

TEACHER AND STUDENT RESOURCE

Matrix Models and Systems



DCS.N.2

Number and Quantity

Seven instructional sections. Thirty multiple-choice and thirty open or performance tasks, original mathematical visuals, generous workspaces, teaching scripts, and worked keys.

Free sample | Contents

DCS.N.2 | NC Discrete Mathematics for Computer Science mathematics

This sample contains 20 complete selected pages from the 93-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand matrices to solve problems.

DCS.N.2.1 Organize data into matrices to solve problems.

DCS.N.2.2 Interpret solutions found using matrix operations including Leslie Models and Markov Chains, in context.

DCS.N.2.3 Represent a system of equations as a matrix equation.

DCS.N.2.4 Use inverse matrices to solve a system of equations with technology.

The decimal codes are the official objectives. All are represented in the item bank. These materials support several sessions, not one 60-task lesson.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

DCS.N.2.1: M01, M02, M03, M16, M17, M18, O01, O02, O03, O04, O05, O06

DCS.N.2.2: M04, M05, M06, M07, M08, M09, M14, M15, M19, M20, M21, M22, M23, M24, M25, M30, O07, O08, O09, O10, O11, O12, O13, O14, O15, O16, O17, O18

DCS.N.2.3: M10, M13, M26, O19, O20, O21, O22

DCS.N.2.4: M11, M12, M27, M28, M29, O23, O24, O25, O26, O27, O28, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Build and interpret representations

Learning target: Organize data into matrices to solve problems. Interpret solutions found using matrix operations including Leslie Models and Markov Chains, in context.

Use selected Warm-up and I Do tasks; compare an initial prediction with the worked model.

Session 2 Apply and explain methods

Learning target: Represent a system of equations as a matrix equation. Use inverse matrices to solve a system of equations with technology.

Use We Do and collaborative tasks across the objectives. Require each learner to explain one mathematical choice.

Session 3 Transfer and collect independent evidence

Learning target: Apply the listed objectives with accurate representations, justified methods, and contextual checks.

Choose independent tasks covering each objective. Reteach an evidenced gap, then use a fresh exit task without answer-supplying cues.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, printed tasks, and optional scratch paper. A spreadsheet or calculator with matrix-inverse support is required for tasks explicitly assessing technology implementation. Use accessible software and record the formula or command and output.

Prerequisites

Matrix multiplication, inverse of a nonsingular 2 by 2 matrix, and interpreting labeled data.

Pacing

Select two to four tasks for a session. Preserve first independent evidence, discuss a model, and revisit with a fresh task. The full banks are reusable choices, not a timed requirement.

Neutral access

Read directions aloud without identifying a method or answer. Enlarge diagrams, allow dictated or typed reasoning, and provide accessible tables. Preserve all mathematical conditions.

Keep evidence interpretable

Record neutral supports separately from answer-supplying cues. Retain first responses. A selected distractor is not by itself a diagnosis.

Conventions

Population models use column vectors: next state = matrix $\times$ current state. Markov matrix columns sum to 1. Leslie first-row entries model fertility; subdiagonal entries model survival. Counts are expected values.

TEACHER MODEL

Read source and destination together

A closed Markov model uses column states [A;B]. Derive P from the transition table and predict the next counts from [60;40].

Destination / source	From A	From B
To A	0.7	0.2
To B	0.3	0.8

The source distributions are columns: $P=[0.7,0.2;0.3,0.8]$. Next $A=0.7(60)+0.2(40)=50$; next $B=0.3(60)+0.8(40)=50$. Total remains 100 because both source columns sum to 1.

Think aloud: I label source and destination before calculating. Compatible dimensions alone cannot detect a reversed model convention.

REUSABLE REASONING GUIDE

Represent justify interpret

Givens and conditions

Model and calculation

Justification and independent check

Conclusion in context

WARM-UP

Teaching script

Core 001 | DCS.N.2.1

Collect a short first response before discussion. Ask learners what is given and which condition controls the method.

