

## TEACHER AND STUDENT RESOURCE

# Multiply the matching probabilities



## NC Math 2 • NC.M2.S-CP.8

### Conditional Probability and the Rules for Probability

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

# Free sample | Contents

NC.M2.S-CP.8 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

## ALIGNMENT

## Standard and learning targets

Apply the general Multiplication Rule  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in context. Include the case where A and B are independent:  $P(A \text{ and } B) = P(A) P(B)$ .

**T1 Apply  $P(A \text{ and } B)=P(A)P(B|A)$  to paths, counts and contextual joint events.**

**T2 Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

**T3 Justify and apply the independent special case  $P(A \text{ and } B)=P(A)P(B)$ , and critique unsupported independence assumptions.**

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

## NAVIGATION

# Choose evidence deliberately

**Warm-up: M01-M04 and O01-O04**

**I Do: M05-M08 and O05-O08**

**We Do: M09-M13 and O09-O13**

**Collaborative practice: M14-M18 and O14-O18**

**Independent practice: M19-M23 and O19-O23**

**Small-group reteach: M24-M27 and O24-O27**

**Exit ticket: M28-M30 and O28-O30**

T1: M01, M03, M07, M11, M12, M14, M16, M18, M25, M27, O02, O07, O16, O17, O21, O29

T2: M05, M06, M09, M10, M15, M17, M20, M21, M23, M28, M30, O01, O05, O06, O08, O12, O15, O20, O22, O23, O25, O26, O28

T3: M02, M04, M08, M13, M19, M22, M24, M26, M29, O03, O04, O09, O10, O11, O13, O14, O18, O19, O24, O27, O30

## DAILY LEARNING TARGETS

# A multi-session teaching path

## Session 1 Identify and represent

Learning target: Apply  $P(A \text{ and } B) = P(A)P(B|A)$  to paths, counts and contextual joint events.

Learning target: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

## Session 2 Explain and connect

Learning target: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.

Learning target: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

## Session 3 Transfer independently

Learning target: Apply  $P(A \text{ and } B) = P(A)P(B|A)$  to paths, counts and contextual joint events.

Learning target: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.

Learning target: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

**PLANNING**

# Materials, access and pacing

## Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

## Prerequisites

Calculate fractions and percentages; interpret two-way tables and conditional groups. List ordered outcomes for simple random experiments. Use independence only when the model states it or evidence in the exact probability model establishes it.

## Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

## Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

## Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

## Review convention

Count-based models select individual outcomes uniformly. A standard 52-card deck contains 13 hearts. Replacement procedures include independent randomization when independence is intended. A conditional factor must be based on the group named by the marginal multiplier. Both general product forms require their respective conditioning groups to have positive probability; use the subset argument for an impossible event.

TEACHER MODEL

# A share of a share is a joint share

In 100 orders, 40 are pickup. Of pickup orders, 75% are prepaid. Another 20 delivery orders are prepaid. Find pickup and prepaid in both directions.

|          | Prepaid | Not prepaid | Total |
|----------|---------|-------------|-------|
| Pickup   | 30      | 10          | 40    |
| Delivery | 20      | 40          | 60    |
| Total    | 50      | 50          | 100   |

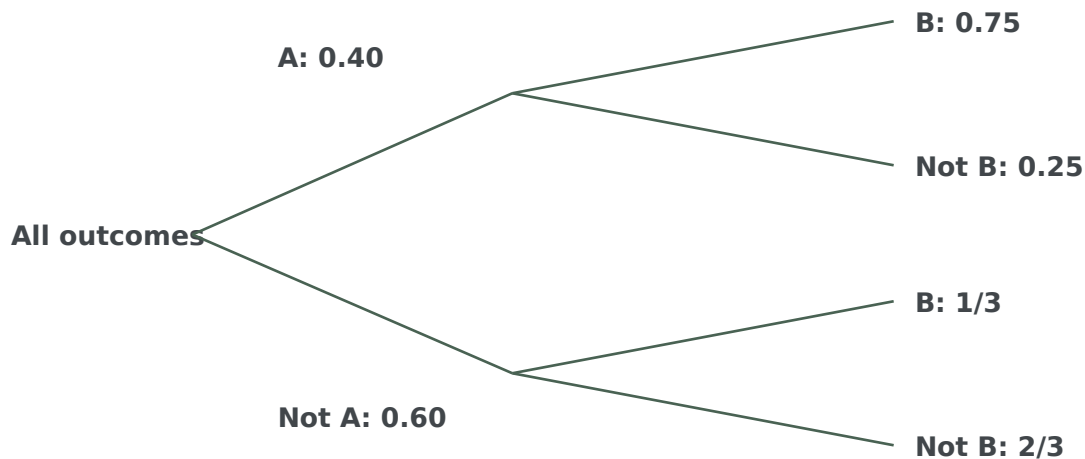
Every record is equally likely to be selected.

