)=x+400/x$ on $5\leq x\leq 50$, how can a graphing minimum be verified globally?



Graph the complete feasible interval and locate the candidate $x=20$, $C=40$. The endpoints give 85 and 58, already larger. More decisively, $C(x)-40=(x-20)^2/x\geq 0$ because $x>0$. Equality occurs exactly at the feasible value $x=20$. Thus 40 dollars per kilogram is not merely a local screen minimum; it is a proven global minimum on the contextual domain.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

For $f(x)=2^x-3$, give domain, range, intercepts, increasing/decreasing and sign intervals, concavity, end limits and asymptotes. Sketch a graph showing the intercepts and asymptotic behavior.

Teacher solution and discussion

Domain is all real x ; range is $(-3, \infty)$. The y -intercept is $(0, -2)$; the x -intercept is $(\log_2 3, 0)$. The function increases everywhere and is concave up. It is negative for $x < \log_2 3$, zero at $\log_2 3$ and positive above that input. As $x \rightarrow -\infty$, $f \rightarrow -3$ from above; as $x \rightarrow \infty$, $f \rightarrow \infty$. The only asymptote is $y = -3$, and the function is continuous on its domain. Points $(1, -1)$ and $(2, 1)$ help locate the graph.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T5: PC.F.4.5 Interpret rational features, including holes, concavity, limits and asymptotes.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

For $r(x)=1+2/(x-1)$, determine domain/range, all intercepts, sign and monotonic intervals, concavity, continuity, one-sided limits at the excluded input, end limits and asymptotes. Sketch both branches.

Teacher solution and discussion

Domain excludes $x=1$; range excludes $y=1$. The x -intercept is $(-1, 0)$, and y -intercept $(0, -1)$. The function is positive on $(-\infty, -1) \cup (1, \infty)$ and negative on $(-1, 1)$. It decreases on each branch $(-\infty, 1)$ and $(1, \infty)$. It is concave down on the left and concave up on the right, and continuous on each branch. As $x \rightarrow 1^-$, $r \rightarrow -\infty$; as $x \rightarrow 1^+$, $r \rightarrow \infty$. At both infinite ends $r \rightarrow 1$, from below on the left and above on the right. Asymptotes are $x=1$ and $y=1$.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T9: PC.F.4.9 Interpret ellipse axes, hyperbola vertices/transverse axes and parabola vertex/symmetry.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

For $(x-1)^2/25 + (y+2)^2/9 = 1$, find the center, major/minor axis lengths and orientation, and all four axis endpoints. Sketch the ellipse with equal coordinate scales.

Teacher solution and discussion

The center is $(1, -2)$. The horizontal semiaxis is 5 and vertical semiaxis is 3, so the major axis is horizontal with length 10 and the minor axis vertical with length 6. The major endpoints are $(-4, -2)$ and $(6, -2)$; minor endpoints are $(1, -5)$ and $(1, 1)$. The graph is an ellipse centered at the stated point, not at the origin.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T5: PC.F.4.5 Interpret rational features, including holes, concavity, limits and asymptotes.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Analyze $r(x) = (x^2 - 1)/(x - 1)$ without discarding its original restriction. Give domain/range, intercepts, sign, monotonicity, continuity, limit at the missing input, end behavior and any vertical asymptote. Sketch the exact set.

Teacher solution and discussion

For $x \neq 1$, $r = x + 1$. Domain excludes 1 and range excludes 2. Intercepts are $(-1, 0)$ and $(0, 1)$. The function is negative for $x < -1$ and positive on $(-1, 1) \cup (1, \infty)$. It increases on each domain interval, lies on a straight line and has neither upward nor downward curvature. It is continuous everywhere in its domain, with a removable discontinuity at 1 and limit 2 as $x \rightarrow 1$. End behavior follows $x + 1$: $-\infty$ at the left and ∞ at the right. There is no vertical asymptote; draw an open circle at $(1, 2)$.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Compare $f(x)=e^x$ with $g(x)=2-e^{-x}$. Both pass through $(0, 1)$. Determine their domains/ranges, increasing/decreasing behavior, concavity, intercepts, end limits and horizontal asymptotes. Explain why one shared point does not make the graphs equivalent.

Teacher solution and discussion

Both domains are all real and both functions increase. For f , range is $(0, \infty)$, concavity is up, y-intercept $(0, 1)$, no x-intercept, left-end limit 0 and right-end limit ∞ , with asymptote $y=0$. For g , range is $(-\infty, 2)$, concavity is down, y-intercept $(0, 1)$, x-intercept $(-\ln 2, 0)$, left-end limit $-\infty$ and right-end limit 2, with asymptote $y=2$. Their ranges, curvature and other values differ despite the shared point.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T6: PC.F.4.6 Solve constrained polynomial and rational optimization problems with technology.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner's prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Use technology to maximize $R(x) = 120x/(x^2 + 9)$ units per hour over control settings $0 \leq x \leq 10$. Record the optimum and endpoint rates, then prove the bound algebraically and explain why choosing the largest setting would be a mistake.

