

MATHEMATICS TEACHING AND ASSESSMENT

Volumes and their relationships



Grade 8 • NC.8.G.9

Teacher models • Seven instructional sections
30 multiple-choice tasks • 30 open/performance tasks
Worked keys • Student workspaces • Observation tools

Reusable across several lessons. Select evidence purposefully; this is not a 60-item single-session assessment.

Free sample | Contents

NC.8.G.9 | Eighth Grade mathematics

This sample contains 18 complete selected pages from the 91-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-14, 29-30, 59, 74

Representative activities

M01: sample PDF page(s) 15; original page(s) 14

M02: sample PDF page(s) 15; original page(s) 14

O01: sample PDF page(s) 16; original page(s) 29

O02: sample PDF page(s) 17; original page(s) 30

Selected teacher prompts and keys: original page(s) 59, 74

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and scope

Understand how the formulas for the volumes of cones, cylinders, and spheres are related and use the relationship to solve real-world and mathematical problems.

Current North Carolina Standard Course of Study, adopted June 2017. Source-verified for the 2026-2027 and 2027-2028 school years. Newly adopted standards begin in 2028-2029.

- Connect cone, cylinder, and sphere volumes through matched dimensions.
- Use radius, diameter, and perpendicular height correctly.
- Model capacity, transfer, empty space, missing dimensions, and scaling.

Tasks include experimental design and analysis of supplied measurements as well as formula-based explanations. A physical trial supports a relationship but does not alone prove a formula for every solid.

PLANNING

Use the bank across sessions

Warm-up: M01-M04 and O01-O04.

I Do: M05-M08 and O05-O08.

We Do: M09-M13 and O09-O13.

Collaborative practice: M14-M18 and O14-O18.

Independent practice: M19-M23 and O19-O23.

Small-group reteach: M24-M27 and O24-O27.

Exit ticket: M28-M30 and O28-O30.

Suggested sequence: launch and model; shared reasoning; collaborative application; independent evidence and responsive reteach. A section may span several meetings. Choose 2-4 tasks for a session, then revisit remaining tasks as needed.

Materials: pencil, eraser, ruler, squared paper, calculator for checking approximations, scissors and plain paper for teacher-supervised models. For volume investigations, use waterproof solids with matching dimensions or the supplied numerical data. No purchased software is required.

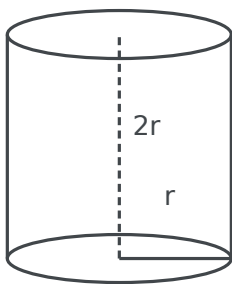
Prerequisites: squares and square roots, positive lengths, area of a square/circle, coordinate differences, and consistent units. Check these orally before scored tasks. Read prompts aloud, enlarge diagrams, and offer tactile or high-contrast grids without supplying the relation being assessed.

TEACHER MODEL

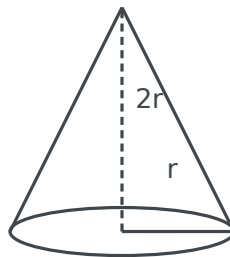
Relate the three formulas

Use a common radius r . Give both cylinder and cone perpendicular height $2r$. Draw the sphere between two horizontal planes $2r$ apart.

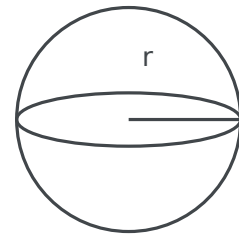
Schematic; h is perpendicular height; r is radius.



cylinder



cone



sphere

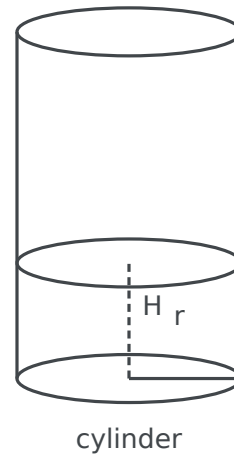
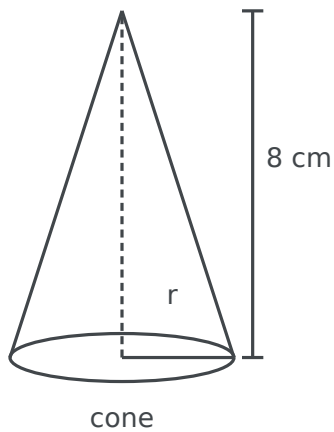
Cylinder: $\pi r^2(2r) = 2\pi r^3$. Cone: $(1/3)\pi r^2(2r) = (2/3)\pi r^3$. Sphere: $(4/3)\pi r^3$. Their volumes are in the ratio cone : sphere : cylinder = 1 : 2 : 3. The sphere is twice the cone and two-thirds of the cylinder. These comparisons depend on the dimensions; they are not true for arbitrary solids.