Pickup-first:  $0.40 \times 0.75 = 0.30$ . Prepaid-first:  $0.50 \times 0.60 = 0.30$ . The conditional fractions differ because pickup and prepaid have different reference-group sizes. Both products count the same 30 shared records among all 100. A product of the two marginals would be 0.20 and is not valid here because the events are dependent.

## VISUAL ANCHOR

## Follow a path not an isolated branch

A first branch has probability 0.40. Inside it, the B branch has conditional probability 0.75.



Second-stage probabilities are conditional on the preceding first-stage branch.

The A-and-B path represents  $0.40 \times 0.75 = 0.30$  of the whole model. The 0.75 label is not a whole-population probability; its denominator is A. The other B path contributes  $0.60 \times 1/3 = 0.20$ , so B has marginal 0.50, and the reverse factorization is  $0.50 \times 0.60 = 0.30$ .

## REUSABLE REASONING GUIDE

# Represent, justify, interpret

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Sample space, events and known information****Representation and calculation****Justification or check****Conclusion in context**

## WARM-UP

# Teaching script

**Core: 001 • T2: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Have learners name what each factor is out of before multiplying. Use a small two-stage experiment to distinguish a full path from a single branch.

## Core task

In 100 equally likely visitor records, 40 arrived by bicycle. Of those 40, 30% carried a backpack; 18 of the 60 nonbicycle visitors carried one. Construct a two-way table and calculate bicycle and backpack in both factorization directions.

## Teacher solution and discussion

The shared count is  $0.30 \times 40 = 12$ . The table has bicycle row [12, 28] and nonbicycle row [18, 42], so backpack total 30. Bicycle-first:  $(40/100)(12/40) = 0.40 \times 0.30 = 0.12$ . Backpack-first:  $(30/100)(12/30) = 0.30 \times 0.40 = 0.12$ . Twelve percent of all records have both properties.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

## Teaching script

**Core: O05 • T2: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model both directions on the same shared cell. Cancel the intermediate group count and explain why the resulting denominator is the whole population.

### Core task

An 80-record uniform model has 20 A records, 32 B records and 12 shared records. Calculate the joint probability in both directions and build a complete membership table as a check.

### Teacher solution and discussion

A-first:  $(20/80)(12/20)=(1/4)(3/5)=3/20$ . B-first:  $(32/80)(12/32)=(2/5)(3/8)=3/20$ . The four counts are both 12, A-only 8, B-only 20, neither 40. Both products equal  $12/80$ , the full-space shared fraction.

### Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

## WE DO

# Teaching script

**Core: 009 • T3: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.**

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Compare replacement, removal and single-experiment event pairs. Ask which conditional probability is needed and whether independence actually makes it equal a marginal.

## Core task

On one fair die, let  $A$  be even and  $B$  be a multiple of 3. Compute the joint by listing outcomes and by both conditional products. Decide whether the independent-marginal shortcut works, and explain why events from one experiment can still be independent.

## Teacher solution and discussion

$A = \{2, 4, 6\}$ ,  $B = \{3, 6\}$ ,  $\text{shared} = \{6\}$ , so joint  $1/6$ . A-first:  $(3/6)(1/3) = 1/6$ ; B-first:  $(2/6)(1/2) = 1/6$ . The marginal product  $(1/2)(1/3) = 1/6$  also agrees: learning either event does not change the other's chance. Independence concerns probabilities, not whether the labels come from different experiments.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

## COLLABORATIVE PRACTICE

# Teaching script

**Core: O14 • T3: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.**

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign a factor matcher, model constructor, probability-bound checker and interpreter. Require all branches or table cells, not only the favored successful path.

## Core task

Model I makes A and B independent with marginals 0.80 and 0.70. Model II has the same marginals but  $P(B|A) = 0.75$ . Compare their joint probabilities and construct a 100-outcome count model showing Model II is feasible.

## Teacher solution and discussion

Model I joint =  $0.80 \times 0.70 = 0.56$ . Model II joint =  $0.80 \times 0.75 = 0.60$ . Model II regions A-only 20, both 60, B-only 10, neither 10 preserve marginals 80 and 70 and total 100. Its greater conditional B share inside A raises the joint by 0.04.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

## INDEPENDENT PRACTICE

# Teaching script

**Core: O19 • T3: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.**

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Collect complete reasoning for changed conditions, inverse factors and decision comparisons. Separate arithmetic accuracy from the justification of independence.