Display and ask

North sold 4 notebooks and 7 pens Monday, then 6 notebooks and 5 pens Tuesday. Build a labeled matrix with days as rows and products as columns. Give a second valid orientation and explain how it is related to the first.

Work and discuss

With rows Monday, Tuesday and columns notebooks, pens, $Q = [4, 7; 6, 5]$. With rows notebooks, pens and columns Monday, Tuesday, $Q^T = [4, 6; 7, 5]$. The second matrix transposes the first: each row-column role is exchanged without changing any data.

Listen for evidence

Two correctly labeled organizations and a transpose explanation.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M01 for contrast after discussion. Select another task from M01-M04 or O01-O04 to check transfer. Record support and the new evidence separately.

I DO

Teaching script

Core 005 | DCS.N.2.1

Make a prediction before modeling. Think aloud through the core task, making each calculation and check visible.

Display and ask

Branches A and B report weekday transactions by rows: $Q = [10, 4; 7, 6; 8, 5]$, where columns are deposits and withdrawals, and rows are A Monday, A Tuesday, B Monday. Construct a matrix R that aggregates A's two rows and retains B's row. Find RQ .

Work and discuss

$R = [1, 1, 0; 0, 0, 1]$. Multiplying gives $RQ = [10 + 7, 4 + 6; 8, 5] = [17, 10; 8, 5]$. Output rows are total for the two listed A days, then the listed B day. Do not describe B's row as a two-day total.

Listen for evidence

Build an aggregation matrix, compute the result, and accurately label unequal reporting windows.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M05 for contrast after discussion. Select another task from M05-M08 or O05-O08 to check transfer. Record support and the new evidence separately.

WE DO

Teaching script

Core 009 | DCS.N.2.2

Ask learners to propose the next step and have a partner verify it against the original conditions.

Display and ask

$P = [0.6, 0.2; 0.4, 0.8]$. Find a stationary column with total 120 and verify it. Then test whether $[120; 0]$ is stationary, explaining why one stationary state does not make every state stationary.

Work and discuss

Solve $a = 0.6a + 0.2b$, so $b = 2a$. With $a + b = 120$, $v = [40; 80]$. $Pv = [24 + 16; 16 + 64] = [40; 80]$. But $P[120; 0] = [72; 48]$, not $[120; 0]$. Stationarity applies only to vectors satisfying $Pv = v$.

Listen for evidence

Stationarity equations, normalization, product verification, and a contrasting state.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M09 for contrast after discussion. Select another task from M09-M13 or O09-O13 to check transfer. Record support and the new evidence separately.

COLLABORATIVE PRACTICE

Teaching script

Core O14 | DCS.N.2.2

Assign reader, modeler, checker, and explainer roles. Require a private first attempt before comparing approaches.

Display and ask

For $L=[0,2;0.5,0]$, start with $[30;8]$. Find one- and two-interval predictions. Explain why the second prediction returns to the initial vector in this idealized model.

Work and discuss

$v_1=[16;15]$; $v_2=[30;8]$. $L^2=[1,0;0,1]=I$, since fertility 2 and survival 0.5 multiply to 1. Thus applying L twice restores every initial vector in this particular two-stage model. This is a property of the assumptions, not a general law of populations.

Listen for evidence

Two computations, the L^2 identity, and a limitation of the interpretation.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M14 for contrast after discussion. Select another task from M14-M18 or O14-O18 to check transfer. Record support and the new evidence separately.

INDEPENDENT PRACTICE

Teaching script

Core O19 | DCS.N.2.3

Provide only neutral access supports. Keep the first response and ask for justification without selecting a method.

Display and ask

Adult tickets cost \$8 and student tickets \$5. A sale includes 11 tickets and \$73 revenue. Define variables, write a matrix equation, and solve with a check in the original context.