TEACHER MODEL

From a model to a capacity

A cone of diameter 6 cm and perpendicular height 8 cm transfers its contents into a cylinder of radius 3 cm. Ignore wall thickness and spills.

Schematic; h is perpendicular height; r is radius.



Say: "Diameter 6 means radius 3. The cone volume is $(1/3)\pi(3^2)(8) = 24\pi \text{ cm}^3$. The cylinder has base area $9\pi \text{ cm}^2$, so water height H satisfies $9\pi H = 24\pi$. Therefore $H = 8/3 \text{ cm}$." Anchor: identify shape → convert diameter to radius → use perpendicular height → apply the appropriate factor → state cubic units → check by a relationship or substitution.

REPRODUCIBLE ORGANIZER

Make reasoning visible

Name: _____ Date: _____

Givens and what I need to find**Representation and calculations****Justification or independent check****Conclusion with units and meaning**

TEACHING SEQUENCE

Warm-up

Optional bank: M01-M04; O01-O04. Core: O01. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Launch

Ask learners to read the givens and sketch the situation without calculating. Invite two explanations of what the unknown measures.

Teacher language

“What quantities must match for this comparison to be fair?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Mean cone transfer = $(60 + 61 + 59)/3 = 60$ mL. Ratio = $60/180 = 1/3$. Trials suggest the cone is one-third of the matching cylinder; the small differences are measurement variability. This is experimental evidence, not a proof for every cone.

TEACHING SEQUENCE

I Do

Optional bank: M05-M08; O05-O08. Core: O05. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Model aloud

Work through the core task, pausing before each equation. Say why the relation applies, what each symbol means, and how the conclusion answers the question.

Teacher language

“Can we derive the ratio instead of memorizing disconnected formulas?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Cylinder = $\pi r^2(2r) = 2\pi r^3$; sphere = $(4/3)\pi r^3$. Ratio $(4/3)/2 = 2/3$. Unoccupied volume is $1 - 2/3 = 1/3$ of the cylinder, equal to a cone of radius r and height $2r$.

TEACHING SEQUENCE

We Do

Optional bank: M09-M13; O09-O13. Core: O09. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Share the work

Ask partners to propose the first representation, then compare equations. Record both a valid approach and a plausible error; have learners explain the difference.

Teacher language

“Which stated length is the radius in this formula?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Radius is 3 cm. $V = \pi(3^2)(7) = 63\pi \text{ cm}^3$. Using 3.14 gives approximately 197.8 cm^3 . Diagram must distinguish radius from diameter and show perpendicular height.

TEACHING SEQUENCE

Collaborative practice

Optional bank: M14-M18; O14-O18. Core: O14. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Partner roles

One learner constructs or labels the model; the other checks the equation and units. Switch roles on the next task. Both write an individual explanation.

Teacher language

“The amount of water is conserved; which dimension changes?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

Cone volume = $(1/3)\pi(3^2)(10) = 30\pi \text{ cm}^3$. In the cylinder $\pi(3^2)H = 30\pi$, so $H = 10/3 \text{ cm}$. With the same base area, the cone’s volume fills a cylinder to one-third the cone height. Assume no loss.

TEACHING SEQUENCE

Independent practice

Optional bank: M19-M23; O19-O23. Core: O19. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Collect independent evidence

Give the core prompt without an example visible. Preserve the first response. Ask only neutral clarifications such as “What does this label represent?”

Teacher language

“What equation makes the unknown dimension visible?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

$112\pi = \pi(4^2)h$. After cancelling π , solve to obtain $h = 7$. Substitution gives $\pi(4^2)(7) = 112\pi \text{ cm}^3$. Height uses cm; volume uses cm^3 .