## Core task

Only  $P(A)=0.40$  and  $P(B)=0.60$  are known. Construct two 50-record complete tables with different joint probabilities. Explain why 0.24 is not forced by the marginal information.

## Teacher solution and discussion

With A total 20 and B total 30, choose overlap 5: the table has B row [5, 25], not B row [15, 5], giving joint 0.10. Choose overlap 15: rows [15, 15] and [5, 15] give joint 0.30. Both preserve the marginals. The value  $0.40 \times 0.60 = 0.24$  would additionally require independence.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

## SMALL-GROUP RETEACH

# Teaching script

**Core: O24 • T3: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.**

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use labeled objects and a share-of-a-share count. Rebuild the conditional denominator after the first result; contrast a possible zero conditional with an impossible condition.

## Core task

Create one pair of events with marginals  $1/2$  and  $1/2$  whose joint is  $1/2$ , and another pair with the same marginals whose joint is  $1/4$ . Explain which pair is independent and show why the marginals alone cannot settle the issue.

## Teacher solution and discussion

On one fair coin, take  $A$ =heads and  $B$ =heads: the joint is  $1/2$  and  $P(B|A)=1$ , so not independent. For two independent fair tosses, take  $A$ =heads first and  $B$ =heads second: joint  $1/4$  and  $P(B|A)=1/2=P(B)$ . The equal marginals describe different intersections in the two models.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

## EXIT TICKET

# Teaching script

**Core: O28 • T2: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Ask for a joint probability, its reversed factorization and an independence verdict in a fresh context. Use the explanation to identify whether factor matching or model choice needs reteaching.

## Core task

A 300-order dataset has 120 pickup orders, 90 prepaid orders and 72 both. Find the shared probability using both directions and write a full contextual sentence for each factor in the two products.

## Teacher solution and discussion

Pickup-first:  $(120/300)(72/120)=0.40 \times 0.60=0.24$ . Here 40% of all orders are pickup and 60% of pickup orders are prepaid. Prepaid-first:  $(90/300)(72/90)=0.30 \times 0.80=0.24$ . Here 30% of all orders are prepaid and 80% of prepaid orders are pickup. Both mean 24% of all orders have both properties.

## Evidence and response

Look for a correct pair of factors, an explicit reference group for each, a joint-probability interpretation and a justified independence decision. Require complete teacher and student representations for construction tasks. A numerical product alone is not evidence that independence was valid.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

## MULTIPLE-CHOICE BANK

**M01 / M02 • Select and justify****M01 | Warm-up | T1** **$P(A)=0.40$  and  $P(B|A)=0.30$ . Find  $P(A \text{ and } B)$ .**

- A. 0.70
- B. 0.12
- C. 0.75
- D. 0.30

Reasoning: \_\_\_\_\_

**M02 | Warm-up | T3** **$A$  and  $B$  are independent, with probabilities 0.60 and 0.50. What is their joint probability?**

- A. 1.10
- B. 0.50
- C. 0.10
- D. 0.30

Reasoning: \_\_\_\_\_

## WARM-UP

**001 • Show your reasoning**

**T2: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

**In 100 equally likely visitor records, 40 arrived by bicycle. Of those 40, 30% carried a backpack; 18 of the 60 nonbicycle visitors carried one. Construct a two-way table and calculate bicycle and backpack in both factorization directions.**

**Representation / calculation**

**Explanation / check**

## WARM-UP

**002 • Show your reasoning**

**T1: Apply  $P(A \text{ and } B) = P(A)P(B|A)$  to paths, counts and contextual joint events.**

**A bag has 2 red and 3 blue labeled tokens. Two are drawn uniformly without replacement. Construct a complete two-stage color tree and find the probability of blue then red. Verify it by counting favorable ordered labeled draws out of all possible draws.**

**Representation / calculation**

**Explanation / check**

## WORKED TEACHER KEYS

**M01****M01 • T1: Apply  $P(A \text{ and } B) = P(A)P(B|A)$  to paths, counts and contextual joint events.****Answer: B. 0.12**

The general Multiplication Rule gives  $0.40 \times 0.30 = 0.12$ . First select the A share, then the B share within A.

**Check each alternative**

- A. Adding the probabilities does not give a joint event.
- B. The general Multiplication Rule gives  $0.40 \times 0.30 = 0.12$ . First select the A share, then the B share within A.
- C. 0.75 divides 0.30 by 0.40 rather than multiplying.
- D. 0.30 describes B among A, not both among all outcomes.