Teacher solution and discussion

Use window $0 \leq x \leq 10$, $0 \leq R \leq 22$. The maximum rate is 20 units/hour at $x = 3$. Endpoints give $R(0) = 0$ and $R(10) = 1200/109 \approx 11.009$. The identity $20 - R(x) = 20(x - 3)^2/(x^2 + 9) \geq 0$ proves the global upper bound, attained at 3. Increasing the setting beyond 3 eventually lowers the rate, so the largest feasible input is not the maximizing input.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T5: PC.F.4.5 Interpret rational features, including holes, concavity, limits and asymptotes.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Analyze $r(x) = 2/(x-1)^2 - 1$. Give domain/range, intercepts, sign and monotonic intervals, concavity, continuity, one-sided limits at 1 and both end limits. Explain why its two branches approach the same infinity near the pole.

Teacher solution and discussion

Domain excludes $x=1$; range is $(-1, \infty)$. The y-intercept is $(0, 1)$, and zeros are $x=1 \pm \sqrt{2}$. It is positive on $(1-\sqrt{2}, 1) \cup (1, 1+\sqrt{2})$, negative outside the two zeros, and zero at them. It increases on $(-\infty, 1)$ and decreases on $(1, \infty)$, and is concave up on both branches. It is continuous on each branch. Both one-sided limits at 1 are $+\infty$ because the squared denominator approaches zero through positive values. Both end limits are -1 from above. Asymptotes are $x=1$ and $y=-1$; the lower range bound is never attained.

Evidence and response

For every feature claim, ask for an algebraic restriction or a visible graph behavior. Separate a limit from a function value, a removable hole from a vertical asymptote, and a numerical candidate from a verified global optimum. For technology tasks, record the formula, domain/window, candidate values, endpoint comparisons and an independent algebraic check.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****For $f(x)=2^x-3$, which feature statement is correct?**

- A. Domain all real x ; range $(-3, \infty)$; horizontal asymptote $y=-3$.
- B. Domain $x>0$; range all real numbers.
- C. Domain all real x ; range $[-3, \infty)$.
- D. Domain all real x ; vertical asymptote $x=-3$.

Reasoning: _____

M02 | Warm-up | T2**A quantity starts at 500 and decreases by 15% each hour. Which exponential model is appropriate for t hours?**

- A. $Q(t)=500(0.85)^t$
- B. $Q(t)=500(0.15)^t$
- C. $Q(t)=500-15t$
- D. $Q(t)=500(1.15)^t$

Reasoning: _____

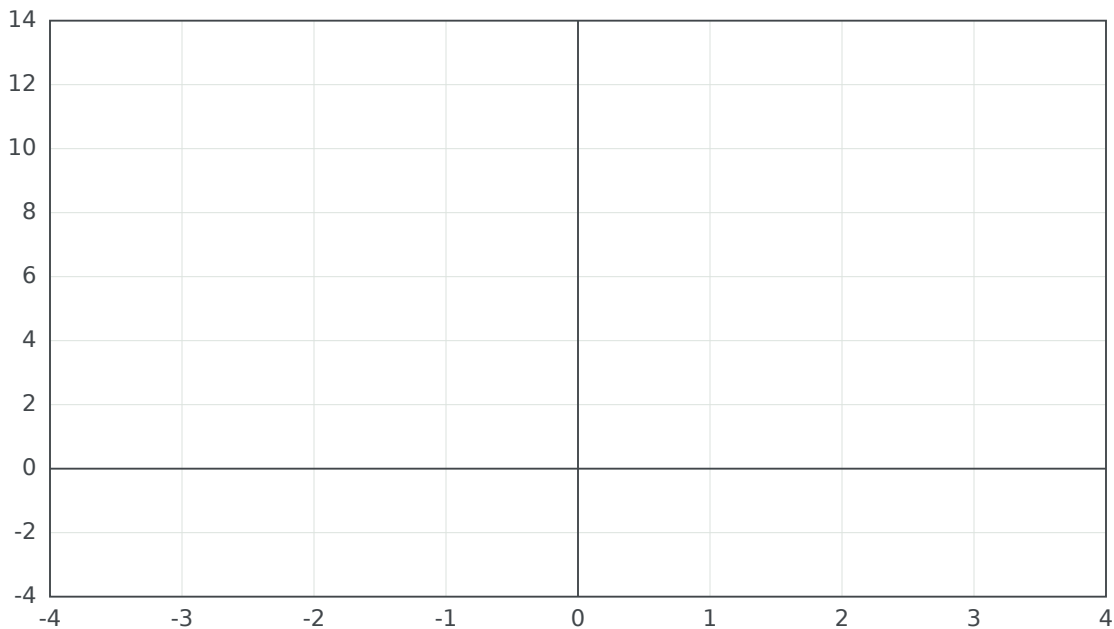
WARM-UP

001 • Show your reasoning

T1: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

For $f(x)=2^x-3$, give domain, range, intercepts, increasing/decreasing and sign intervals, concavity, end limits and asymptotes. Sketch a graph showing the intercepts and asymptotic behavior.