Work and discuss

Let x =adult and y =student tickets. $x+y=11$, $8x+5y=73$, so $[1,1;8,5][x;y]=[11;73]$. Subtract 5 times the first equation from the second: $3x=18$, $x=6$, $y=5$. Check $6+5=11$ and $48+25=73$.

Listen for evidence

Named variables, aligned equations and matrix, feasible integer solution, and context check.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M19 for contrast after discussion. Select another task from M19-M23 or O19-O23 to check transfer. Record support and the new evidence separately.

SMALL-GROUP RETEACH

Teaching script

Core O24 | DCS.N.2.4

Use the earlier work to identify a specific gap. Model that connection, then remove the cue on a fresh item.

Display and ask

$A = \begin{bmatrix} 2 & 1 & 1 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 7 & 2 \end{bmatrix}$. A student's calculator reports $\begin{bmatrix} 2 & 3 \end{bmatrix}$. Use matrix technology to test the reported vector, find the correct solution with $A^{-1}b$, and identify what the evidence can and cannot tell you about the mistake.

Work and discuss

The reported vector gives $Ax = \begin{bmatrix} 7 & -1 \end{bmatrix}$, residual $\begin{bmatrix} 0 & -3 \end{bmatrix}$, so it fails. $A^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 & -2/3 \end{bmatrix}$; $A^{-1}b = \begin{bmatrix} 3 & 1 \end{bmatrix}$. Its residual is $\begin{bmatrix} 0 & 0 \end{bmatrix}$. Record the inverse/product commands used. The evidence establishes the incorrect output but does not uniquely identify whether a sign, entry, or command error caused it.

Listen for evidence

Technology verification, corrected solution, and no unsupported diagnosis from one wrong answer.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M24 for contrast after discussion. Select another task from M24-M27 or O24-O27 to check transfer. Record support and the new evidence separately.

EXIT TICKET

Teaching script

Core O28 | DCS.N.2.4

Collect a concise claim and supporting computation before showing the key. Use the evidence to select a next task.

Display and ask

Use matrix technology to solve $[1,1,1;2,-1,1;1,2,-1][x;y;z]=[6;3;2]$. Record the matrix dimensions, inverse-product command, output, and checks in all three equations.

Work and discuss

A is 3 by 3 and b 3 by 1. Use `=MMULT(MINVERSE(A1:C3),E1:E3)`, after entering A and b in those ranges. $A^{-1}=[-1/7,3/7,2/7;3/7,-2/7,1/7;5/7,-1/7,-3/7]$. Output $[1;2;3]$. Checks: $1+2+3=6;2-2+3=3;1+4-3=2$.

Listen for evidence

Three-dimensional technology use, recorded inverse/output, and every equation check.

Probe and respond

Ask what each row, column, and output represents. If a result is numerically correct but unlabeled, request a unit and state-order check. A matrix model requires stated assumptions.

Next: use M28 for contrast after discussion. Select another task from M28-M30 or O28-O30 to check transfer. Record support and the new evidence separately.

MULTIPLE-CHOICE BANK

M01 Select and justify

M01 | Warm-up | DCS.N.2.1

Rows of $M = \begin{bmatrix} 9 & 5 \\ 7 & 8 \end{bmatrix}$ are Monday and Tuesday; columns are notebooks and pens sold. How many total items were sold Monday? Explain the row choice.

Day	Notebooks	Pens
Monday	9	5
Tuesday	7	8

- A. 14
- B. 16
- C. 15
- D. 13

Reasoning: _____

MULTIPLE-CHOICE BANK

M02 / M03 Select and justify**M02 | Warm-up | DCS.N.2.1**

Rows of $Q = \begin{bmatrix} 6 & 3 & 2 & 5 \end{bmatrix}$ are shops A and B; columns are packs and pens. The price column is $p = \begin{bmatrix} 4 & 2 \end{bmatrix}$ dollars. Find revenue Qp in shop order.