**Evidence to assess**

Require the marginal paired with its matching conditional.

## WORKED TEACHER KEYS

**M02**

**M02 • T3: Justify and apply the independent special case  $P(A \text{ and } B) = P(A)P(B)$ , and critique unsupported independence assumptions.**

**Answer: D. 0.30**

Independence means  $P(B|A) = P(B) = 0.50$ . Thus the joint probability is  $0.60 \times 0.50 = 0.30$ .

**Check each alternative**

- A. 1.10 is the marginal sum and exceeds 1.
- B. 0.50 is B's marginal, not the probability of both.
- C. 0.10 is the difference, not the joint probability.
- D. Independence means  $P(B|A) = P(B) = 0.50$ . Thus the joint probability is  $0.60 \times 0.50 = 0.30$ .

**Evidence to assess**

Require using the stated independence assumption.

WORKED TEACHER KEYS

001

**001 • T2: Use both product directions, recover missing factors and interpret each marginal, conditional and joint probability.**

The shared count is  $0.30 \times 40 = 12$ . The table has bicycle row [12, 28] and nonbicycle row [18, 42], so backpack total 30. Bicycle-first:  $(40/100)(12/40) = 0.40 \times 0.30 = 0.12$ . Backpack-first:  $(30/100)(12/30) = 0.30 \times 0.40 = 0.12$ . Twelve percent of all records have both properties.

|             | Backpack | No backpack | Total |
|-------------|----------|-------------|-------|
| Bicycle     | 12       | 28          | 40    |
| Not bicycle | 18       | 42          | 60    |
| Total       | 30       | 70          | 100   |

Every record is equally likely to be selected.

**Evidence to assess**

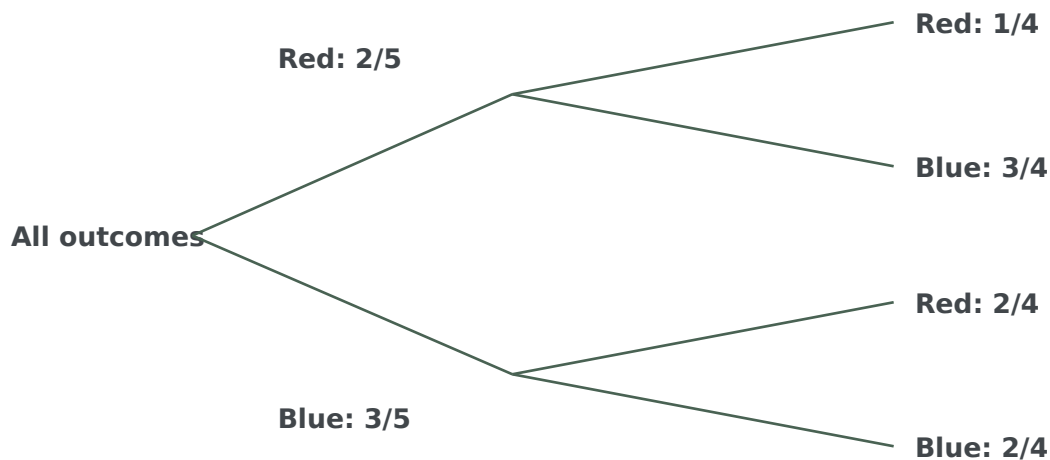
Item evidence: complete table; two matching products; joint 0.12; full-group interpretation. Award one point per named requirement, up to 4; accept equivalent exact fractions and valid alternative models.

## WORKED TEACHER KEYS

## 002

**002 • T1: Apply  $P(A \text{ and } B) = P(A)P(B|A)$  to paths, counts and contextual joint events.**

The first branches are red  $\frac{2}{5}$  and blue  $\frac{3}{5}$ . After red, remaining color fractions are red  $\frac{1}{4}$ , blue  $\frac{3}{4}$ ; after blue, red  $\frac{2}{4}$ , blue  $\frac{2}{4}$ . Blue then red  $= (\frac{3}{5})(\frac{2}{4}) = \frac{3}{10}$ . There are  $5 \times 4 = 20$  equally likely ordered labeled draws and  $3 \times 2 = 6$  favorable, so  $\frac{6}{20} = \frac{3}{10}$ .



Second-stage probabilities are conditional on the preceding first-stage branch.

**Evidence to assess**

Item evidence: complete conditional tree; product  $\frac{3}{10}$ ; 6 favorable of 20; agreement explanation. Award one point per named requirement, up to 4; accept equivalent exact fractions and valid alternative models.