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

002 • Show your reasoning

T2: PC.F.4.2 Build exponential growth and decay models from contextual information.

An amount starts at 640 grams and halves every 5 days. Build an exponential model for $t \geq 0$, determine the one-day factor, predict the amount after 15 days and explain why subtracting 64 grams each day is not equivalent.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

Answer: A. Domain all real x ; range $(-3, \infty)$; horizontal asymptote $y = -3$.

Since $2^x > 0$ for every real x , $f > -3$ and approaches -3 as x tends to negative infinity.

Check each alternative

- A. Correct. Since $2^x > 0$ for every real x , $f > -3$ and approaches -3 as x tends to negative infinity.
- B. This confuses an exponential with a logarithm.
- C. The positive exponential never equals zero, so -3 is not attained.
- D. The vertical shift creates a horizontal, not vertical, asymptote.

M02 • T2: PC.F.4.2 Build exponential growth and decay models from contextual information.

Answer: A. $Q(t) = 500(0.85)^t$

Each hour leaves $1 - 0.15 = 0.85$ of the preceding quantity, so repeated multiplication gives the stated model.

Check each alternative

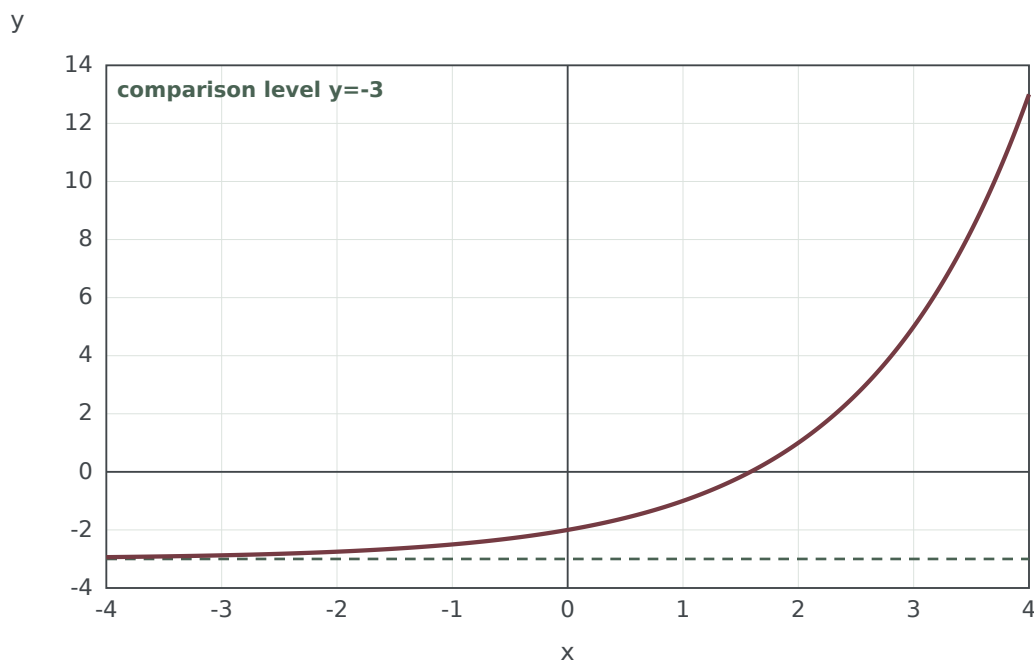
- A. Correct. Each hour leaves $1 - 0.15 = 0.85$ of the preceding quantity, so repeated multiplication gives the stated model.
- B. This retains only 15% each hour rather than losing 15%.
- C. This subtracts a fixed amount, not a fixed percentage.
- D. This models growth by 15%, not decay.

WORKED TEACHER KEYS

001

001 • T1: PC.F.4.1 Interpret exponential features, including concavity, limits and asymptotes.

Domain is all real x ; range is $(-3, \infty)$. The y -intercept is $(0, -2)$; the x -intercept is $(\log_2 3, 0)$. The function increases everywhere and is concave up. It is negative for $x < \log_2 3$, zero at $\log_2 3$ and positive above that input. As $x \rightarrow -\infty$, $f \rightarrow -3$ from above; as $x \rightarrow \infty$, $f \rightarrow \infty$. The only asymptote is $y = -3$, and the function is continuous on its domain. Points $(1, -1)$ and $(2, 1)$ help locate the graph.

**Evidence to assess**

Full evidence: All listed features with correct strict range/sign endpoints, both limits and a graph matching the intercepts and horizontal asymptote. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002

002 • T2: PC.F.4.2 Build exponential growth and decay models from contextual information.

A model is $A(t) = 640 \cdot 2^{(-t/5)} = 640(2^{(-1/5)})^t$. The one-day factor is $2^{(-1/5)}$, not a fixed daily loss. After 15 days there are three half-lives, so $A(15) = 640/8 = 80$ grams. Subtracting 64 daily would make the same absolute loss each day and eventually negative amounts; the exponential loses the same proportion over equal time spans and stays positive.

Evidence to assess

Full evidence: Correct time-scaled model, one-day factor, exact prediction with units and a proportional-versus-additive explanation. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.