TEACHING SEQUENCE

Small-group reteach

Optional bank: M24-M27; O24-O27. Core: O24. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Diagnose and reteach

Use one previously observed error to select a model. Reconnect the diagram to each term, then remove the model and offer a different item in this range.

Teacher language

“How does a sphere relate to its exactly enclosing cylinder?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

$A = \pi(3^2)(8) = 72\pi$. B has $r = 6$, $h = 4$, so $V = \pi(6^2)(4) = 144\pi$. Radius doubles, multiplying base area by 4; height halves, so total multiplier is 2.

TEACHING SEQUENCE

Exit ticket

Optional bank: M28-M30; O28-O30. Core: O28. Suggested time: 15-25 minutes; extend construction and discussion as needed.

Close and decide

Ask learners to complete the core task independently and explain one check. Use the remaining exit tasks later to distinguish a one-time slip from a persistent gap.

Teacher language

“What dimensions make your proposed equality true?”

Elicit and examine

Ask: “Why does your equation match this diagram?” Look for labels linked to quantities, a justified operation, and a conclusion that addresses the original question.

Respond to a gap

If the response uses an unconnected formula, ask the learner to identify each quantity on the model. Record any formula or numerical hint you supply as a cue, then reassess with a new task.

Closure

Have a learner summarize the key relationship in words and identify one condition needed for it to apply.

Core solution and look-fors

For common r , equality $\pi r^2 H = (1/3)\pi r^2 h$ gives $h = 3H$. Example: cylinder volume $\pi(4)(5) = 20\pi$; cone volume $(1/3)\pi(4)(15) = 20\pi$. Require dimensions, labeled sketches, and equality.

MULTIPLE-CHOICE BANK

M01 - M02**M01 | Warm-up**

Target: Calculate cylinder volume from its dimensions.

A cylinder has radius 2 cm and perpendicular height 7 cm. What is its exact volume?

- A. $14\pi \text{ cm}^3$
 - B. $112\pi \text{ cm}^3$
 - C. $28\pi \text{ cm}^3$
 - D. $84\pi \text{ cm}^3$
-

M02 | Warm-up

Target: Calculate cylinder volume from its dimensions.

A cylinder has diameter 6 cm and perpendicular height 5 cm. What is its exact volume?

- A. $135\pi \text{ cm}^3$
 - B. $(45/4)\pi \text{ cm}^3$
 - C. $45\pi \text{ cm}^3$
 - D. $15\pi \text{ cm}^3$
-

WARM-UP

001 | Show your reasoning

Target: Investigate matched solid capacities.

A cone and cylinder have the same circular base and height. In three careful trials, a full cone transfers 60 mL, 61 mL, and 59 mL. The cylinder capacity is 180 mL. Organize the data, estimate the cone-to-cylinder ratio, and explain what relationship the experiment supports and why measurement error matters.