- A. $\begin{bmatrix} 30 & 22 \end{bmatrix}$
- B. $\begin{bmatrix} 60 & 36 \end{bmatrix}$
- C. $\begin{bmatrix} 32 & 18 \end{bmatrix}$
- D. $\begin{bmatrix} 30 & 18 \end{bmatrix}$

Reasoning: _____

M03 | Warm-up | DCS.N.2.1

Inventory matrix $M = \begin{bmatrix} 4 & 2 & 1 & 6 \end{bmatrix}$ records stores by rows and products by columns. Delivery matrix $R = \begin{bmatrix} 1 & 2 & 2 & 1 \end{bmatrix}$ uses the same labels. Find the new inventory.

- A. $\begin{bmatrix} 6 & 14 & 9 & 10 \end{bmatrix}$
- B. $\begin{bmatrix} 5 & 4 & 3 & 7 \end{bmatrix}$
- C. $\begin{bmatrix} 3 & 0 & -1 & 5 \end{bmatrix}$
- D. $\begin{bmatrix} 5 & 3 & 4 & 7 \end{bmatrix}$

Reasoning: _____

WARM-UP

001 Show your reasoning

DCS.N.2.1 | Organize data into matrices to solve problems.

North sold 4 notebooks and 7 pens Monday, then 6 notebooks and 5 pens Tuesday. Build a labeled matrix with days as rows and products as columns. Give a second valid orientation and explain how it is related to the first.

Representation and calculation

Explanation and check

WARM-UP

002 Show your reasoning

DCS.N.2.1 | Organize data into matrices to solve problems.

Three shops sold 2 books and 5 pens; 4 books and 1 pen; and 3 books and 6 pens. Books cost \$7 and pens \$2. Organize the raw data and prices into matrices, calculate each shop’s revenue, and label the output.

Representation and calculation

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 | DCS.N.2.1****Answer A: 14**

Monday is row 1, so add $9+5=14$ items. Adding a column would combine days for one product.

A. Correct. Monday is row 1, so add $9+5=14$ items. Adding a column would combine days for one product.

B. This totals the notebook column across two days.

C. This totals Tuesday's row, not Monday's.

D. This totals the pen column across two days.

Evidence: Select the labeled row and add both product counts.

M02 | DCS.N.2.1**Answer D: [30; 18]**

Shop A: $4(6)+2(3)=30$ dollars. Shop B: $4(2)+2(5)=18$ dollars. $Qp=[30; 18]$.

A. Output entry (2,1) is 22, but its row-column calculation gives 18. The labeled output is therefore incorrect.

B. Output entry (1,1) is 60, but its row-column calculation gives 30. The labeled output is therefore incorrect.

C. Output entry (1,1) is 32, but its row-column calculation gives 30. The labeled output is therefore incorrect.

D. Correct. Shop A: $4(6)+2(3)=30$ dollars. Shop B: $4(2)+2(5)=18$ dollars. $Qp=[30; 18]$.

Evidence: Quantity-price products, row ordering, and dollar units.

WORKED TEACHER KEYS

001 / 002**001 | DCS.N.2.1**

With rows Monday,Tuesday and columns notebooks,pens, $Q=[4,7;6,5]$. With rows notebooks,pens and columns Monday,Tuesday, $Q^T=[4,6;7,5]$. The second matrix transposes the first: each row-column role is exchanged without changing any data.

Evidence: Two correctly labeled organizations and a transpose explanation.

002 | DCS.N.2.1

$Q=[2,5;4,1;3,6]$, with shop rows and book/pen columns; $p=[7;2]$.

$Qp=[2 \cdot 7 + 5 \cdot 2; 4 \cdot 7 + 1 \cdot 2; 3 \cdot 7 + 6 \cdot 2]=[24;30;33]$ dollars in shop order.

Evidence: Data organization, price alignment, three dot products, and labeled dollar output.