Model, construction, or calculations

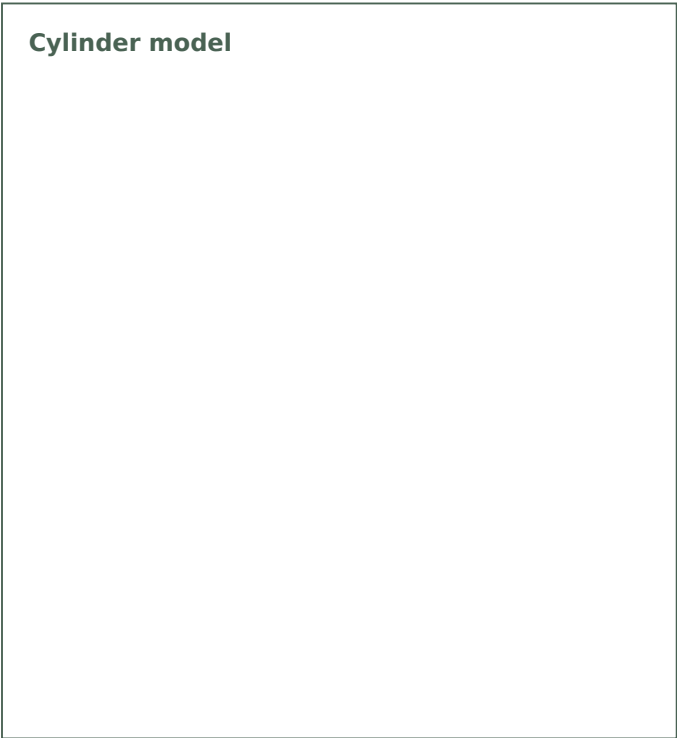
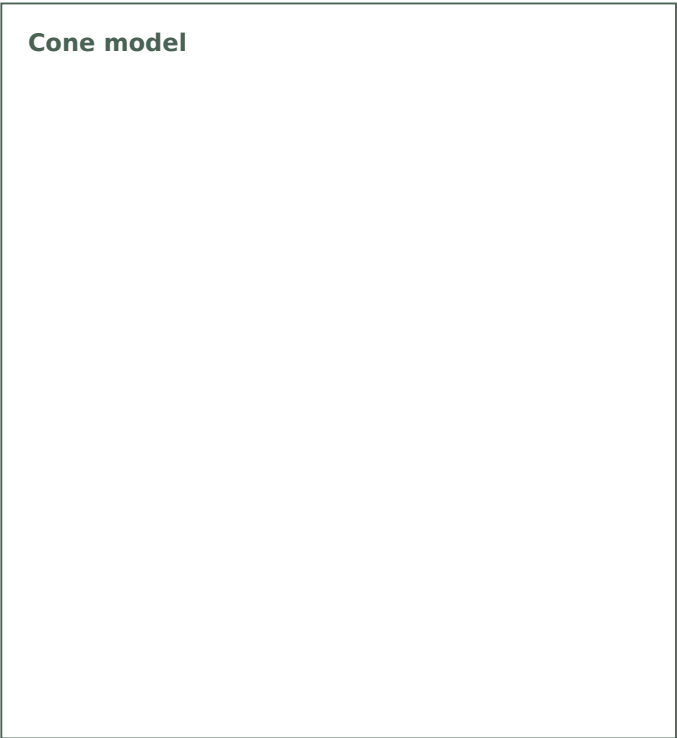
Explanation, check, and conclusion

WARM-UP

002 | Show your reasoning

Target: Connect formulas through shared dimensions.

Draw a cylinder and cone with common radius r and height h . Label the radius and perpendicular height. Use their volume formulas to determine their ratio and explain why matching BOTH dimensions matters.

Cylinder model 	Cone model 
Calculations, explanation, and check 	

WORKED KEYS

M01 - M02**M01 | Warm-up**

Target: Calculate cylinder volume from its dimensions.

Answer: C • $28\pi \text{ cm}^3$

$$r = 2 \text{ cm. } V = \pi r^2 h = \pi \times 2^2 \times 7 = 28\pi \text{ cm}^3.$$

A: This uses r rather than r^2 for the circular base.

B: This doubles the radius before squaring.

D: This triples the height before finding volume.

A selection alone does not establish a misconception; ask for the reasoning.

M02 | Warm-up

Target: Calculate cylinder volume from its dimensions.

Answer: C • $45\pi \text{ cm}^3$

$$r = 3 \text{ cm. } V = \pi r^2 h = \pi \times 3^2 \times 5 = 45\pi \text{ cm}^3.$$

A: This triples the height before finding volume.

B: This halves the radius before squaring.

D: This uses r rather than r^2 for the circular base.

A selection alone does not establish a misconception; ask for the reasoning.

WORKED KEYS

001 - 002**001 | Warm-up**

Target: Investigate matched solid capacities.

Result: About 1/3

Mean cone transfer = $(60 + 61 + 59)/3 = 60$ mL. Ratio = $60/180 = 1/3$. Trials suggest the cone is one-third of the matching cylinder; the small differences are measurement variability. This is experimental evidence, not a proof for every cone.

Assess the requested representation, reasoning, accuracy, and interpretation separately. Equivalent exact forms and valid alternative constructions are acceptable.

002 | Warm-up

Target: Connect formulas through shared dimensions.

Result: Cone : cylinder = 1 : 3

$V_c = \pi r^2 h$ and $V_k = (1/3)\pi r^2 h$. $V_k/V_c = 1/3$ when r, h are the same. A change in radius changes the base area quadratically, and a change in height changes volume linearly; without matched dimensions the ratio need not be $1/3$.

Assess the requested representation, reasoning, accuracy, and interpretation separately. Equivalent exact forms and valid alternative constructions are acceptable.